Shock Waves

We have shown earlier that disturbances in plasmas tend to steepen. In the presence of dissipation (viscosity etc) the steepening will halt to form a propagating discontinuity in which ~~dissipation~~ some ~~is localized~~ parameters jump across the discontinuity. Dissipative processes are highly localized

$\Rightarrow$ called a shock

$\Rightarrow$ jump in parameters can be obtained by integrating across the shock

$\Rightarrow$ Rankine - Hugoniot conditions

$\Rightarrow$ broadly important

~~Basic MHD equations~~

$$\cancel{\nabla \times \underset{\sim}{B} = \frac{4\pi}{c}\underset{\sim}{J} + \frac{1}{c}\frac{\partial \underset{\sim}{E}}{\partial t}}$$

Basic MHD Eqns

$$\frac{1}{c}\frac{\partial B}{\partial t} + \nabla \times \underset{\sim}{E} = 0$$

$$\nabla \times \underset{\sim}{B} = \frac{4\pi}{c}\underset{\sim}{J} + \cancel{\frac{1}{c}\frac{\partial E}{\partial t}}^{\,0}$$

$$\nabla \cdot \underset{\sim}{B} = 0$$

1. $$\frac{\partial n}{\partial t} + \nabla\cdot n\underset{\sim}{V} = 0 \qquad \text{continuity}$$

$$m n \frac{d\underset{\sim}{V}}{dt} = -\nabla P + \frac{1}{c}\underset{\sim}{J}\times\underset{\sim}{B}$$

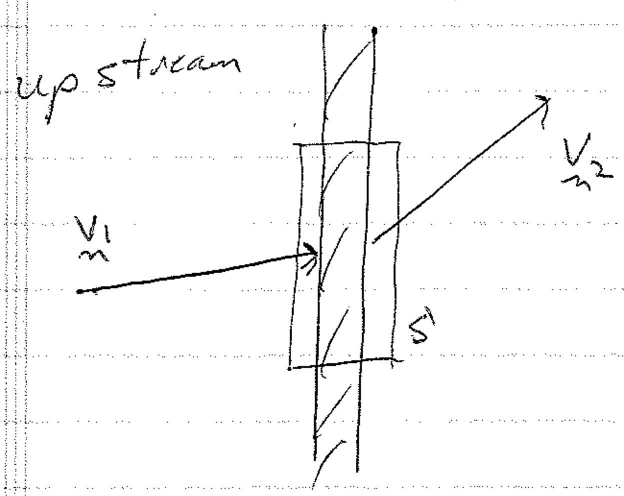
$$\underset{\sim}{E} + \frac{\underset{\sim}{V}\times\underset{\sim}{B}}{c} = 0 \qquad \frac{\partial P}{\partial t} + \underset{\sim}{V}\cdot\nabla P + \Gamma P\,\nabla\cdot\underset{\sim}{V} = 0$$

$$\frac{d}{dt}\left(\frac{P}{n^{\Gamma}}\right) = 0 \qquad \frac{d}{dt} = \frac{\partial}{\partial t} + \underset{\sim}{V}\cdot\nabla$$

$\Rightarrow$ write equations in conservation form

$$\frac{\partial}{\partial t} Q + \nabla\cdot\underset{\sim}{F} = 0$$

~~in steady state~~



down stream

$\Rightarrow$ steady state

$$\int_V dV\,\nabla\cdot\underset{\sim}{F} = 0$$

jump condition

$$\int_{S'} d\underset{\sim}{A}\cdot\underset{\sim}{F}$$

$$[F_n] = 0$$

$$(\underset{\sim}{F_2} - \underset{\sim}{F_1})\cdot\underset{\sim}{n} = 0 \qquad \int_S dA\,\underbrace{\underset{\sim}{F}\cdot\underset{\sim}{n}}_{F_n} = 0$$

momentum

$$n \frac{d\mathbf{v}}{dt} = n \frac{\partial \mathbf{v}}{\partial t} + n\, \mathbf{v} \cdot \nabla \mathbf{v}$$

$$= \frac{\partial}{\partial t}(n\mathbf{v}) + \nabla \cdot (n\mathbf{v}\mathbf{v})$$

$\Longrightarrow$ using continuity

$$\frac{1}{c}\, \mathbf{J} \times \mathbf{B} = \frac{1}{4\pi}\left[(\nabla \times \mathbf{B}) \times \mathbf{B} \right]$$

$$= -\frac{1}{4\pi}\left[\nabla\left(\frac{B^2}{2}\right) - \mathbf{B}\cdot\nabla\mathbf{B} \right]$$

