Homework #3 — PHYS 625 — Spring 2017
Deadline: Wednesday, April 12, 2017, in class

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Office: PSC 2270 and Toll Physics Bld. 2330

Relevant textbook: Abrikosov, A.A., Gor'kov, L.P., and Dzyaloshinskii, I.Ye., *Methods of Quantum Field Theory in Statistical Physicss*, Dover Publications Inc, New York, ISBN-10: 0486632288

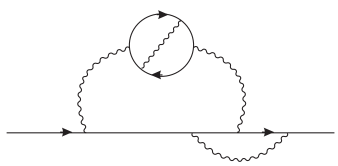
Web page: `http://terpconnect.umd.edu/~galitski/PHYS625/`

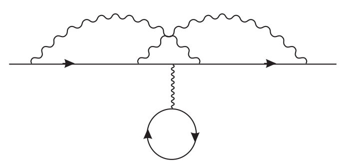
Do not forget to write your name, the homework number and staple your pages together!

Feynman diagrams; scrreening; phonons

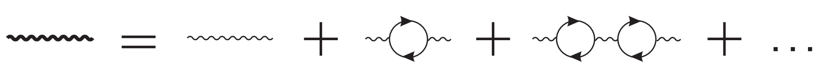
1. Draw all possible topologically non-equivalent diagrams (connected and disconnected) in second order perturbation theory with respect to a two-particle interaction $V(\mathbf{r}_1 - \mathbf{r}_2)$.

2. Write down the analytical expressions (in momentum representation) corresponding to the following diagrams:





3. The screening of Coulomb interaction in an electron gas is described by the following diagram series,



where the wavy line is the Coulomb interaction and the bubble corresponds to the polarization operator

$$\Pi(\omega, \mathbf{q}) = 2i \int G\left(\varepsilon + \frac{\omega}{2}, \mathbf{p} + \frac{\mathbf{q}}{2}\right) G\left(\varepsilon - \frac{\omega}{2}, \mathbf{p} - \frac{\mathbf{q}}{2}\right) \frac{d^d\mathbf{p}}{(2\pi)^d} \frac{d\varepsilon}{2\pi},$$

where $G(\varepsilon, \mathbf{p})$ is the free electron Green's function and d is the dimensionality of the system.

The dynamically screened Coulomb interaction is described by the following formula

$$V(\omega, \mathbf{q}) = \frac{v(\mathbf{q})}{1 - v(\mathbf{q})\Pi(\omega, \mathbf{q})},$$

where $v(\mathbf{q})$ is the bare Coulomb potential.

Consider a *two-dimensional* ($d = 2$) electron gas and...

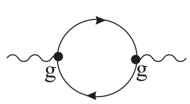
(a) Calculate the bare (unscreened) Coulomb interaction in momentum space, by Fourier-transforming the potential $v(r) = e^2/r$.

(b) Calculate the polarization operator at $\omega = 0$ and $\mathbf{q} = \mathbf{0}$, and find the analog of the Debye screening length in two dimensions (write the screened Coulomb potential at $q = 0$ and $\omega = 0$).

(c) Calculate the screened Coulomb potential in real space at large separation ($r \to \infty$). Use the static screening limit ($\omega = 0$). Note: the asymptotic analysis of the integrals may be somewhat cumbersome, if you are unable to extract the asymptote, just leave a formal exact result. However, those of you who are theorists should spend a little more time on the problem and try to get the right asymptote.

(d) Calculate the polarization operator at small but finite frequencies and momenta $\omega \ll E_{\mathrm{F}}$ and $q \ll p_{\mathrm{F}}$. Find the spectrum of collective modes (two-dimensional plasmons). This spectrum is the solution of the equation $v(\mathbf{q})\Pi\left[\omega(\mathbf{q}), \mathbf{q}\right] = 1$. What is the main difference between the two-dimensional and three-dimensional plasmons?

4. The leading-order correction to the phonon propagator due to electron-phonon coupling in a metal is given by the diagram,



$$\Longrightarrow \quad D^{-1}(\omega,\mathbf{k}) = D_0^{-1}(\omega,\mathbf{k}) + g^2\,\Pi(\omega,\mathbf{k})$$

where the wavy line corresponds to the phonon propagator (use the Debye model), the expression for the bubble was calculated in class (for a three-dimensional electron gas), and each vertex corresponds to the electron-phonon coupling constant g.

Find the leading order correction to the speed of sound in three and two dimensions. Assume that the bare speed of sound is much smaller than the Fermi velocity in a metal, $c \ll v_{\mathrm{F}}$.

Reading: Abrikosov, Gor'kov, and Dzyaloshinskii, Mahan, and Lectures

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