Magnetic Reconnection

The dominant mechanism for dissipating magnetic energy in the universe. Converts magnetic energy into high-speed flows and energetic particles. Underlies important phenomena in the laboratory and nature

⇒ disruptions in tokamaks and other fusion experiments.

⇒ solar and stellar flares
⇒ flares from pulsar magnetospheres
⇒ magnetospheric substorms

⇒ gamma ray bursts?

⇒ driver of cosmic rays?

Required for
⇒ the dynamo
⇒ accretion around compact objects etc.

Basic questions

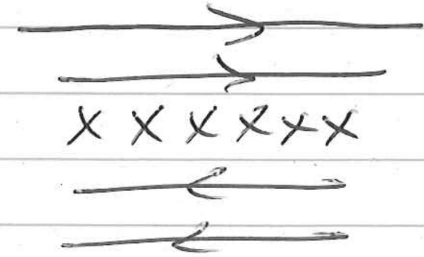
1) What is the time scale for energy release

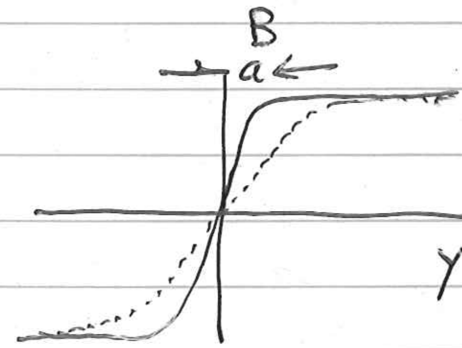
2) Why is the onset so sudden?

3) When does it occur?

4) What is the mechanism for the production of energetic particles?
⇒ flares, tokamaks, substorms, cosmic rays?

Magnetic energy can be released as a result of diffusion





$$\frac{\partial}{\partial t}B - \frac{\eta c^2}{4\pi}\frac{\partial^2}{\partial y^2}B = 0$$

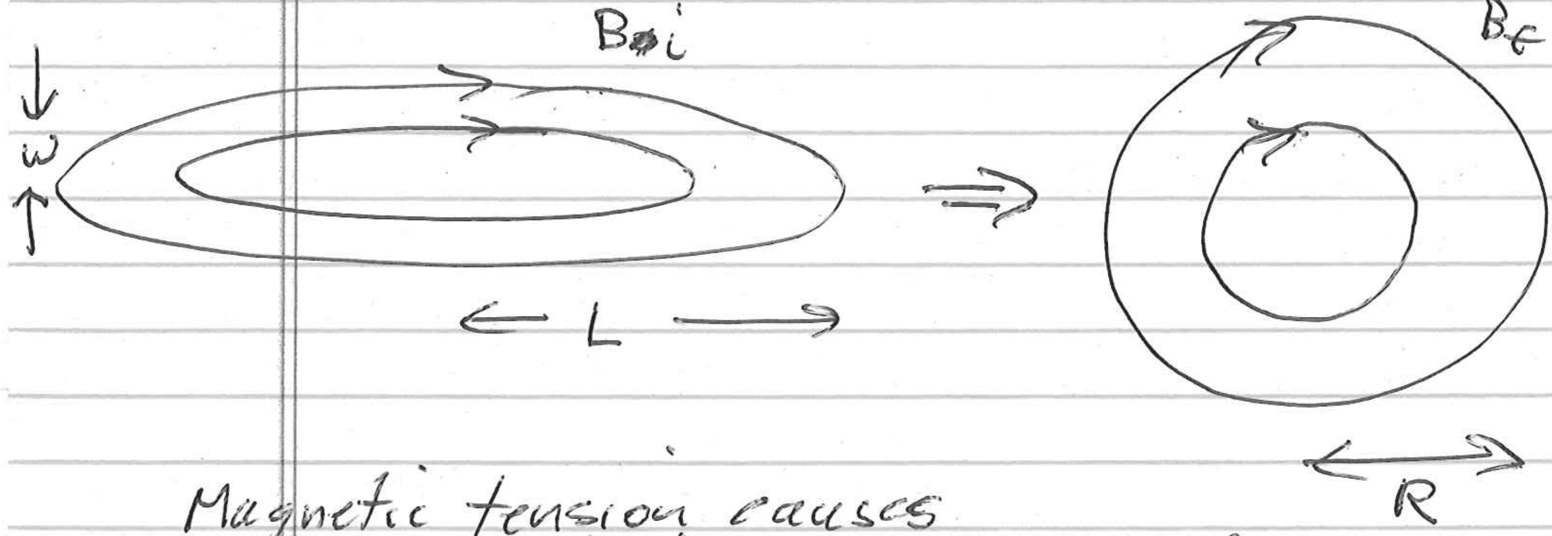
$$\tau_r \sim \frac{4\pi a^2}{\eta c^2} = \text{resistive time}$$

Diffusion time scales:

	Resistive Time	Observed Release Time
tokamaks	1–10 s	100 μsec
solar flares	~ 10^4 yrs.	~ 2 minutes
magnetospheric substorms	~ ∞	30 minutes

Basic Physics

Energy release from squashed bubble



Magnetic tension causes squashed bubble to become round

$$\underset{\sim}{F} = -\nabla\left(P + \frac{B^2}{8\pi}\right) + \frac{1}{4\pi}\,\underset{\sim}{B}\cdot\nabla\underset{\sim}{B}$$

Energy release

$\Rightarrow$ use conservation for ideal motion
flux, area

flux: $B_i w \sim B_f R$

area: $L w \sim R^2$

$$W_f = \frac{B_f^2}{8\pi} A = \frac{B_i^2}{8\pi} \frac{w^2}{R^2} A$$

$$= \frac{w^2}{R^2} W_i = \frac{w}{L} W_i$$

$$\ll W_i$$

$\Rightarrow W_f \ll W_i$

$\Rightarrow$ most magnetic energy is released.

