The Zakharov Eqns and strong Langmuir Turbulence

Consider a system in which strong electron plasma waves (Langmuir waves) have been produced by some mechanism. What is the nonlinear behavior of the waves?

The ponderomotive force due to the high frequency waves can drive sound waves which in turn trap the Langmuir waves to produce nonlinear structures called cavitons

$\Longrightarrow$ these collapse to smaller and smaller spatial scales.

Represent perturbed quantities by high and low field components

e.g. $n_e = n_0 + \tilde{n}_e^h + \tilde{n}_e^L$

$n_i \cong n_0 + \tilde{n}_i^L$

High Freq. Eqns

$\Longrightarrow$ neglect high freq. convective nonlinearities studied earlier in large amplitude plasma wave

$\Longrightarrow$ consider Langmuir wave in a modulated density profile

As in nonlinear plasma wave but keep pressure to obtain dispersion

$$\frac{\partial}{\partial x}E = -4\pi e(n-n_i) \qquad \frac{\partial}{\partial t}n + \frac{\partial}{\partial x}nv = 0$$

$$\frac{\partial}{\partial x}\dot{E} = -4\pi e(\dot{n}) = +4\pi e\frac{\partial}{\partial x}nv$$

$$\dot{E} = 4\pi e n v$$

momentum

$$m\frac{dv}{dt} = -eE - \frac{1}{n_0}\frac{\partial}{\partial x}P_e$$

pressure

$$\frac{P_e}{n^\Gamma} = \text{const} \Longrightarrow \Gamma = 3 \text{ for adiabatic in 1-D system}$$

$$m\frac{d^2v}{dt^2} = -e\left(\cancel{4\pi e n v} + v(-4\pi e)(\cancel{n}-n_i)\right)$$

$$-\frac{1}{n_0}\frac{\partial}{\partial x}\dot{P}_e$$

$$P_e\frac{\Gamma \cancel{n^\Gamma}}{n\,\cancel{n^\Gamma}}\dot{n}$$

$$\frac{P_e\Gamma}{\cancel{n_e}}\left(-n_e\frac{\partial v}{\partial x}\right)$$

$$\frac{\partial^2}{\partial t^2}v = -\frac{4\pi e^2}{m}v(n_0+\tilde{n}^2) + \cancel{P_e}\,\frac{T_e\Gamma}{m}\frac{\partial^2}{\partial x^2}v$$

Let $V = V_0 e^{-i\omega_{pe0} t} + cc$

$$V_0 = V_0(x,t)$$

$\Rightarrow$ ~~time variation of V_0 weak~~

$$\left(\cancel{-\omega_{pe}^2} + 2(-i\omega_{pe0})\frac{\partial}{\partial t}\right)V_0 - \frac{3T_e}{m}\frac{\partial^2}{\partial x^2}V_0$$

$$= -\omega_{pe0}^2 V_0 \frac{\tilde{n}^L}{n_0}$$

$$\boxed{+2i\omega_{pe0}\frac{\partial}{\partial t}V_0 + \frac{3T_e}{m}\frac{\partial^2}{\partial x^2}V_0 = +\omega_{pe0}^2 V_0 \frac{\tilde{n}^L}{n_0}}$$

Low frequency Eqns

$$0 = -eE^L - \frac{1}{n_0}\frac{\partial}{\partial x}P_e^L + F_p$$

$$\Rightarrow E^L = -\frac{1}{\cancel{n_0}e}\Gamma \cancel{n_0} T_e \frac{\partial}{\partial x} n_e^L + \frac{1}{e}F_p$$

$$m_i \frac{dv_i}{dt} = -\Gamma T_e \frac{\partial}{\partial x} n_e^L + F_p$$

$$n_e^L \cong n_i^L \qquad \frac{\Gamma T_e}{m_i} = c_s^2$$

$$\frac{\partial}{\partial t} n_i^L = -n_0 \frac{\partial}{\partial x} v_i$$

$$\ddot{n}^L - c_s^2 \frac{\partial^2}{\partial x^2} n^L = -\frac{n_0}{m_i}\frac{\partial}{\partial x}F_p$$

~~Pondermotive~~ Force

Want to find the low frequency response of electrons to high frequency waves for the case of electro static waves. As in the case of EM waves find that the electrons are expelled from the regions of large amplitude by an effective force.