(2) $$\frac{\partial}{\partial t}(mn\mathbf{v}) + \nabla \cdot \left[mn\, \mathbf{v}\mathbf{v} + \left(P + \frac{B^2}{8\pi}\right) \mathbf{I} - \frac{1}{4\pi} \mathbf{B}\mathbf{B} \right]$$

Energy

$$n\mathbf{v} \cdot \frac{d}{dt}\mathbf{v} = n\left(\frac{\partial}{\partial t}\frac{v^2}{2} + \mathbf{v}\cdot\nabla \frac{v^2}{2} \right)$$

$$+ \frac{v^2}{2}\left(\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{v} \right)$$

$$= \frac{\partial}{\partial t} \frac{nv^2}{2} + \nabla \cdot \left(\mathbf{v} \frac{nv^2}{2} \right)$$

$$\frac{\partial}{\partial t} P + (1-\Gamma)\, \mathbf{v}\cdot\nabla P + \Gamma\, \nabla \cdot P\mathbf{v} = 0$$

$$\mathbf{V}\cdot\nabla P = \frac{1}{\Gamma-1}\left(\frac{\partial P}{\partial t} + \Gamma\,\nabla\cdot P\mathbf{V}\right)$$

$$\frac{1}{c}\,\mathbf{V}\cdot(\mathbf{J}\times\mathbf{B}) = -\frac{1}{c}\,(\mathbf{V}\times\mathbf{B})\cdot\mathbf{J}$$

$$= \mathbf{E}\cdot\frac{c}{4\pi}\,\nabla\times\mathbf{B}$$

$$= -\frac{c}{4\pi}\left[\nabla\cdot(\mathbf{E}\times\mathbf{B}) - \mathbf{B}\cdot\nabla\times\mathbf{E}\right]$$

$$= -\frac{c}{4\pi}\,\nabla\cdot(\mathbf{E}\times\mathbf{B}) - \frac{1}{8\pi}\frac{\partial}{\partial t}\left(B^2\right)$$

(3)
$$\frac{\partial}{\partial t}\left(\frac{1}{2}mnV^2 + \frac{P}{\Gamma-1} + \frac{B^2}{8\pi}\right)$$

$$+\ \nabla\cdot\left[\frac{1}{2}mnV^2\,\mathbf{V} + \frac{\Gamma}{\Gamma-1}P\,\mathbf{V} + c\,\underbrace{\frac{\mathbf{E}\times\mathbf{B}}{4\pi}}_{\frac{1}{4\pi}\mathbf{B}\times(\mathbf{V}\times\mathbf{B})}\right] = 0$$

$$\frac{1}{4\pi}\left(B^2\,\mathbf{V} - \mathbf{B}\cdot\mathbf{V}\,\mathbf{B}\right)$$

Jump Conditions

continuity : from (1)

(4)
$$[nV_n] = 0$$

energy : from (3)

(5)
$$\left[\frac{1}{2}mnV^2\,V_n + \frac{\Gamma}{\Gamma-1}P\,V_n + \frac{1}{4\pi}B^2V_n - \frac{1}{4\pi}\mathbf{B}\cdot\mathbf{V}\,B_n\right] = 0$$

<u>momentum</u>: from 2 normal

6 $$\left[m n V_n^2 + P + \frac{B_t^2}{8\pi} \right] = 0$$

$\underset{\sim}{B}_t = \underset{\sim}{B}$ tangent to plane of shock

$= -\hat{n} \times (\hat{n} \times \underset{\sim}{B})$

$[B_n] \Rightarrow$

from 2 tangent

7 $$\left[m n V_n \underset{\sim}{V}_t - \frac{1}{4\pi} B_n \underset{\sim}{B}_t \right] = 0$$

<u>Electromagnetic Conditions</u>:

from $\nabla \cdot \underset{\sim}{B} = 0$

8 $$[B_n] = 0$$

tangential $\underset{\sim}{E}$: $\underset{\sim}{E} = -\frac{1}{c} \underset{\sim}{V} \times \underset{\sim}{B}$

$$[\hat{n} \times (\underset{\sim}{V} \times \underset{\sim}{B})] = 0 \qquad B_n \underset{\sim}{V} - V_n \underset{\sim}{B} = 0$$

