

Some equations and formulae that might or might not be useful:

$$\Delta E = h\nu; \quad \nu = \frac{c}{\lambda}; \quad \lambda = \frac{h}{p}$$

$$\hat{p}_x = -i\hbar \frac{\partial}{\partial x}; \quad E_{kinet} = K = \frac{p^2}{2m}$$

$$E = \frac{\hbar^2 k^2}{2m}; \quad \Psi_+(x) = Ae^{ikx}; \quad \Psi_-(x) = Ae^{-ikx}$$

$$E_n = \frac{\hbar^2 n^2}{8ma^2}; \quad \Psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi n}{a}x\right)$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\nu; \quad \Psi_n(x) = A_n H_n(\sqrt{\alpha}x) e^{-\alpha x^2/2}$$

$$\nu = \frac{1}{2\pi} \sqrt{\frac{k}{\mu}}; \quad A_n = \frac{1}{\sqrt{2^n n!}} \left(\frac{\alpha}{\pi}\right)^{1/4}$$

$$\alpha = \sqrt{k\mu}/\hbar; \quad \mu = m_1 m_2 / (m_1 + m_2)$$

$$E_{m_l} = \frac{\hbar^2}{2I} m_l^2; \quad \Psi_{m_l}(\phi) = \frac{1}{\sqrt{2\pi}} e^{im_l \phi}$$

$$E_l = \frac{l(l+1)\hbar^2}{2I}; \quad \Psi_{l,m_l} = Y_{l,m_l}(\theta, \phi);$$

$$|\vec{l}|^2 = l(l+1)\hbar^2; \quad l_z = m_l \hbar$$

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}; \quad \Delta A \times \Delta B \geq \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle \right|$$

$$h = 6.626 \times 10^{-34} \text{ J s}; \quad \hbar = h/(2\pi)$$

$$c = 3 \times 10^8 \text{ m/s}; \quad k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

$$\Delta p_x \cdot \Delta x \geq \frac{\hbar}{2}$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}; \quad 1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$$

$$H_0(\sqrt{\alpha}x) = 1; \quad H_1(\sqrt{\alpha}x) = 2\sqrt{\alpha}x$$

$$H_2(\sqrt{\alpha}x) = 2(2\alpha x^2 - 1)$$

$$H_3(\sqrt{\alpha}x) = 4\sqrt{\alpha}x(2\alpha x^2 - 3)$$

$$\hat{l}_z = -i\hbar \frac{\partial}{\partial \phi}; \quad l_z = m_l \hbar; \quad I = \mu r^2$$

$$Y_{0,0}(\theta, \phi) = 1/\sqrt{4\pi}; \quad Y_{1,0}(\theta, \phi) = (3/4\pi)^{1/2} \cos \theta$$

$$Y_{1,\pm 1}(\theta, \phi) = \mp (3/8\pi)^{1/2} \sin \theta e^{\pm i\phi}$$

$$Y_{2,0}(\theta, \phi) = (5/16\pi)^{1/2} (3\cos^2 \theta - 1)$$

$$Y_{2,\pm 1}(\theta, \phi) = \mp (15/8\pi)^{1/2} \cos \theta \sin \theta e^{\pm i\phi}$$

$$Y_{2,\pm 2}(\theta, \phi) = (15/32\pi)^{1/2} \sin^2 \theta e^{\pm 2i\phi}$$

Helpful Math:

$$e^{i\alpha} = \cos \alpha + i \sin \alpha;$$

$$\cos \alpha = (e^{i\alpha} + e^{-i\alpha})/2; \quad \sin \alpha = (e^{i\alpha} - e^{-i\alpha})/2i$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha; \quad \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha;$$

$$2 \sin^2 \alpha = 1 - \cos 2\alpha; \quad 2 \cos^2 \alpha = 1 + \cos 2\alpha$$

$$\int_0^\infty e^{-ax^2} x^{2n} dx = \sqrt{\frac{\pi}{a}} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n} \quad (\text{for } n \geq 1);$$

$$\int_0^\infty e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}; \quad \int_0^\infty e^{-ax^2} x^2 dx = \frac{1}{4a} \sqrt{\frac{\pi}{a}}$$

$$\int_0^a \sin^2\left(\frac{\pi n}{a}x\right) dx = \frac{a}{2}; \quad \int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$n! = 1 \cdot 2 \cdots n$$