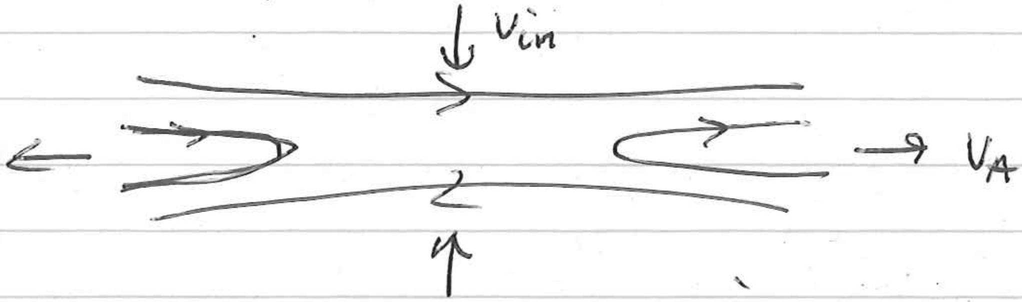
$\Rightarrow$ appears as flow energy

$$\frac{1}{2} \rho v^2 \sim \frac{B_i^2}{8\pi}$$

$$\Rightarrow v \sim v_A = \left(\frac{B_i^2}{4\pi\rho}\right)^{\frac{1}{2}}$$

$\Rightarrow$ characteristic time $\tau_A \sim \frac{L}{c_A}$

Basic phenomenon:



newly reconnected field lines release tension by expanding outward.

$\Rightarrow$ magnetic slingshot

Produces pressure drop around the magnetic x-line.

$\Rightarrow$ pressure drop pulls in plasma from above and below

$\Rightarrow$ new field lines break and reconnect

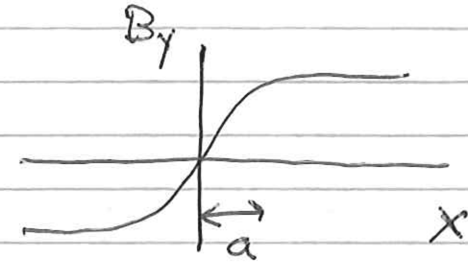
$\Rightarrow$ need dissipation

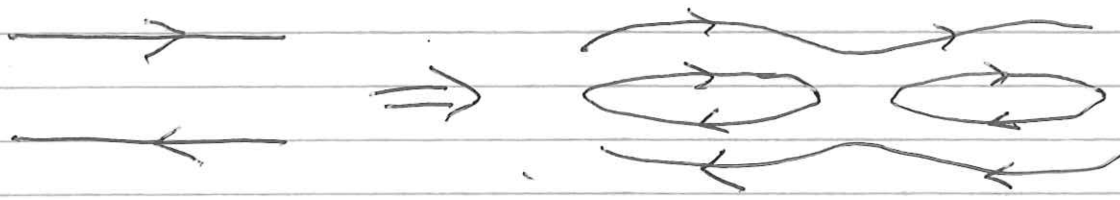
$\Rightarrow$ newly reconnected field lines expand outward

Reconnection is self-driven

Outline

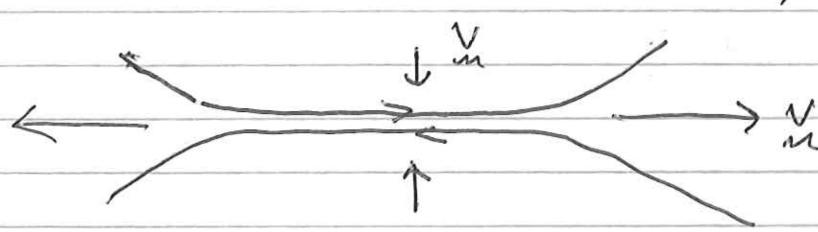
1) Linear theory





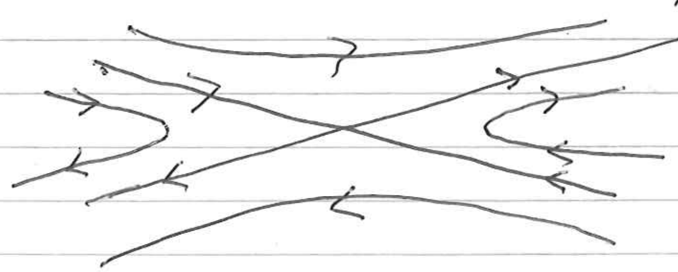
⇒ tearing mode

2) Sweet-Parker theory



macroscopic current layer

3) Petschek theory



open outflow

2-D reduced equations

$$\rho \frac{d\underset{\sim}{v}}{dt} = \frac{1}{c} \underset{\sim}{J} \times \underset{\sim}{B} - \nabla P$$

$$\underset{\sim}{E} + \frac{1}{c} \underset{\sim}{v} \times \underset{\sim}{B} = \eta \underset{\sim}{J}$$

$$\frac{1}{c} \frac{\partial}{\partial t} \underset{\sim}{B} + \nabla \times \underset{\sim}{E} = 0$$

$$\nabla \times \underset{\sim}{B} = \frac{4\pi}{c} \underset{\sim}{J}$$

In 2-D

$$\underset{\sim}{B} = \hat{z} \times \nabla \psi + B_z \hat{z} \implies \nabla \cdot \underset{\sim}{B} = 0$$

$$\left(\nabla \times \underset{\sim}{B}\right)_z = \boxed{\nabla^2 \psi = \frac{4\pi}{c} J_z}$$

From Faradays law: $\hat{z} \times (\;)$

$$-\frac{1}{c} \frac{\partial}{\partial t} \nabla \psi + \underbrace{\hat{z} \times \left(\nabla \times \underset{\sim}{E}\right)}_{\nabla E_z - \cancel{\hat{z} \cdot \nabla \underset{\sim}{E}}^{\,0}} = 0$$

$$\nabla \left(-\frac{1}{c} \frac{\partial}{\partial t} \psi + E_z \right) = 0$$

$$E_z = \frac{1}{c} \frac{\partial}{\partial t} \psi$$

Ohm's Law

$$\hat{z}\cdot\left(\underset{\sim}{E} + V \times (\hat{z}\times\nabla\psi)\frac{1}{c} = \eta \underset{\sim}{J}\right)$$

$$E_z + \underset{\sim}{V}\cdot\nabla\psi\frac{1}{c} = \eta J_z$$

$$\boxed{\frac{\partial\psi}{\partial t} + V\cdot\nabla\psi = \frac{\eta c^2}{4\pi}\nabla^2\psi}$$

Convective Diffusion Eqn. for flux ψ.