9 $$\Longrightarrow \quad [V_n \underset{\sim}{B}_t - \underset{\sim}{V}_t B_n] = 0$$

$\Longrightarrow$ <u>total of 8 conditions</u>

Classes of solutions:

(a) contact on tangential discontinuity

no mass flux through surface

$$V_{n_1} = 0, \; V_{n_2} = 0 \; , \; B_n = 0$$

(b) non-compressible shock

no density jump $\Rightarrow n_1 = n_2$

non-zero mass flux (in shock frame)

$$V_{1n} = V_{2n}$$

(c) compressible shock

$\Rightarrow$ density jump $n_1 \neq n_2$

$$V_{1n}, V_{2n} \neq 0$$

From (4), (7), (9)

$$[V_t] = \frac{1}{4\pi} \frac{B_n}{mnV_n} [B_t] \Rightarrow$$

$$\frac{1}{B_n} [V_n B_t] = [V_t]$$

$$[V_n B_t] = \frac{B_n^2}{4\pi m n V_n} [B_t]$$

$$[\underset{\sim}{B}_t] = \alpha\,[V_n \underset{\sim}{B}_t] \qquad \alpha = \frac{4\pi m n V_n}{B_n^2}$$

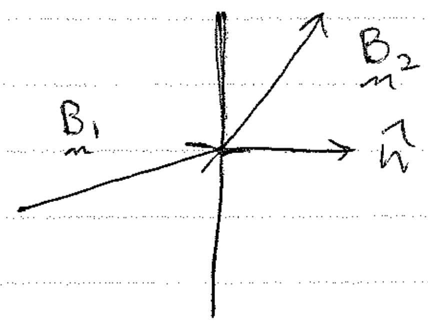
$$\underset{\sim}{B}_{t2} - \underset{\sim}{B}_{t1} = \alpha\left(V_{n2}\underset{\sim}{B}_{t2} - V_{n1}\underset{\sim}{B}_{t1}\right)$$

$$\underset{\sim}{B}_{t2}\left(1 - \alpha V_{n2}\right) = \underset{\sim}{B}_{t1}\left(1 - \alpha V_{n1}\right)$$

$$\underset{\sim}{B}_{t2} = \underset{\sim}{B}_{t1}\left(\frac{1-\alpha V_{n1}}{1-\alpha V_{n2}}\right)$$

$\Rightarrow$ $\underset{\sim}{B}_{t2}$ in same direction as $\underset{\sim}{B}_{t1}$

$\Rightarrow$



$\hat{n}$, $\underset{\sim}{B}_1$, $\underset{\sim}{B}_2$ are in same plane

$$\boxed{\hat{n}\cdot\left(\underset{\sim}{B}_1 \times \underset{\sim}{B}_2\right) = 0}$$

co-planarity theorem

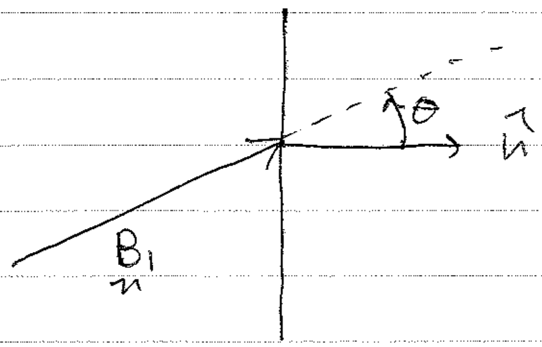
$\Rightarrow$ not true for contact discontinuity where $B_n = 0$

Choose shock frame in which

$\underset{\sim}{V}_{t1} = 0$

$\Rightarrow$ inflow is parallel to $\hat{n}$

$\Rightarrow$ free choice.



$$r = \frac{n_2}{n_1}$$

$$M_A = \frac{V_1}{V_{1A}}$$

$$V_{1A} = \frac{B_1^2}{4\pi m n_1}$$

from 4 $\qquad n_2 V_{n2} = n_1 V_1$

From 3

$$n_1 V_1 \; \frac{1}{2} m \cancel{n} V_2^2 \; \cancel{V_{n2}} + \frac{\Gamma}{\Gamma - 1} P_2 V_{n2} + \frac{B_2^2}{4\pi} V_{n2}$$

$$- \frac{1}{4\pi} \underline{B}_2 \cdot \underline{V}_2 \; B_n$$

$$= \frac{1}{2} m V_1^3 n_1 + \frac{\Gamma}{\Gamma - 1} P_1 V_1 + \frac{B_1^2}{4\pi} V_1$$

$$- \frac{1}{4\pi} B_n^2 V_1$$

Comes later to form eqn for $V_2 = \frac{V_{n2}}{[illegible]}$

From 6

$$\frac{n_1^2 V_1^2}{\cancel{n_2}} \frac{m}{n} \cancel{V_{n2} n_2} + P_2 + \frac{B_{t2}^2}{8\pi} = m n_1 V_1^2 + P_1 + \frac{B_1^2 \sin^2\theta}{8\pi}$$

