Feynman diagrams

1. Draw all possible topologically non-equivalent diagrams (connected and disconnected) in second order perturbation theory with respect to a two-particle interaction $V(r_1 - r_2)$.

2. Write down the analytical expressions (in momentum representation) corresponding to the following diagrams:

3. The screening of Coulomb interaction in an electron gas is described by the following diagram series,

$$\quad = \quad + \quad + \quad + \ldots$$

where the dashed line is the Coulomb interaction and the bubble corresponds to the polarization operator

$$\Pi(\omega, q) = 2i \int \frac{d^d p \, d\varepsilon}{(2\pi)^d} \frac{d^d p \, d\varepsilon}{(2\pi)^d} G(\varepsilon + \frac{\omega}{2}, p + \frac{q}{2}) G(\varepsilon - \frac{\omega}{2}, p - \frac{q}{2})$$

where $G(\varepsilon, p)$ is the free electron Green’s function and $d$ is the dimensionality of the system.

The dynamically screened Coulomb interaction is described by the following formula

$$V(\omega, q) = \frac{v(q)}{1 - v(q)\Pi(\omega, q)}$$

where $v(q)$ is the bare Coulomb potential.

Consider a two-dimensional ($d = 2$) electron gas and...
(a) Calculate the bare (unscreened) Coulomb interaction in momentum space, by Fourier-transforming the potential $v(r) = e^2/r$.

(b) Calculate the polarization operator at $\omega = 0$ and $\mathbf{q} = \mathbf{0}$, and find the analog of the Debye screening length in two dimensions (write the screened Coulomb potential at $q = 0$ and $\omega = 0$).

(c) Calculate the screened Coulomb potential in real space.

(d) Calculate the polarization operator at small but finite frequencies and momenta $\omega \ll E_F$ and $q \ll p_F$. Find the spectrum of collective modes (two-dimensional plasmons). This spectrum is the solution of the equation $v(q)\Pi[\omega(q), q] = 1$. What is the main difference between the two-dimensional and three-dimensional plasmons?

Reading: Abrikosov, Gor’kov, and Dzyaloshinskii, Mahan, and Lectures

Due Wednesday, May 1 (in class)