If $\frac{\partial}{\partial t} \ll \frac{1}{\tau_{ms}} \Rightarrow$ nearly incompressible

Take curl of momentum equation

$$\nabla\cdot\underset{\sim}{V} \simeq 0$$

$$\Rightarrow \underset{\sim}{V} = \hat{z}\times\nabla\varphi \quad \varphi \text{ stream function}$$

$$\nabla\times\underset{\sim}{V} = \hat{z}\,(\nabla^2\varphi)$$ — vorticity

$$\rho_0\,\hat{z}\cdot\nabla\times\frac{d}{dt}\underset{\sim}{V} = -\rho_0\nabla\cdot\left(\hat{z}\times\frac{d}{dt}\underset{\sim}{V}\right)$$

$$= +\rho_0\nabla\cdot\frac{d}{dt}\nabla\varphi$$

$$= \rho_0\left(\frac{\partial}{\partial t}\nabla^2\varphi + \underbrace{\nabla\cdot(\hat{z}\times\nabla\varphi\cdot\nabla)\nabla\varphi}\right)$$

$\varphi_i = \partial_i\varphi$

$$\underbrace{\partial_i(\hat{z}\times\nabla\varphi\cdot\nabla)\varphi_i}$$

$$\hat{z}\times\nabla\varphi\cdot\nabla\,\varphi_{ii}$$

$$+\,\cancel{\hat{z}\times\nabla\varphi_i\cdot\nabla\varphi_i} \to 0$$

$$\rho_0 \frac{d}{dt} \nabla^2 \varphi = \frac{1}{c} \hat{z} \cdot \nabla \times (\underset{\sim}{J} \times \underset{\sim}{B})$$

$$= \frac{1}{c} \left(B \cdot \nabla J_z - \underset{\sim}{J} \cdot \nabla B_z \right)$$

Since $\nabla \cdot \underset{\sim}{B} = \nabla \cdot \underset{\sim}{J} = 0$

$$\underset{\sim}{J} \cdot \nabla B_z = \frac{c}{4\pi} \nabla B_z \times \hat{z} \cdot \nabla B_z = 0$$

$$\boxed{4\pi \rho_0 \frac{d}{dt} \nabla^2 \varphi = \underset{\sim}{B} \cdot \nabla \nabla^2 \psi}$$

Relation between φ and $\underset{\sim}{E}_\perp$ (x, y direction)

$$\underset{\sim}{v} = \hat{z} \times \nabla\varphi$$

$$\underset{\sim}{E} + \frac{1}{c}\underset{\sim}{v} \times \underset{\sim}{B} = 0$$

$$\underset{\sim}{E} + \frac{1}{c}(\hat{z} \times \nabla\varphi) \times \underset{\sim}{B} = 0$$

$$\hat{z} \times \underset{\sim}{E} + \frac{B_z}{c}\hat{z} \times \nabla\varphi = 0$$

$$\underset{\sim}{E} = -\frac{B_z}{c}\nabla\varphi$$

$$E_\parallel = \frac{B_z E_z - \frac{B_z}{c} B\cdot\nabla\varphi}{B}$$

$$= \frac{B_z}{B}\left(\frac{1}{c}\frac{\partial\psi}{\partial t} - \frac{1}{c}B\cdot\nabla\varphi\right)$$

$$= \frac{B_z}{cB}\left(\frac{\partial\psi}{\partial t} - B\cdot\nabla\varphi\right)$$

$\varphi \longrightarrow$ electrostatic potential

$$\frac{\partial\psi}{\partial t} + \underset{\sim}{v}\cdot\nabla\psi = \frac{3c^2}{4\pi}\nabla^2\psi$$

$$\frac{Bc}{B_z}E_\parallel = \frac{3c^2}{4\pi}\frac{4\pi}{c}J_z$$

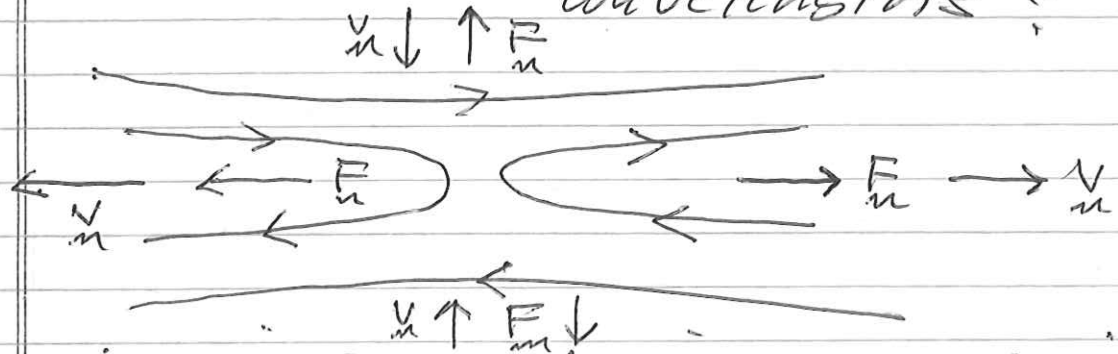
$$\boxed{E_\parallel \frac{B}{B_z} = J_z} \Longrightarrow \text{Ohm's law}$$

183c

Magnetic energy

when do we expect reconnection to release magnetic energy ?