From 7

$$4\pi \, m \, n_1 V_1 \, \underline{V}_{t2} - \cancel{\frac{1}{4\pi}} B_n \, \underline{B}_{t2} = - \cancel{\frac{1}{4\pi}} B_n \, \underline{B}_{t1}$$

From 9

$$\frac{n_2 V_{n2} \, \underline{B}_{t2}}{n_2} - \underline{V}_{t2} B_n = V_1 \, \underline{B}_{t1}$$

$$\underline{B}_{t2} = \frac{4\pi m n_1 V_1}{B_n} \underline{V}_{t2} + \underline{B}_{t1}$$

$$M_{An}^2 = \frac{V_1^2}{B_n^2/4\pi m n_1}$$

$$\frac{\cancel{n_1} V_1}{r\,\cancel{n_2}}\left[\frac{4\pi m n_1 V_1^2}{B_n^2}\frac{\mathbf{V}_{t2}}{V_1} + \frac{\mathbf{B}_{t1}}{B_n}\right] - \cancel{B_n}\,\mathbf{V}_{t2}/V_1$$

$$= \cancel{V_1}\,\frac{\mathbf{B}_{t1}}{B_n}$$

$$\Rightarrow \boxed{\frac{\mathbf{V}_{t2}}{V_1}\left(\frac{M_{An}^2}{r} - 1\right) = \frac{\mathbf{B}_{t1}}{B_n}\left(1 - \frac{1}{r}\right)}$$

$$\mathbf{B}_{t2} = \frac{4\pi m n_1 V_1}{B_n}\left(\frac{\cancel{n_1} V_1}{r\,\cancel{n_2}}\frac{\mathbf{B}_{t2}}{B_n} - V_1\frac{\mathbf{B}_{t1}}{B_n}\right) + \mathbf{B}_{t1}$$

$$\Rightarrow \boxed{\mathbf{B}_{t2}\left(1 - \frac{M_{An}^2}{r}\right) = \mathbf{B}_{t1}\left(1 - M_{An}^2\right)}$$

$$\frac{|\mathbf{V}_{t2}|}{V_1} = \frac{\cancel{B_1}\sin\theta}{\cancel{B_1}\cos\theta}\left(1 - \frac{1}{r}\right)\frac{1}{\frac{M_A^2}{r\cos^2\theta} - 1}$$

$$\boxed{\frac{|\mathbf{V}_{t2}|}{V_1} = \frac{(r-1)\sin\theta\cos\theta}{M_A^2 - r\cos^2\theta}}$$

$$\boxed{\frac{|\mathbf{B}_{t2}|}{|\mathbf{B}_{t1}|} = \frac{r\left(M_A^2 - \cos^2\theta\right)}{M_A^2 - r\cos^2\theta}}$$

From jump conditions (energy and momentum)

$$\left(\frac{a M_A^2}{r} - \beta\right)\left(\frac{M_A^2}{r} - \cos^2\theta\right)^2 - \frac{M_A^2}{r}\sin^2\theta \left\{\frac{M_A^2}{r}\left(\frac{a}{r} - \frac{1-r}{2}\right) - a\cos^2\theta\right\} = 0$$

$$a = \frac{\Gamma + 1 - (\Gamma - 1) r}{2}$$

$$r = \frac{n_2}{n_1} \qquad M_A = \frac{V_1}{C_{A1}}$$

$$\beta = \frac{C_{S1}^2}{C_{A1}^2}$$

$$C_S^2 = \frac{\Gamma P}{m n}$$

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note that this equation is related to the ~~[illegible]~~ dispersion relation for small amplitude MHD waves