① $$\frac{\partial}{\partial t} V + V \frac{\partial}{\partial x} V = -\frac{eE}{m}$$

E is a high frequency wave which can be represented as

$$E = E_0(x,t)\, e^{-i\omega_0 t} + cc$$

and V as $\quad V = V_0 e^{-i\omega_0 t} + V_L(x,t)$

The high frequency part of the equation to lowest order is given by

$$-i\omega_0 V_0 = -\frac{e}{m} E_0$$

$$V_0 = -i \frac{e}{m\omega_0} E_0(x,t)$$

$\Longrightarrow$ time variation of E_0 is over a longer time scale than ω_0^{-1}.

The low frequency component of ① is given by

$$\frac{\partial}{\partial t} V_L = -\frac{\partial}{\partial x}\left(\frac{V^2}{2}\right)_L$$

where $(\;)_L$ means only the low frequency component is retained. Thus

$$\frac{\partial}{\partial t} v_L = - \frac{\partial}{\partial x} \frac{1}{2}\left[\left(-\frac{ie}{m\omega_0} E_0 e^{-i\omega_0 t} + cc\right)^2\right]_L$$

$$m \frac{\partial v_L}{\partial t} = - \frac{e^2}{m\omega_0^2} \frac{\partial}{\partial x} |E_0|^2$$

$$\boxed{F_p = - \frac{e^2}{m_e\omega_0^2} \frac{\partial}{\partial x} |E_0|^2}$$

$\Longrightarrow$ ponderomotive force.

$$\ddot{n}^L - c_s^2 \frac{\partial^2}{\partial x^2} n^L = + \frac{n_0}{m_i} \frac{e^2}{m_e\omega_0^2} \frac{\partial^2}{\partial x^2} |E_0|^2$$

(1) $$\boxed{\ddot{n}^L - c_s^2 \frac{\partial^2}{\partial x^2} n^L = \frac{n_0 m_e}{m_i} \frac{\partial^2}{\partial x^2} |V_0|^2}$$

(2) $$\boxed{2i\omega_{peo} \frac{\partial}{\partial t} V_0 + \frac{3T_e}{m} \frac{\partial^2}{\partial x^2} V_0 = \omega_{peo}^2 V_0 \frac{\tilde{n}^L}{n_0}}$$

Zakharov Eqns

Modulational Instability

Consider perturbation of a long wavelength pump

$$V_0 = V_{00} e^{ik_0 x} e^{-i\omega_{pe} t} + V_{00}^* e^{-ik_0 x} e^{i\omega_{pe} t}$$

$\Rightarrow$ local increase in V_0 causes density reduction $\Rightarrow$ traps waves $\Rightarrow$ unstable

Coupling: low frequency $\tilde{n}^L = \tilde{n}_0^L e^{ikx} e^{-i\omega t}$

$$\tilde{n}^L \overset{\text{pump}}{\Longrightarrow} \tilde{V}^+ e^{i(k+k_0)x} e^{-i(\omega+\omega_{pe})t}$$

$$\tilde{V}^- e^{i(k-k_0)x} e^{-i(\omega-\omega_{pe})t}$$

$$\tilde{V}^\pm \overset{\text{pump}}{\Longrightarrow} \tilde{n}^L$$

Momentum eqn: driven by $V_0 \tilde{V}^L$ (Eq. 2)

Eq 2 for $\tilde{V}^+ \left(\sim e^{-i(\omega+\omega_{pe})t}\right)$

$$2i\omega_{pe}(-i\omega)\tilde{V}^+ - \frac{3T_e}{m_e}(k+k_0)^2 \tilde{V}^+ = \omega_{pe}^2 V_{00} \frac{\tilde{n}_0}{n_0}$$

cc of 2 for $\tilde{V}^- \left(\sim e^{-i(\omega-\omega_{pe})t}\right)$

$$-2i\omega_{pe}(-i\omega)\tilde{V}^- - \frac{3T_e}{m_e}(k-k_0)^2 \tilde{V}^- = \omega_{pe}^2 V_{00}^* \frac{\tilde{n}_0^L}{n_0}$$

from 1

$$-\omega^2 \tilde{n}_0^L + C_s^2 k^2 \tilde{n}_0^L = \frac{n_0 m_e}{m_i}(-k^2)$$