$\Longrightarrow$ range of unstable wavelengths ?



tension force drives reconnection : $\underline{\xi}$

$$\underline{F} = \underline{B} \cdot \nabla \underline{B}$$

Reconnection driven where $\underline{F} \cdot \underline{V} > 0$

$\Longrightarrow$ inflow energetically unfavorable

$\Longrightarrow$ outflow energetically favorable

$\Rightarrow$ Calculate energy released

$\Rightarrow$ displace plasma and magnetic field to evaluate energy change

$\Longrightarrow$ displacement increases from $0 \longrightarrow \xi$

$$dW_B = - \int d\underline{x} \; d\underline{\xi} \cdot \underline{F}(\underline{\xi})$$

$\Longrightarrow$ integrate $\underline{\xi}$ from 0 to ξ $\Rightarrow \underline{F} \sim \underline{\xi}$

$$\Delta W_B = - \frac{1}{2} \int d\underline{x} \; \underline{\xi} \cdot \underline{F}(\underline{\xi})$$

183d

$$\underset{\sim}{B} = \hat{z}\times\nabla\psi + \hat{z}B_z$$

$$\underset{\sim}{F} = \frac{1}{c}\underset{\sim}{J}\times\underset{\sim}{B} \qquad \underset{\sim}{J} = \hat{z}\,\frac{c}{4\pi}\nabla^2\psi$$

$$\Delta W_B = -\frac{1}{2}\int d\underset{\sim}{x}\;\underset{\sim}{\xi}\cdot\frac{1}{c}\,\underset{\sim}{J}\times\underset{\sim}{B}$$

$$= -\frac{1}{2}\int d\underset{\sim}{x}\;\underset{\sim}{\xi}\cdot\cancel{\frac{1}{c}}\frac{\cancel{c}}{4\pi}\;\underbrace{\hat{z}\times(\hat{z}\times\nabla\psi)}_{-\nabla\psi}\nabla^2\psi$$

$$= \frac{1}{8\pi}\int d\underset{\sim}{x}\;\underset{\sim}{\xi}\cdot\left(\nabla\psi\,\nabla^2\psi\right)$$

$$= \frac{1}{8\pi}\int d\underset{\sim}{x}\;\underset{\sim}{\xi}\cdot\left(\nabla\psi_0\,\nabla^2\tilde{\psi} + \nabla\tilde{\psi}\,\underbrace{\nabla^2\psi_0}_{B_{y0}'}\right)$$

$$\frac{\partial\psi}{\partial t} + \underset{\sim}{v}\cdot\nabla\psi = 0$$

$$\frac{\partial}{\partial t}\tilde{\psi} + \tilde{v}\cdot\nabla\psi_0 = 0 \implies \tilde{\psi} = -\underset{\sim}{\xi}\cdot\nabla\psi_0$$

$$B_y^0 = \frac{\partial}{\partial x}\psi_0 \qquad = -\xi_x\psi_0' = -\xi_x B_{0y}$$

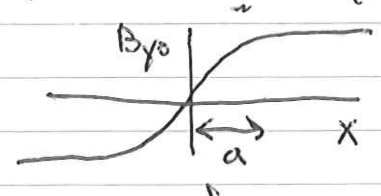
$$= \frac{1}{8\pi}\int d\underset{\sim}{x}\left(-\tilde{\psi}\,\nabla^2\tilde{\psi} + B_{y0}'\,\underbrace{\underset{\sim}{\xi}\cdot\nabla\tilde{\psi}}\right)$$

second order magnetic perturbation

$$= \frac{1}{8\pi}\int d\underset{\sim}{x}\left(|\nabla\tilde{\psi}|^2 - B_{y0}''\,\xi_x\tilde{\psi}\right)$$

$$\tilde{\psi}^{(2)} = -\underset{\sim}{\xi}\cdot\nabla\tilde{\psi}$$

$$= \frac{1}{8\pi}\int d\underset{\sim}{x}\left(|\nabla\tilde{\psi}|^2 + \frac{B_{y0}''}{B_{y0}}\tilde{\psi}^2\right)$$



$$\Delta W_B = \frac{1}{8\pi}\int d\underset{\sim}{x}\left(\underbrace{|\nabla\psi|^2}_{\sim k^2} + \underbrace{\frac{B_{y0}''}{B_{y0}}}_{\sim \frac{1}{a^2}}\tilde{\psi}^2\right)$$

$$\implies k^2a^2 < 1 \text{ for } \Delta W_B < 0$$

Linearized Equations
(slab, $\partial/\partial z = 0$)

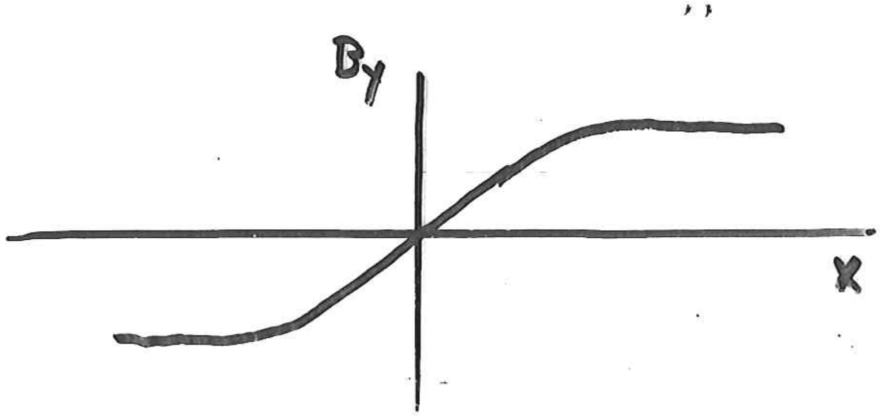
$$\nabla_\perp^2 \tilde{\psi} = \frac{4\pi}{c} \tilde{J}_z$$

$$\gamma \rho \nabla_\perp^2 \tilde{\varphi} = i \underset{\sim}{k} \cdot \underset{\sim}{B} \frac{1}{c} \tilde{J}_z \underbrace{- i k_y \tilde{\psi} \frac{1}{c} J_z'}_{\tilde{B}_x J_z'}$$

$$\gamma \tilde{\psi} - i \underset{\sim}{k} \cdot \underset{\sim}{B} \tilde{\varphi} = c \eta \tilde{J}_z$$

γ = growth rate

$$\underset{\sim}{k} \cdot \underset{\sim}{B} = k_y B_y(x) \quad \Rightarrow 0 \text{ at } x = 0$$



Role of Resistivity

- Ohm's Law

$$\tilde{E}_{\parallel} = \gamma \tilde{\psi} - i k_y B_y(x) \tilde{\varphi} = c \eta \tilde{J}_z$$

- to have reconnection require
$$\tilde{B}_x = -i k_y \tilde{\psi} \neq 0 \text{ at } x = 0$$

$$\Rightarrow \gamma \tilde{\psi} = c \eta \tilde{J}_z \text{ at } x \approx 0$$

$\Rightarrow$ resistivity required for reconnection

$\Rightarrow$ small resistivity $\Rightarrow$ small growth rate

- away from $x \approx 0$ where $B_y \neq 0$ can neglect resistivity

$$\tilde{E}_{\parallel} = \gamma \tilde{\psi} - i k_y B_y(x) \tilde{\varphi} \simeq 0$$