$\Longrightarrow$ take $r = 1 \Longrightarrow a = 1$

$$M_A = \frac{\omega}{K C_A}$$

$$\left(\frac{\omega^2}{k^2 C_A^2} - \beta\right)\left(\frac{\omega^2}{k^2 C_A^2} - \cos^2\theta\right)^2 - \frac{\omega^2}{k^2 C_A^2}\sin^2\theta\left(\frac{\omega^2}{k^2 C_A^2} - \cos^2\theta\right) = 0$$

Generally has three solutions:

$$\omega = k C_A \cos\theta$$ intermediate or Alfven wave

$$\left(\frac{\omega^2}{k^2 C_A^2} - \beta\right)\left(\frac{\omega^2}{k^2 C_A^2} - \cos^2\theta\right) = \frac{\omega^2}{k^2 C_A^2}\sin^2\theta$$

$\Rightarrow$ fast and slow modes

$\theta = 0$

$$\omega^2 = \beta k^2 C_A^2 \Longrightarrow$$ sound wave

$$\omega^2 = k^2 C_A^2 \Longrightarrow$$ ~~Alfven~~ Alfven wave

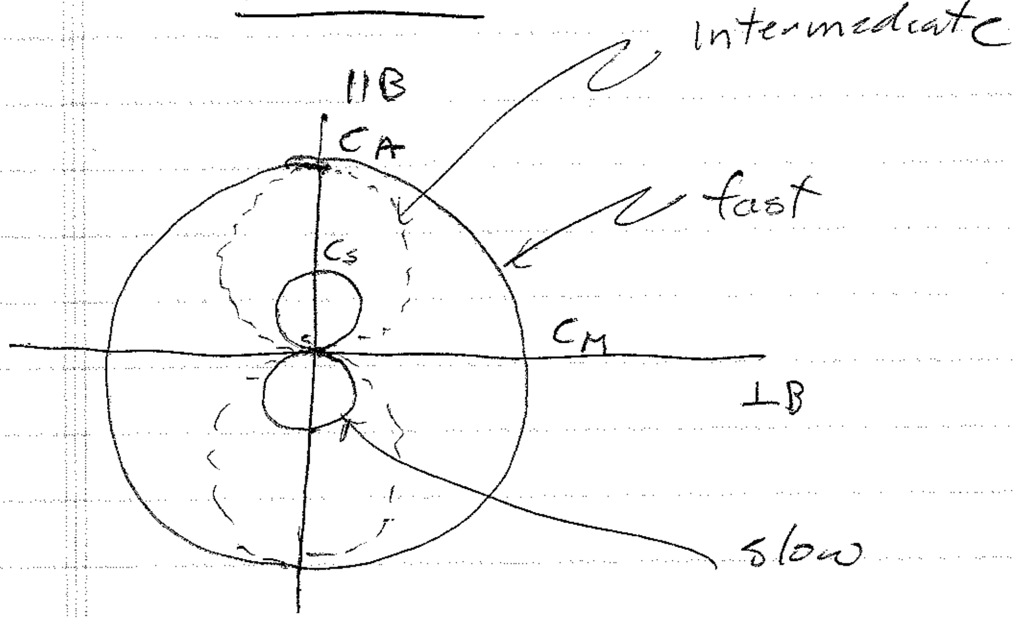
$\theta = \frac{\pi}{2}$

$$\omega^2 = 0 \Rightarrow$$ slow mode

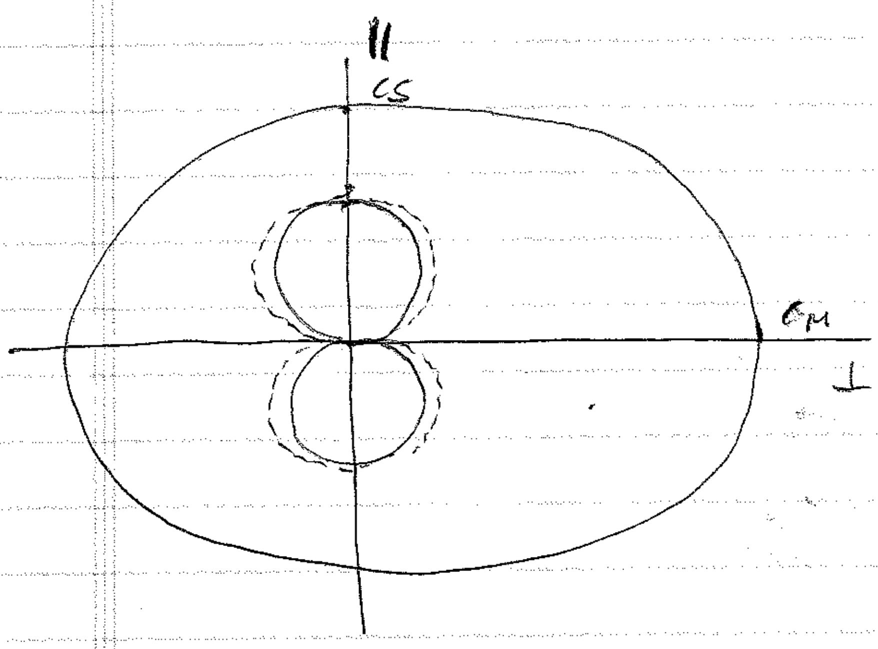
$$\omega^2 = k^2 C_A^2 (1+\beta) \Rightarrow$$ fast magnetosonic mode.

