BCHM485 Graded HW#1 (25 points) Due date/time: Feb 27, 2026 by noon (11:59 am)

Unless specified otherwise, you don't need to show all your calculations – simple answers written in the allocated spaces will be sufficient, unless you want me to follow your math. For problems 1-2, if your answers contain $h$ or $\hbar$ or other constants, keep them as letters, no need to substitute them with their numeric values.

**<u>Problem 1</u> (10 points).**

A quantum mechanical particle of mass $m$ is in a state described by the following wave function: $\Psi = \left(7e^{-i6x} + 6ie^{i3x} - 5ie^{i6x}\right) / \sqrt{2L}$ Assume it's a free particle defined in the interval $(-L, L)$, where $L \to \infty$. Using the postulates of QM, answer the following questions:

(**A**) List all values of the kinetic energy of the particle that can be obtained in a single measurement.

(**B**) List all values of the linear momentum $p$ that can be obtained in a single measurement and their probabilities.

(**C**) Calculate the average value of the linear momentum $p$ that you expect to obtain as a result of a very large number of measurements.

(**D**) Is the particle more likely to be found moving in the positive or negative direction of the x axis? *Explain your reasoning*.

**Problem 2 (6 points).** A particle of mass $m$ in the 1D box ($0 \le x \le a$) is in the state corresponding to the quantum number $n$=6. Answer the following questions:

(**A**) Write the complete expression for the wave function $\Psi(x,t)$ describing the particle. Make sure the wave function is normalized and *explicitly* includes the time dependence.

(**B**) In which points inside the box you are unlikely to find the particle? List their coordinates.

(**C**) Consider the following two intervals inside the box: (1) $a/6 \le x \le a/3$ and (2) $a/12 \le x \le a/4$. Calculate and compare the probabilities to find the particle within each interval: which one is more likely? *Explain your reasoning.*

**Problem 3 (3 points).** A particle in a 1D box ($0 \le x \le a$) is in a state described by the wave function $\Psi = A\,(ax - x^2)$. This is an acceptable wave function for a particle in a 1D box, see, for example, Problem 4.16 in the textbook. Is the particle in a stationary state? *Explain your reasoning and support it with calculations.*

**Problem 4 (6 points).** As a simplified model of the hydrogen atom, consider an electron in a 1D box with the size of 1 Å (=$10^{-10}$ m) (approximately the diameter of the lowest orbit in Bohr's model).

(**A**) Calculate the energy difference, $\Delta E$, between the ground state and the first excited state of the electron. *Show your calculation.*

(**B**) Now consider the same box as a model for the nucleus of the hydrogen atom. Perform the same calculation as in question A, now for the proton.

(**C**) Compare your results in A and B with the average kinetic energy (½ $k_B T$ per degree of freedom) at room temperature (assume $T = 300$ K) to answer the question whether classical or quantum mechanics should be used to describe each of these particles in this model.