$\Rightarrow$ region around $x = 0$ is a boundary layer of scale size $\Delta \ll a$

Ideal Region ($\tilde{E}_{\parallel} = 0$)
($x > \Delta$)

- resistivity not important and inertia not important

$$\tilde{E}_{\parallel} = 0 \qquad \text{Ohm's Law}$$

$$\tilde{J}_z = \frac{\tilde{\psi}}{B_y} J_z' \qquad \text{Vorticity Eqn}$$

- $\tilde{E}_{\parallel} = 0$ can be rewritten as

$$\frac{\partial \tilde{\psi}}{\partial t} + \tilde{v}_x \frac{\partial \psi}{\partial x} = 0$$

$\Rightarrow$ magnetic flux is frozen into the fluid

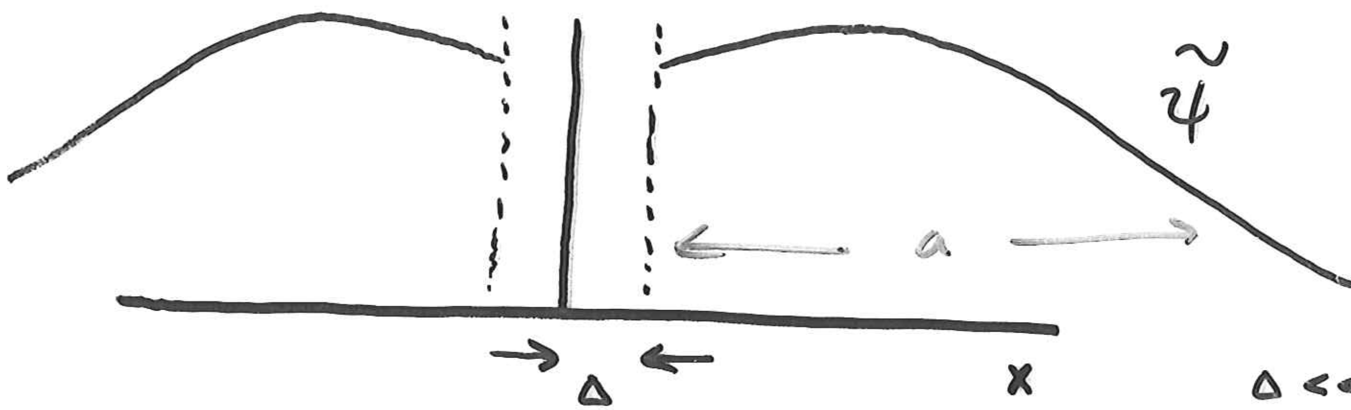
- expression for $\tilde{J}_z$ can be rewritten as

$$\tilde{J}_z = - \xi_x \frac{\partial J_z}{\partial x}$$

$\Rightarrow$ current perturbation arises from displacement of equilibrium current

- Equation for $\tilde{\psi}$ (slab)

$$\nabla_\perp^2 \tilde{\psi} - \frac{B_y''}{B_y} \tilde{\psi} = 0$$



$\Delta \ll a$

- solve equation for $\tilde{\psi}$ subject to $\tilde{\psi}(\infty) = 0$
- solution not valid around $x = 0$
- characterize solution by jump in slope across $x = 0$.

$$\Delta' \equiv \frac{1}{\tilde{\psi}(0)} \left. \frac{\partial \tilde{\psi}}{\partial x} \right|_{-\Delta}^{+\Delta}$$

- can show

$$\Delta' = -8\pi\, \delta W_B \Big/ \int dx\, |\tilde{\psi}|^2$$

$$\Rightarrow \Delta' > 0 \text{ for instability}$$

- for Harris equilibrium (slab)

$$\Delta' a = 2(1 - k_y^2 a^2) / |k_y| a$$

$$B_y = B_0 \tanh\left(\frac{x}{a}\right)$$

$\Rightarrow$ note $\Delta' > 0$ for $k_y a < 1$

$\Rightarrow$ consistent with previous scaling argument

Resistive Region ($\tilde{E}_{\parallel} \neq 0$)
($x \sim \Delta << a$)

- J_z' can be neglected
- $\nabla_\perp^2 \approx \dfrac{\partial^2}{\partial x^2}$
- $\underset{\sim}{k} \cdot \underset{\sim}{B} \simeq k_y B_y' x \Rightarrow$ since $|x| << a$

$$\Delta^2 \tilde{\psi}'' = \left(\frac{4\pi \gamma \Delta^2}{\eta c^2}\right)\left(\tilde{\psi} - \frac{x}{\Delta}\tilde{\varphi}\right)$$

$$\Delta^2 \tilde{\varphi}'' = -\frac{x}{\Delta}\left(\tilde{\psi} - \frac{x}{\Delta}\tilde{\varphi}\right)$$

$$\frac{\Delta}{a} = \left(\frac{\gamma \tau_{Ay}}{k_y^2 a^2}\frac{1}{S}\right)^{1/4}$$

$$\tau_{Ay} = \frac{a}{C_{Ay}} \qquad C_{Ay}^2 = \frac{B_y'^2 a^2}{4\pi \rho}$$

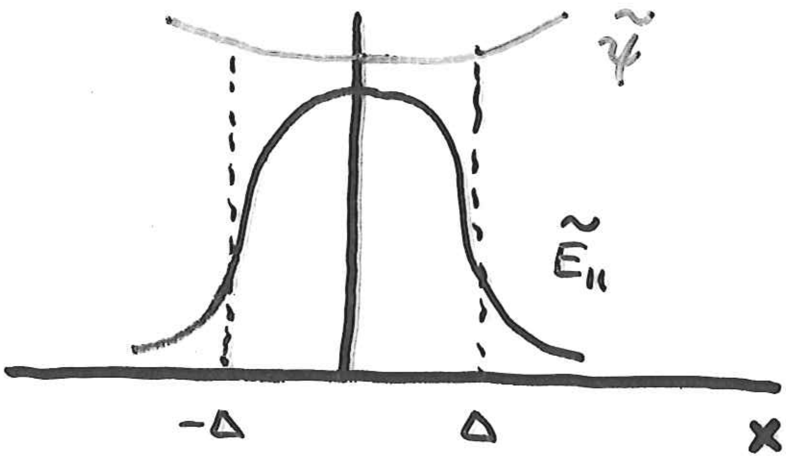
$$\tau_r = \frac{4\pi a^2}{\eta c^2} \qquad S = \frac{\tau_r}{\tau_{Ay}}$$