Friedrichs diagrams

low β



high β



Shock solutions

Generally have three classes of solutions analagous to waves.

* fast and slow shocks are true shocks.

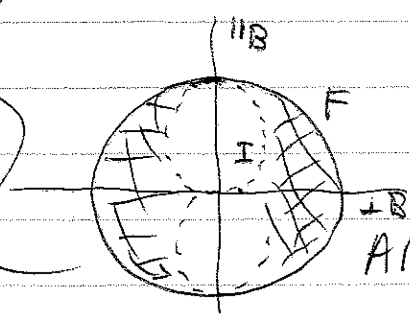
* Alfven or intermediate wave does not steepen
  $\Rightarrow$ no compression

$\Rightarrow$ plasma must compress across shock corresponding to an increase in the entropy.
$\Rightarrow$ increase in entropy defines shock.

Existence conditions

fast shock $\rightarrow$ upstream normal velocity must exceed fast wave speed. downstream normal velocity must fall below down speed fast wave speed and above intermediate speed.

low β



Allowed downstream flow speeds.

Intermediate or Alfvén solution

$r = 1, \; a = 1 \implies$ no compression

$$(M_A^2 - \beta)(M_A^2 - \cos^2\theta)^2 - M_A^2 \sin^2\theta \left[M_A^2 - \cos^2\theta \right] = 0$$

$$\implies M_A^2 = \cos^2\theta \; , \quad M_{An} = 1 \qquad V_1 = \frac{B_n}{\sqrt{4\pi m n}}$$

$\implies$ equation for $\underset{\sim}{B}_{t_2}$ ambiguous

$\implies$ use (6) $\implies [B_t^2] = 0$ for $\underbrace{[P] = 0}_{\text{no compression}}$

$\underset{\sim}{V} \cdot \nabla P + \Gamma P \cancel{\nabla \cdot \underset{\sim}{V}} = 0$

$[P] = 0$

Look for solution with

$$\underset{\sim}{B}_{t_2} = - \underset{\sim}{B}_{t_1}$$

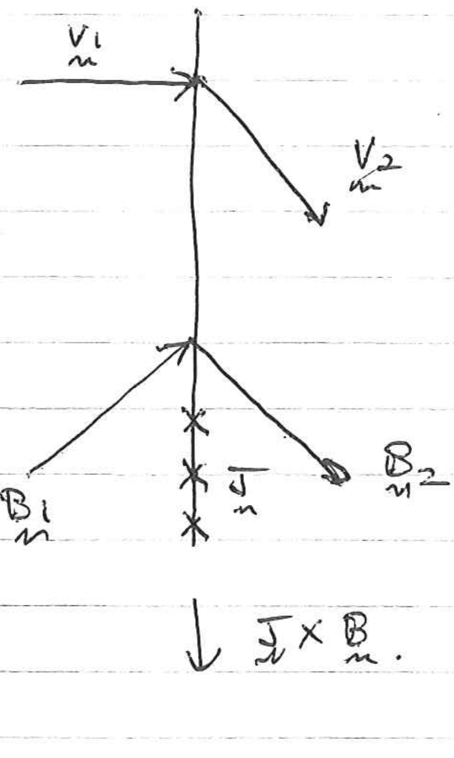
From (7)

$$[\underset{\sim}{V}_t] = \frac{B_n}{4\pi m n V_1} [B_t]$$

$$[\underset{\sim}{V}_t] = [C_{At}]$$

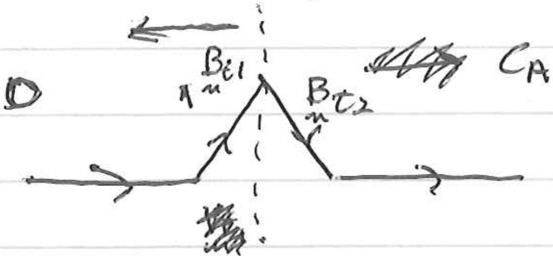
$$\underset{\sim}{V}_{t_2} = -2 \, \underset{\sim}{C}_{At_1}$$