$$\otimes \Big[\underbrace{\tilde{v}^+ V_{00}^* + \tilde{v}^- V_{00}}_{\text{must be low frequency.}} \Big]$$

$$e^{-i\omega t}\, e^{-i\omega_{pe}t}\, e^{+i\omega_{pe}t} \quad e^{-i\omega t}\, e^{i\omega_{pe}t}\, e^{-i\omega_{pe}t}$$

$$\cancel{-\omega^2 + k^2 C_s^2 \; z = -k^2 \frac{n_0 m_e}{m_i}} \qquad K \gg k_0$$

$$\tilde{v}^+ = \omega_{pe}^2 V_{00} \frac{\tilde{n}^L}{n_0} \frac{1}{2\omega\omega_{pe} - \frac{3k^2 T_e}{m_e}}$$

$$\tilde{v}^- = \omega_{pe}^2 V_{00}^* \frac{\tilde{n}^L}{n_0} \frac{1}{-2\omega\omega_{pe} - \frac{3k^2 T_e}{m_e}}$$

$$\underbrace{-\omega^2 + k^2 C_s^2}_{\omega_s^2} = -\frac{n_0 m_e}{m_i} k^2 \frac{\omega_{pe}^2 |V_{00}|^2}{\cancel{n_0}}$$

$$\otimes \left[\frac{1}{2\omega\omega_{pe} - \frac{3k^2 T_e}{m_e}} - \frac{1}{2\omega\omega_{pe} - \frac{3k^2 T_e}{m_e}} \right]$$

$$\omega^2 - k^2 C_s^2 - \frac{k^2 \omega_{pi}^2 |V_{00}|^2}{2\omega_{pe}} \Big[\underbrace{\frac{1}{\omega - \delta} - \frac{1}{\omega + \delta}}_{\frac{2\delta}{\omega^2 - \delta^2}} \Big] = 0$$

$$\delta = \frac{3k^2 T_e}{2\omega_{pe} m_e}$$

strong pump: $\omega^4 \sim V_{00}^2 \Rightarrow \gamma \sim V_{00}^{1/2}$

threshold:

$$(\omega^2-\omega_s^2)(\omega^2-\delta^2) = \frac{\omega_{pi}^2\,\delta}{\omega_{pe}} k^2 |V_{00}|^2$$

$$\omega^4 - (\omega_s^2+\delta^2)\omega^2 + \omega_s^2\delta^2 - \frac{\omega_{pi}^2\delta}{\omega_{pe}} k^2 |V_0|^2 = 0$$

$$\omega^2 = \frac{(\omega_s^2+\delta^2) \pm \sqrt{(\omega_s^2+\delta^2)^2 - 4\omega_s^2\delta^2 + 4\frac{\omega_{pi}^2\delta k^2 |V_{00}|^2}{\omega_{pe}}}}{2}$$

unstable for

$$\frac{\omega_{pi}^2\delta k^2 |V_{00}|^2}{\omega_{pe}} > \omega_s^2\delta^2$$

$$\frac{\omega_{pi}^2 \cancel{k^2} |V_{00}|^2}{\cancel{\omega_{pe}}} > \cancel{k^2} c_s^2 \frac{3k^2 T_e}{2\cancel{\omega_{pe}} m_e}$$

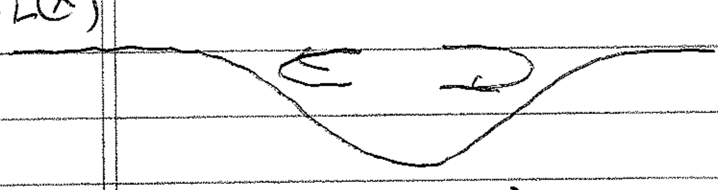
$$\frac{|V_{00}|^2}{T_e/m_e} > \frac{3}{2}k^2 \frac{T_e \cancel{m_i}}{\cancel{m_i}\, 4\pi n_0 e^2}$$

$c_s^2 = \Gamma T_e / m_i \Rightarrow T_e/m_i$ isothermal

$$\boxed{\frac{|V_0|^2}{T_e/m_e} > \frac{3}{2} k^2/k_{De}^2}$$

$\Rightarrow$ leads to Langmuir wave condensation

$n_L(x)$



Langmuir waves reflect from high density

$$\omega \sim \omega_{pe} + \frac{3}{2}\frac{k^2 T_e}{m_e \omega_{pe}} \Rightarrow n \uparrow \Rightarrow k \downarrow$$

Development of "Cavitons" and Trapping of rf Field*

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A new kind of parametric instability is observed in a nonuniform plasma in which a resonant rf electric field and a density cavity grow simultaneously through mutual enhancements. The threshold is observed to be virtually zero and the growth rate is linearly dependent on pump intensity, E_0^2. Ponderomotive force and wave trapping are observed to be the mechanisms which drive the instability.