$\Rightarrow$ Δ is the scale length of the resistive region

$\Delta / a << 1$

- Layer equations have exact solutions

$$\tilde{E}_{\parallel} \sim \tilde{\psi} - \frac{x}{\Delta}\tilde{\varphi}$$



$\tilde{E}_{\parallel} \to 0$ for $|x| > \Delta$

$\Downarrow$

matches MHD solution

- For $\gamma \ll \eta c^2/4\pi\Delta^2$, $\tilde{\psi}$ is nearly constant across region Δ so

$$\tilde{\psi}(x) \simeq \tilde{\psi}(0)$$

$\Rightarrow$ constant ψ approximation

- Integrate $\tilde{\psi}$ eqn across layer to calculate Δ' from resistive region

$$\Delta' \simeq \frac{4\pi\gamma}{\eta c^2}\Delta$$

Dispersion Relation for TM

- Equate Δ' from resistive region to that from ideal region

$$\gamma \tau_{A_y} = S^{-3/5} \left[\frac{\Delta' \Gamma(\frac{1}{4})}{2\pi \Gamma(\frac{3}{4})} \right]^{4/5} (k_y a)^{2/5} \ll 1$$

$$\gamma \sim S^{-3/5}$$

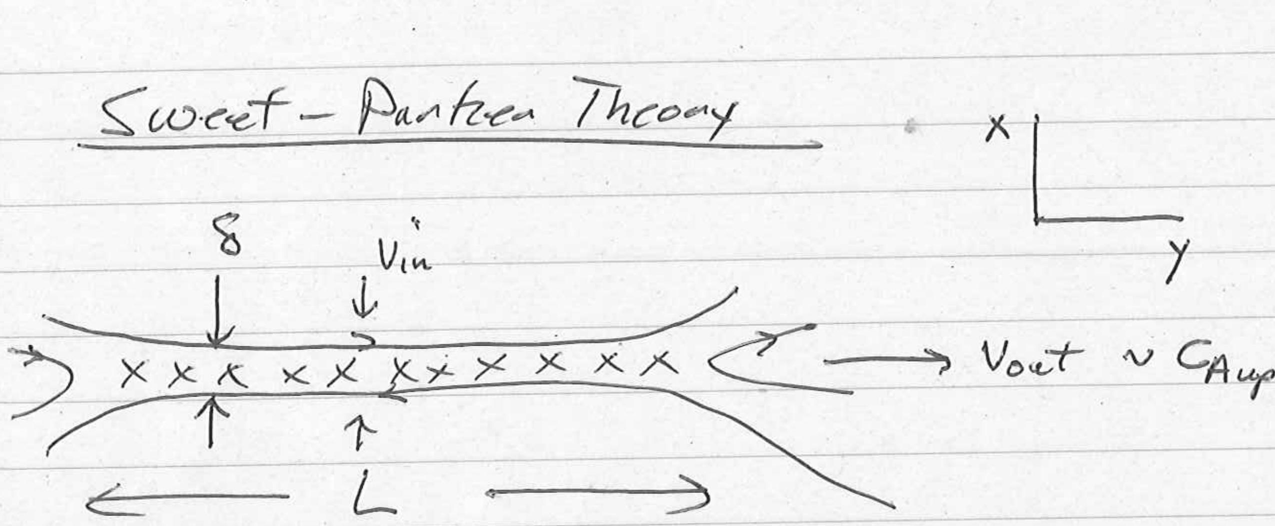
$$\Rightarrow \frac{\Delta}{a} \sim S^{-2/5} \ll 1$$

$\Rightarrow$ linear tearing mode theory tells little about reconnection or island evolution

$\Rightarrow$ theory breaks down when island width w is of order of the tearing layer width Δ.

$\Rightarrow$ very small w !!

Sweet-Parker Theory



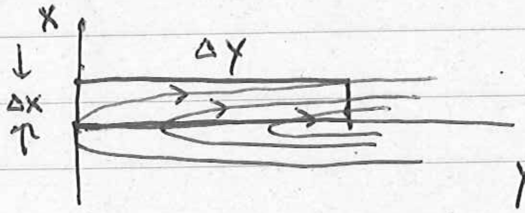
In the S-P mode plasma flows into a ~~macroscopic~~ current layer of macroscopic scale length L and a microscopic width δ which depends on the plasma resistivity

outflow velocity

Easiest to work directly from momentum equation along outflow direction. In steady state

$$\rho_0 V_y \frac{\partial}{\partial y} V_y = \frac{1}{c} J_z B_x = \frac{1}{4\pi} B_x \frac{\partial}{\partial x} B_y$$

$$\frac{\partial}{\partial y} \rho_0 \frac{V_y^2}{2} = \frac{1}{4\pi} B_x \frac{B_y}{\Delta x} \sim \frac{B_y^2}{4\pi \Delta y} \sim \frac{1}{8\pi} \frac{\partial}{\partial y} B_y^2$$



$$B_x \Delta y \sim B_y \Delta x$$

$$\rho_0 V_y^2 \sim \int dy \frac{1}{4\pi} \frac{\partial}{\partial y} B_y^2$$

$$\sim \frac{1}{4\pi} B_y^2$$

$\boxed{V_y \sim C_{A\,up}}$ $\Longrightarrow$ B_y is the magnetic field just upstream of the current layer

$\Longrightarrow$ consistent with earlier squashed bubble model.

Structure of current layer

Use the flux diffusion equation to look at the structure of the current layer along the inflow direction

$$\frac{\partial \psi}{\partial t} + V_x \frac{\partial}{\partial x} \psi = \frac{\eta c^2}{4\pi} \frac{\partial^2}{\partial x^2} \psi$$

In steady state flux is convected into the current layer at a ~~constant~~ constant rate.