$\implies$ rotational discontinuity



Consider an ~~a~~ Alfven kink

$V_n = 0$

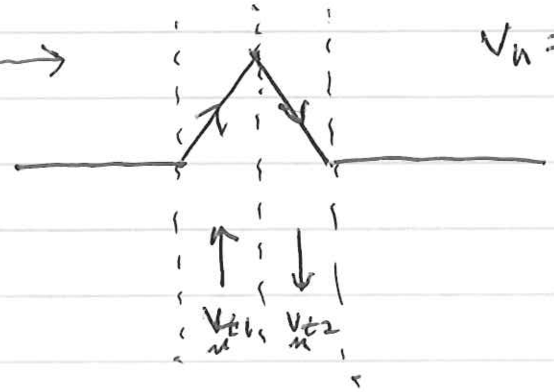


$$C_{An} = \frac{B_n}{\sqrt{4\pi m n}} \qquad \underset{\sim}{B}_{t1} = -\underset{\sim}{B}_{t2}$$

$\underset{\sim}{B}_n$

In wave frame $\Rightarrow$ stationary $\underset{\sim}{B}$ $\Rightarrow$ plasma flow to the right

$V_n \longrightarrow$



$$V_n = \frac{B_n}{\sqrt{4\pi m n}}$$

$\underset{\sim}{V}_t = \underset{\sim}{C}_{At}$ = for Alfven wave

$\underset{\sim}{V}_{t1} = -\underset{\sim}{V}_{t2}$

Wave frame $\qquad \underset{\sim}{V} = \underset{\sim}{V}_n + \underset{\sim}{V}_t$, $\underset{\sim}{B} = \underset{\sim}{B}_n + \underset{\sim}{B}_t$

$$\underset{\sim}{V} \times \underset{\sim}{B} = \underset{\sim}{C}_{At} \times \underset{\sim}{B}_n + \underset{\sim}{V}_n \times \underset{\sim}{B}_t$$

$$= \underset{\sim}{B}_t \times \underset{\sim}{V}_n + \underset{\sim}{V}_n \times \underset{\sim}{B}_t = 0$$

$\Rightarrow$ $\underset{\sim}{V}$ parallel with $\underset{\sim}{B}$

$\Rightarrow$ $\underset{\sim}{E} = 0$

$\Rightarrow$ de Hoffmann-Teller frame

How does this frame relate to the frame of our Alfven solution?

$\Rightarrow$ shift to frame moving with $\underset{\sim}{V}_{t1}$

$\Rightarrow$ $\underset{\sim}{V}'_{t1} = 0$, $\underset{\sim}{V}'_{t2} = -2\underset{\sim}{C}_{At1}$ $\Rightarrow$ our shock solution

Fast Mode Shock

$\Rightarrow$ consider low β

$$a \cancel{\frac{M_A^2}{r}}\left(\frac{M_A^2}{r} - \cos^2\theta\right)^2 - \cancel{\frac{M_A^2}{r}} \sin^2\theta \left\{\frac{M_A^2}{r}\left(\frac{a}{r} - \frac{1-r}{2}\right) - a\cos^2\theta\right\} = 0$$

$$a = \frac{\Gamma+1 - (\Gamma-1)r}{2}$$

$$r = \frac{n_2}{n_1}$$

$$\left(\frac{M_A^2}{r} - \cos^2\theta\right)^2 - \sin^2\theta \left\{\frac{M_A^2}{r}\left(\frac{1}{r} - \frac{1-r}{2a}\right) - \cos^2\theta\right\} = 0$$

consider what happens when M_A is very large $\Rightarrow$ how large does r become?

$$\left(\frac{M_A^2}{r}\right)^2 - \sin^2\theta \, \frac{M_A^2}{\cancel{r}}\left(\frac{1}{r} - \frac{1-r}{2a}\right) = 0$$

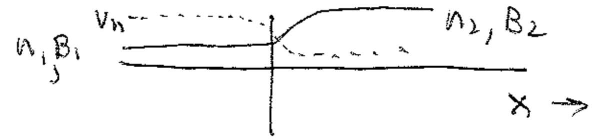
$$\cancel{M_A} \quad M_A^2 - \sin^2\theta\left(1 - \frac{(1-r)r}{2a}\right) = 0$$

Note that $a < 0$ for large r so second term ~~can't be~~ is positive and can't balance first.

