Physics 761 Homework #6 Fall '17

Dr. Drake

1. Write the quasilinear equations for the ion acoustic instability with cold ions and Maxwellian electrons with thermal velocity v_{te} drifting with a velocity $U < 2c_s/3$. What is the maximum phase speed of the unstable waves? Minimum phase velocity?

   Hint: the ions remain linear so no equation for the ion distribution function is needed. You can use the cold ion limit. You need to calculate the growth rate for the instability based on the local slope of the electron distribution function, which will vary with time. This differs from the earlier assignment which simply had a drifting Maxwellian of electrons. The diffusion equation for electrons is unchanged from that of the bump-on-tail problem.

2. Calculate the final electron distribution function after the instability has saturated. What is the minimum velocity of the plateau? The maximum velocity v_0? Why was an upper limit given for U above?

3. Calculate the energy density $|E_k|^2/8\pi$ in the waves as a function of their phase speed after saturation. Show that the wave amplitude goes to zero at either end of the plateau. Sketch the wave energy versus the phase speed of the wave. Hint: the final answer is

$$\frac{|E_k|^2}{8\pi} = \frac{1}{3\sqrt{\pi}} \frac{\Delta k}{k} \frac{\omega_0^2}{\omega_{pi}^2} \frac{v_p^2(v_p - v_g)}{v_{te}^3} \frac{n_0 m_e}{2} (v_p - 3U/2)^2, \tag{1}$$

   where v_p is the wave phase speed, $v_g = d\omega/dk$ is the group velocity and $\Delta k = 2\pi/L$ is the mode spacing.