$E_{rec} = \frac{1}{c} \frac{\partial \psi}{\partial t}$ is the reconnection electric field

$\Longrightarrow$ rate of flux reconnection

Resistivity again only important in a narrow region of scale length δ. Just upstream of δ, flux is simply convected by flow

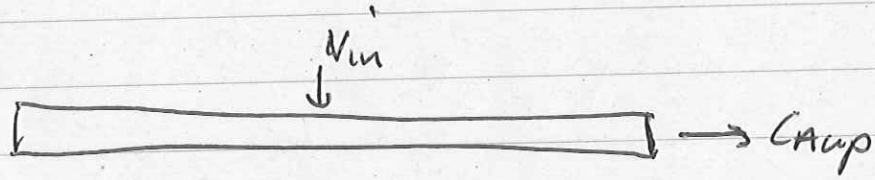
$$\frac{\partial \psi}{\partial t} \sim V_x \frac{\partial \psi}{\partial x} \sim V_{in} \frac{\Delta \psi}{\delta} \sim$$

Within the current layer ($J_z \sim \frac{\partial^2}{\partial x^2}\psi$ large)
flux diffuses due to resistivity

$$\frac{\partial \psi}{\partial t} \sim \frac{\eta c^2}{4\pi} \frac{\Delta\psi}{\delta^2}$$

$$\Longrightarrow \quad V_{in} \cancel{\frac{\Delta\psi}{\delta}} \sim \frac{\eta c^2}{4\pi} \cancel{\frac{\Delta\psi}{\delta^2}}$$

$$\boxed{V_{in} \sim \frac{\eta c^2}{4\pi} \frac{1}{\delta}}$$



On the downstream edge of the current layer

$$\frac{\partial \psi}{\partial t} \sim V_y \frac{\partial}{\partial y}\psi \sim C_{A up} \frac{\Delta\psi}{L}$$

$$\sim V_{in} \frac{\Delta\psi}{\delta}$$

$$\Longrightarrow \boxed{C_{A up}\, \delta \sim L\, V_{in}}$$

$\Longrightarrow$ continuity of flow
since nearly incompressible

$$C_{A up}\, \delta \sim L \frac{\eta c^2}{4\pi} \frac{1}{\delta}$$

$$\frac{1}{\tau_A} \equiv \frac{C_{Aup}}{L} , \quad \frac{3c^2}{4\pi}\frac{1}{L^2} = \frac{1}{\tau_r}$$

$$\left(\frac{\delta}{L}\right)^{?} = \left(\frac{\tau_A}{\tau_r}\right)^{\frac{1}{2}} \ll 1$$

$$V_{in} = \frac{C_{Aup}}{L}\delta \sim C_{Aup}\left(\frac{\tau_A}{\tau_r}\right)^{\frac{1}{2}} \ll C_{Aup}$$

$\Rightarrow$ too slow to explain observations

$\Rightarrow$ e.g. solar flares $L \sim$ ~~[illegible]~~ 10^4 km, $T \sim 100$ eV

$B \sim 200\,G$, $n \sim 10^9/cm^3$

$$S = \frac{\tau_r}{\tau_A} \approx 10^{12} = \text{Lundquist number.}$$

$\tau_A \sim 10$ sec

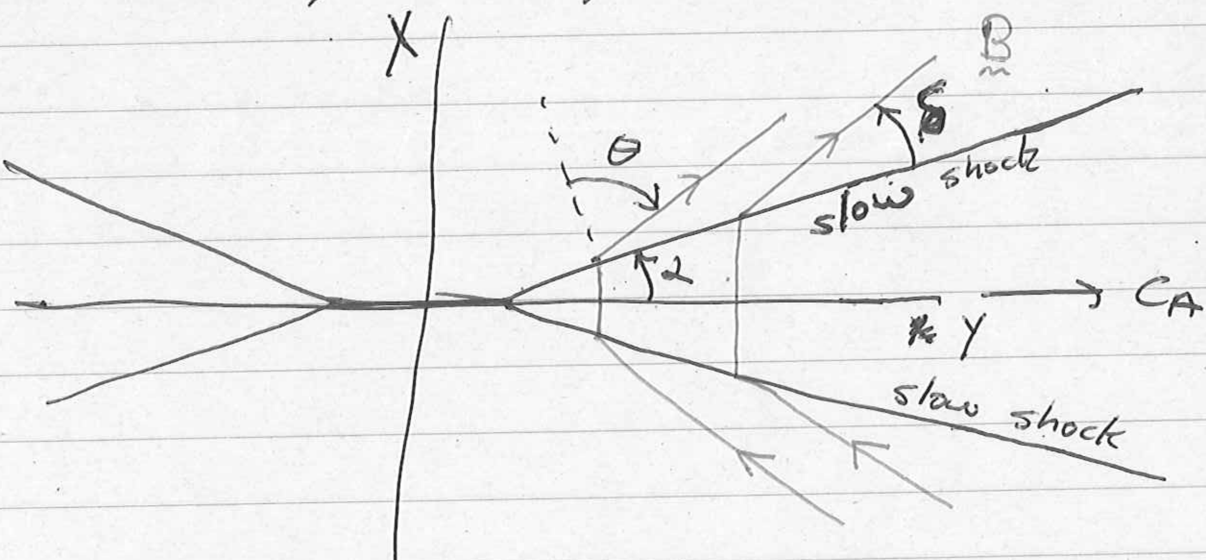
$\tau_{sp} \sim 10\,(10^6)$ sec $\sim 10^7$ s

observed times ~ 100 sec

Petchek Theory

S-P model is slow because of the length of the current sheet. Can shorten current sheet to microscopic scale η and thereby increase reconnection rate?