$\Rightarrow$ must ~~that~~ have $a \to 0^+$ as $M_A \to \infty$

$$a \to 0 \Rightarrow r = \frac{\Gamma+1}{\Gamma-1} = \frac{\frac{5}{3}+1}{\frac{5}{3}-1} = 4$$

(over)



$$V_{2n} = \frac{V_{1n}}{n_2}\, n_1 = \frac{V_{1n}}{4} = \frac{V_1}{4}$$

$$B_2 = \left(B_n^2 + B_{t_2}^2\right)^{\frac{1}{2}} = \left(B_n^2 + 16 B_{t_1}^2\right)^{\frac{1}{2}}$$

$$= B_1 \left(\cos^2\theta + 16\sin^2\theta\right)^{\frac{1}{2}}$$

skip

$$C_{A2} = C_{A1} \frac{B_2}{B_1} \sqrt{\frac{n_1}{n_2}}$$

$$\frac{V_{2n}}{C_{A2}} = \frac{V_1}{4\, C_{A1} \left(\cos^2\theta + 16\sin^2\theta\right)^{\frac{1}{2}}} \quad 2$$

$$= M_A \frac{1}{2\cancel{4}\left(\cos^2\theta + 16\sin^2\theta\right)^{\frac{1}{2}}}$$

$$= \frac{M_A}{2\cancel{4}\left(1 + 15\sin^2\theta\right)^{\frac{1}{2}}} < 1$$

$$M_A < 2\cancel{4}\left(1 + 15\sin^2\theta\right)^{\frac{1}{2}}$$

Limit $\theta = 0$

$$M_A^2 = r\cos^2\theta$$

$$\frac{|\underset{\sim}{B}_{t_2}|}{|\underset{\sim}{B}_{t_1}|} \longrightarrow \infty \quad \text{but} \quad |\underset{\sim}{B}_{t_1}| \longrightarrow 0$$

($|\underset{\sim}{B}_{t_1}| = B_1\sin\theta$)

$\Longrightarrow \underset{\sim}{B}_{t_2} \neq 0$

$\Longrightarrow$ switch-on shock.

Slow shocks

$\Rightarrow$ low β

take $M_A^2 \sim \beta$

v_1 must exceed projection of sound speed normal to shock.

$$\left(\frac{a}{r} M_A^2 - \beta\right) \cos^{2}\theta + \frac{M_A^2}{r} \sin^2\theta (+a) \cancel{\cos^2\theta} = 0$$

$$\boxed{M_A^2 = \beta \cos^2\theta \, \frac{r}{a}}$$

$\Rightarrow \frac{v_1^2}{c_{s1}^2 \cos^2\theta}$

$\frac{M_A^2}{\beta \cos^2\theta} > 1$ for shock to form.

$\Rightarrow \theta = 0$ B drops out.

$\Rightarrow$ pure sound ~~wave~~ shock

$$\frac{|B_{t2}|}{|B_{t1}|} = \cancel{r} \frac{\left(\beta \frac{r}{a} - 1\right)}{\left(\beta \frac{r}{a} - \cancel{r}\right)} = \frac{1 - \beta \frac{r}{a}}{1 - \frac{\beta}{a}}$$

$$\simeq 1 - \frac{\beta}{a}(r-1) < 1$$

$n_2 > n_1$

$B_2 < B_1$

$$\cancel{|B_{t2}| \simeq B_1 \sin\theta \left(1 - \frac{\beta}{a}(r-1)\right)}$$

$$\cancel{\Delta\left(\frac{B^2}{8\pi}\right) \sim \frac{B_t \Delta B_t}{4\pi} \sim \frac{B_1^2}{4\pi} \sin^2\theta \, \frac{\beta}{a}(r-1) \, \frac{n_1 T_1}{B_1^2}}$$

$$\cancel{\sim \Delta n \, T_1}$$

$\Rightarrow$ pressure increases across shock

$\Rightarrow$ magnetic pressure decreases