What is Statistics?

Elie Gurarie

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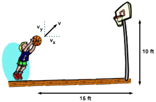
Statistics is not physics ...

In physics, we take information and make predictions:



With speed V and angle θ and distance d and height h, we can say "exactly" where the ball will go.
But that's an easy problem!

The world is incredibly complex...

- There are very very few things in the world that we can *describe* with certainty,
- There are even fewer things that we can *predict* with certainty.

Real world problems ...

- How do we describe the weather today?
- How do we describe the climate tomorrow / next year?
- How many polar bears are there today?
- How many will there be next year (given how the climate might change next year)?
- Does drug X cure disease Y?
- How will a government policy (e.g. regarding drug X) affect society / the economy?
- What side will a flipped coin land on?

What about a coin flip?

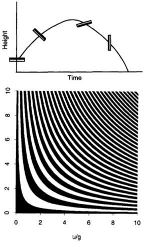
what do you need to know to predict how a coin will land?

What about a coin flip?

what do you need to know to predict how a coin will land?

- angle
- strength
- speed
- fluctuations in air movement
- notches/kinks on the coin
- landing surface
- more ...?

What about a coin flip? [1]



[1] Stochastic Modeling of Scientific Data, P. Guttorp and V. N. Minin

Is it feasible to “predict” exactly?

No.

Is it feasible to “predict” probabilistically?

Easy!

50% chance Heads
50% chance Tails

This “random” result weirdly tells us just about everything we need to know about the very complex problem of the coin-flip.

Is it feasible to “predict” probabilistically?

Easy!

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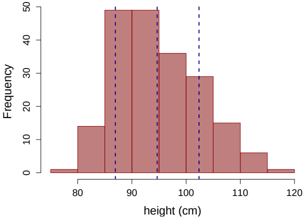
Probability Theory is the science of understanding mathematical “randomness”.

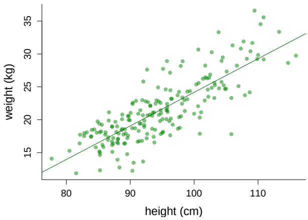
Complex or Random?

Pupils at Anshu Academy Nepal

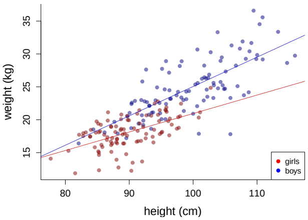
Complex or Random?



Complex or Random?



Complex or Random?



Possible definition of statistics

Statistics is the art of sweeping **complexity** under the rug of **randomness**
(not because we're lazy, but because its not practical)
((ok, sometimes it's because we're lazy)).

Definitions

- The **research question**.
- A **population** is the collection of all possible individuals of interest.
- A **sample** is a subset of the population.
- An **individual** (or **experimental unit**) is an object on which observations are made.
- A **variable** is any quantity that can be measured on an individual.
  - A **dependent** or **response** variable the focus of our research question, or very closely linked to it
  - An **independent** variable or **covariate** or **explanatory factor** is an additional factor we consider for explaining the **response**

Example: Assessing health/growth of children

A question: How big are 2nd graders at Anshu County School in Nepal?

- **Population**: The 2nd graders in Anshu County School.
- **Sample**: The 2nd graders in Anshu County School.
- **Individual**: A 2nd grader in Anshu County School.
- **Variable**: Height, weight,
- **Covariate**: Sex, age, etc...

Definition

Descriptive statistics are used to describe a population that had been completely sampled.

Sample = **Population**

Example of inference

Question: What factors impact health of children in "general"?

- **Population**: All pupils in class? school? town? region? country? world?
- **Sample**: The pupils in the class.
- **Individual**: A child
- **Variable**: Height, weight,
- **Covariate**: Age, income level, diet, size of family, ...

This is a complex question! How do we best collect data? What's the most we can learn from these data? How do we tease apart confounding effects in the variables? How do we even define some of these variables?

Definition

Inference is when you use knowledge gained about a *sample* to extrapolate (infer) something about a larger *population*. Statistics that allow us to infer about greater populations are referred to as **inference statistics**.

Sample < **Population**

Broad outline of what we'll learn in this course

1. **Descriptive statistics**: visualizations and statistical summaries
2. **Probability theory**: random processes and distributions
3. **Inferential analysis**: hypothesis testing, modeling, prediction

A statistical model:

$$Y = f(X|\theta)$$

- Y - response variable(s)
- X - explanatory variable(s)
- θ - parameter(s)
- $f(\cdot)$ - a (probabilistic) function

Inference

Specifying a model:

- What should $f(\cdot)$ be?

Parameterizing / model-fitting:

- Given Y_i and X_i (data), what is our best guess for θ? (called $\widehat{\theta}$)

Model selection:

- Which of available X do we need?
- Or, which of several possible $f(\cdot)$ is best

For each these tasks, there are different more or less rigorous procedures - but in ALL cases - good judgment and reasoning are the MOST important step!

Prediction

Predictive statistics (or *machine learning*) has a lot of overlap / equivalence with inference, but differs in its goals.

- You DO NOT care about the model $f(\cdot)$
- you DO NOT care about $\widehat{\theta}$

You ONLY care about the **prediction**:

$$\widehat{Y_j} = E(f(X_j, ...))$$

where $f(\cdot)$ was obtained using a *training* subset X_i, Y_i and is predicted onto a *validation* subset X_j, Y_j. More specifically - you ONLY care about minimizing:

$$Y_j - \widehat{Y_j}$$

Using whatever algorithms you can (parameteric / non-parametric / supervised / unsupervised).
In practice (varieties of) REGRESSION are most commonly used.