Petchek proposed that the dissipation region (where η important) was short and that slow shocks bound the outflow region, which opens as a fan



The slow shocks are stationary ~~[illegible]~~ and divert the plasma inflow into the outflow direction. Down stream of the SS. $V_x = 0$ and $B_x = 0$. ~~The jump conditions can be obtained from the reduced equations~~

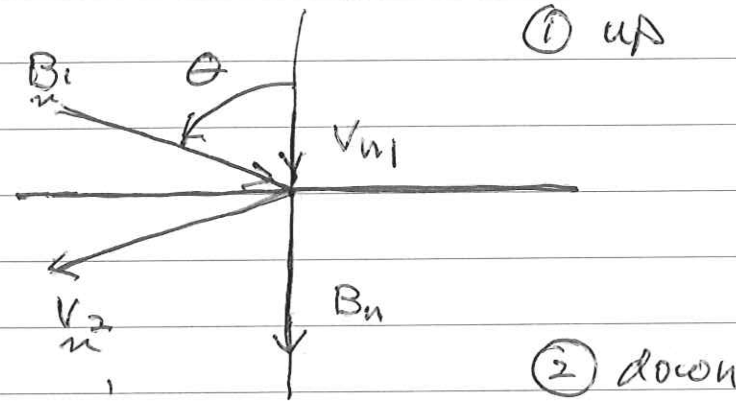
$\Rightarrow$ typically α, δ small and $\theta \sim \frac{\pi}{2}$

The reduced eqns are incompressible

$\Rightarrow$ don't properly describe the slow shock

$\Rightarrow$ return to full MHD equations

Key assumptions:



~~$\Rightarrow$ so~~

$\Rightarrow$ switch off shock

$B_{t2} = 0$

$\Rightarrow$ slow shock

$\Rightarrow$ $V_n \ll$ magnetosonic velocity

$\Rightarrow$ Can use earlier master equation

$\Rightarrow$ instead look at jump conditions

* Tangential force balance (Eq 7)

$$m n_1 V_1 \underset{\sim}{V}_{t2} = -\frac{B_n}{4\pi} \underset{\sim}{B}_{t1}$$

* Tangential E

$$V_1 \underset{\sim}{B}_{t1} = -\underset{\sim}{V}_{t2} B_n$$

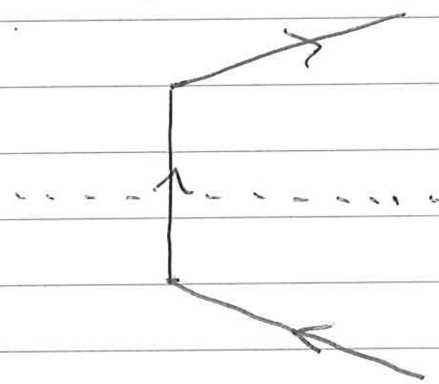
$$\Rightarrow \quad V_1 \underset{\sim}{B}_{t1} = -B_n \left(-\frac{B_n}{4\pi} \frac{\underset{\sim}{B}_{t1}}{m n_1 V_1} \right)$$

$$V_1^2 = \frac{B_n^2}{4\pi m n_1} \quad \Rightarrow M_n^2 = \frac{V_1^2 \, 4\pi m n_1}{B_n^2}$$

$$= 1$$

$$V_{t2} = -\frac{B_{t1}}{\sqrt{4\pi m n_1}} \quad \Rightarrow \quad V_{t2} = C_{At1}$$

$\Rightarrow$ Alfvenic outflow



Kink in B propagates up

$$C_{An} = \frac{B_n}{\sqrt{4\pi m n_1}}$$

Inflow down flows at V_1

$\Rightarrow V_1 = C_{An} \Rightarrow$ stationary

$\Rightarrow$ standing slow shock

Across the shock magnetic energy is dissipated.

$$B_{t1} \rightarrow B_{t2} = 0$$

* pressure balance $\Rightarrow$ inertia small

$\Rightarrow$ sub magnetosonic flow

$$P_2 = \frac{B_{t1}^2}{8\pi}$$

* energy flux

$$n_1 V_1 \tfrac{1}{2} m V_{t2}^2 + \frac{\Gamma}{\Gamma - 1} P_2 \frac{V_1}{r} = \frac{B_{t1}^2}{4\pi} V_1$$

with $r = \dfrac{n_2}{n_1}$

$$\cancel{n_1} V_1 \frac{1}{2} \cancel{m} \frac{B_{t1}^2}{4\pi \cancel{m} \cancel{n_1}} + \frac{\Gamma}{\Gamma-1} \frac{B_{t1}^2}{8\pi} \frac{\cancel{V_1}}{r} = \frac{B_{t1}^2}{4\pi} V_1$$

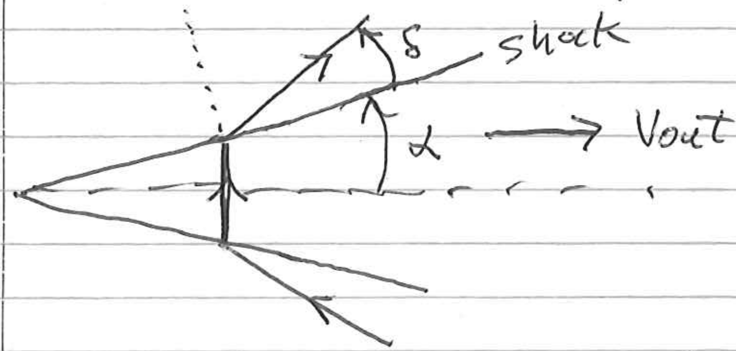
$$V_1 \frac{B_{t1}^2}{8\pi} + \frac{\Gamma}{\Gamma-1} \frac{B_{t1}^2}{8\pi} \frac{V_1}{r} = \frac{B_{t1}^2}{4\pi} V_1$$

kinetic docon | thermal docon | mag up

$$r = \frac{\Gamma}{\Gamma-1} = \frac{5/3}{2/3} = \frac{5}{2}$$

r = compression ratio $= \frac{5}{2}$

Shock angles α, δ



Downstream flow is horizontal
$\Rightarrow$ vertical flow is zero

$$V_{t2} \sin\alpha - V_{n2} \cos\alpha = 0$$

$\alpha \ll 1$

$$V_{t2}\,\alpha = V_{n2} = \frac{n_1 v_1}{n_2} = \frac{v_1}{r}$$

$$\boxed{\alpha = \frac{V_1}{C_{A1}} \frac{1}{r}}$$

$$\delta \sim \frac{B_n}{B_{t1}} = \frac{V_1}{C_{At1}}$$

$$\boxed{\delta \sim \frac{V_1}{C_{A1}}} \quad \Rightarrow \delta \sim \alpha r > \alpha$$