We wish to report an observation of a new kind of parametric instability in a driven nonuniform plasma with a pump electric field directed parallel to the density gradient. The ponderomotive force[1] of the linearly enhanced electric field[2] first generates a density cavity[3] at the resonant location, $\omega_{pe}(z_0)=\omega_0$. The cavity in turn traps the rf field and causes mutual enhancements between the rf field and the density perturbation.

The experiment is performed in a unmagnetized dc-discharge plasma produced in a vacuum chamber of 60 cm length and 30 cm diam with base pressure 2×10^{-7} Torr [Fig. 1(a)]. The axial density gradient is produced by distributing hot filaments in only one half of the chamber. Typical parameters are argon pressure 3×10^{-4} Torr, $n_0\approx10^9$ cm^{-3}, noise level $n/n_0 \lesssim 0.1\%$, $n_0/(dn_0/dz) = 20$ cm, $T_e=1$ eV, $T_i=0.1$ eV, and $\nu_{en}/\omega_0\approx3\times10^{-3}$. A quasistatic external rf field ($\omega_0/2\pi = 360$ MHz) is imposed on the plasma by an electrode located at the low-density region in the chamber; the excited region is well separated from the exciter on account of the density inhomogeneity. The amplitude of this external field E_0 decreases exponentially (scale length ≈ 4 cm) along the axial direction as a result of the finite size of the exciter. We have used the following "remote" diagnostic techniques: An electron-beam probing technique[4] [Fig. 1(a)] to measure the rf field, a thin Langmuir probe to measure the density perturbation which propagates out after the rf is terminated, and a small rf resonant dipole probe [Fig. 1(b)] to supplement the electron-beam technique. Comparisons between the two methods are summarized in Table I.

The steady-state field E_T in the plasma is monitored by observing the lateral spread of a narrow (0.5 mm diam) electron beam (5–9 keV, 0.1 μA, $n_b'=10^4$ cm^{-3}) traversing the resonant region along the radial direction in approximately $\frac{1}{2}$ to $\frac{1}{4}$ of the rf period. Using the probe to map out the radial extent of the resonant region ($\Delta r \simeq 5$ cm) and taking into account the amplification of the electron beam deflection in the drift space, we establish the correlation between the rms value of E_T and the maximum lateral spread of the electron beam ($E_T=5$ V/cm per 1 mm spread). The phase relationship between the total field E_T and the externally imposed field E_0 is conveniently measured by generating Lissajous figures. The electron beam is modulated vertically by $E_0\times\cos\omega_0 t$ through a set of plates at the source and after the horizontal modulation by the resonant

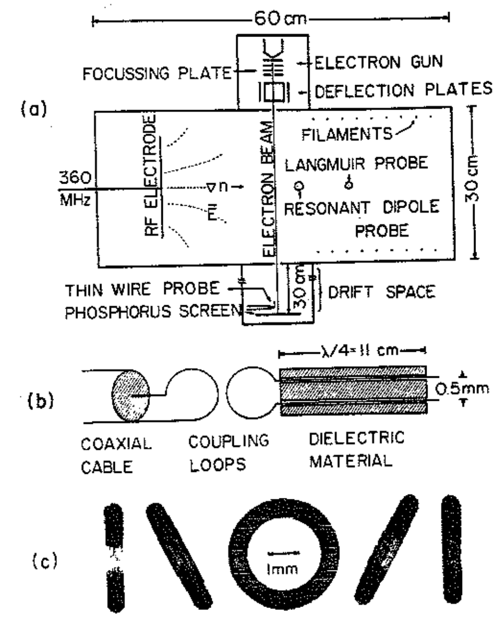

FIG. 1. (a) Schematic diagram of the apparatus, showing plasma device and differentially pumped electron beam device. (b) Resonant-dipole probe consisting of a λ/4 resonator in a ceramic shield. The potential difference measured between the exposed probe tips is magnetically coupled to a 50-Ω coaxial cable. (c) Sketches of the observed variation of the Lissajous pattern when the density is increased: (from left to right) $\omega_{pe}^2(z_0)/\omega_0^2 = 0.85$, 0.97, 1.00, 1.03, and 1.10. $P_0=10$ W, $\omega_0/2\pi=360$ MHz, steady state. The horizontal deflection is caused by the plasma field and the vertical deflection by a constant external field.

