Homework #5 — PHYS 625 — Spring 2021

Deadline: Monday, April 17 2021, by email to masoudma@umd.edu before class

Professor Victor Galitski
Office: 2270 PSC

TA: Masoud Arzanagh
masoudma@umd.edu

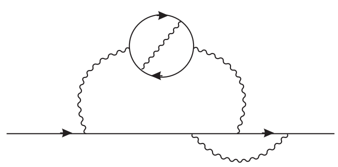
Web page: https://terpconnect.umd.edu/ galitski/PHYS625/index.html

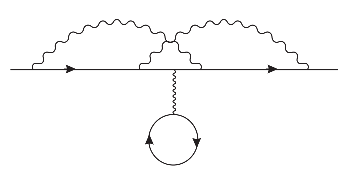
Do not forget to write your name and the homework number!

Feynman diagrams

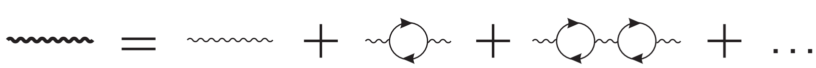
1. Draw all possible topologically non-equivalent diagrams (connected and disconnected) in second order perturbation theory with respect to a two-particle interaction $V(\mathbf{r}_1 - \mathbf{r}_2)$.

2. Write down the analytical expressions (in momentum representation) corresponding to the following diagrams:





3. The screening of Coulomb interaction in an electron gas is described by the following diagram series,



where the thin wavy line is the Coulomb interaction and the bubble corresponds to the polarization operator

$$\Pi(\omega, \mathbf{q}) = 2i \int G\left(\varepsilon + \frac{\omega}{2}, \mathbf{p} + \frac{\mathbf{q}}{2}\right) G\left(\varepsilon - \frac{\omega}{2}, \mathbf{p} - \frac{\mathbf{q}}{2}\right) \frac{d^d\mathbf{p}}{(2\pi)^d} \frac{d\varepsilon}{2\pi},$$

where $G(\varepsilon, \mathbf{p})$ is the free electron Green's function and d is the dimensionality of the system.

The dynamically screened Coulomb interaction is described by the following formula

$$V(\omega, \mathbf{q}) = \frac{v(\mathbf{q})}{1 - v(\mathbf{q})\Pi(\omega, \mathbf{q})},$$

where $v(\mathbf{q})$ is the bare Coulomb potential.

Consider a *two-dimensional* ($d = 2$) electron gas and...

- **(a)** Calculate the bare (unscreened) Coulomb interaction in momentum space, by Fourier-transforming the potential $v(r) = e^2/r$.
- **(b)** Calculate the polarization operator at $\omega = 0$ and $\mathbf{q} = \mathbf{0}$, and find the analog of the Debye screening length in two dimensions (write the screened Coulomb potential at $q = 0$ and $\omega = 0$).
- **(c)** Calculate the screened Coulomb potential in real space.
- **(d)** Calculate the polarization operator at small but finite frequencies and momenta $\omega \ll E_{\rm F}$ and $q \ll p_{\rm F}$. Find the spectrum of collective modes (two-dimensional plasmons). This spectrum is the solution of the equation $v(\mathbf{q})\Pi\left[\omega(\mathbf{q}), \mathbf{q}\right] = 1$. What is the main difference between the two-dimensional and three-dimensional plasmons?

Reading: Abrikosov, Gor'kov, and Dzyaloshinskii and Lectures