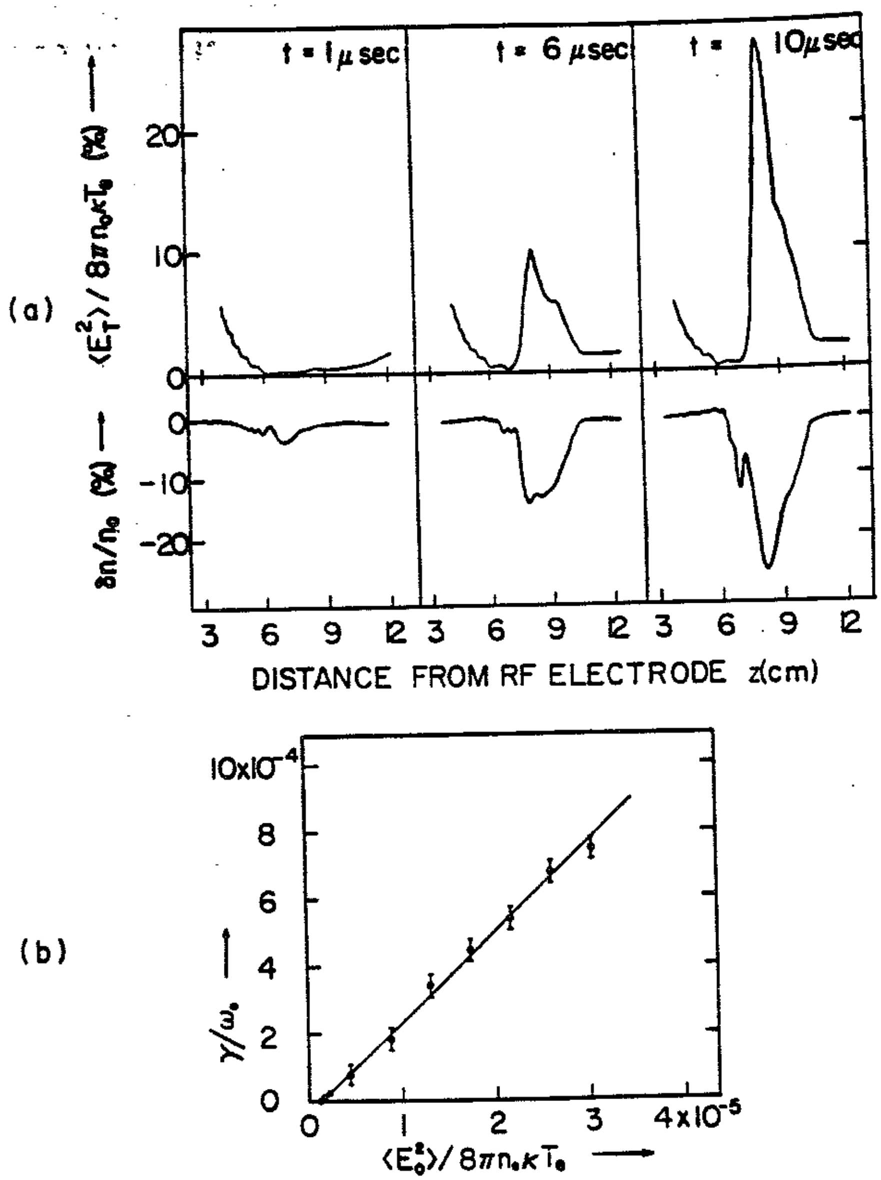

FIG. 2. (a) Profiles of the normalized field intensity (top traces) and density perturbation (bottom traces) at different times t after turn-on of 10-W rf pump (measured with probe). rf pressure and density perturbation mutually enhance each other. (b) Growth rate of $\langle E_T{}^2\rangle$ versus applied pump intensity $\langle E_0{}^2\rangle$.