Thermal Fluctuations

$$\nabla \cdot \underset{\sim}{E} = 4\pi e\,(n_e - n_i)$$

discrete transform

$$\underset{\sim}{E}_K = \int \frac{d^3x}{L^3}\, e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}}\, \underset{\sim}{E}(x)$$

$$\underset{\sim}{E}(\underset{\sim}{x}) = \sum_j e^{i\underset{\sim}{k}_j\cdot\underset{\sim}{x}}\, \underset{\sim}{E}_{k_j}$$

$\sum_j \left(\frac{2\pi}{L}\right)^3 \Rightarrow \int d\underset{\sim}{k}_j$

$$= \sum_j e^{i\underset{\sim}{k}_j\cdot\underset{\sim}{x}} \int \frac{d^3x'}{L^3}\, e^{-i\underset{\sim}{k}_j\cdot\underset{\sim}{x}'}\, \underset{\sim}{E}(x')$$

$$= \int d^3x'\, \delta(\underset{\sim}{x}-\underset{\sim}{x}')\, \underset{\sim}{E}(\underset{\sim}{x}')$$

$$= \underset{\sim}{E}(x)$$

Consider the plasma response to a discrete particle moving with some velocity $\Rightarrow$ dressed particle.

$$k^2 \phi_{k\omega} = +4\pi e\, n_{k\omega} + 4\pi e\, \delta n_{k\omega}$$

$$k^2 \epsilon_{k\omega} \phi_{k\omega} = +4\pi e\, \delta n_{k\omega} = +4\pi e \int d\underset{\sim}{v}\, \delta f_{k\omega}(\underset{\sim}{v})$$

$n_{k\omega}$ response to $\phi_{k\omega}$

$\delta n_{k\omega}$ background source $\Rightarrow$ each particle acts as a source

$$f(\underset{\sim}{x},\underset{\sim}{v},t) = \sum_i \delta(\underset{\sim}{x}-\underset{\sim}{x}_i')\, \delta(\underset{\sim}{v}-\underset{\sim}{v}_i)$$

$$f_k(v,t) = \sum_i \frac{1}{L^3} e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}_i'}\, \delta(\underset{\sim}{v}-\underset{\sim}{v}_i')$$

$$\underset{\sim}{x}_i' = \underset{\sim}{x}_{io} + \underset{\sim}{v}_i' t$$

$$f_{k\omega}(v) = \sum_i \int_{-T/2}^{T/2} \frac{dt}{T} \frac{1}{L^3}\, e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}_{io}}\, e^{i\omega t}\, e^{-i\underset{\sim}{k}\cdot\underset{\sim}{v}_i t}\, \delta(v-v_i)$$

$$= \frac{1}{L^3} \sum_i e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}_{io}}\, \cancel{\delta(\omega - \underset{\sim}{k}\cdot\underset{\sim}{v}_i)}\, \delta(\underset{\sim}{v}-\underset{\sim}{v}_i)$$

$$\frac{\sin \bar{\omega}_i T/2}{\bar{\omega}_i T/2}$$

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{dt}{T}\, e^{i\bar{\omega} t} = 2\,\frac{e^{i\bar{\omega}\frac{T}{2}} - e^{-i\bar{\omega}\frac{T}{2}}}{2i\bar{\omega}T}$$

$$= \frac{\sin \bar{\omega}_i \frac{T}{2}}{\bar{\omega}_i T/2} \qquad \bar{\omega}_i = \omega - \underset{\sim}{k}\cdot\underset{\sim}{v}_i$$

$$k^2 a_{k-\omega} k^2 a_{k\omega}$$

$$\shortparallel$$

$$k^4 |a_{k\omega}|^2 = \frac{1}{L^6}\Big\langle \sum_{i,j} \int d\underset{\sim}{v}\, d\underset{\sim}{v}'\, e^{-i\underset{\sim}{k}\cdot \underset{\sim}{x}_{i0}}\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}_{j0}}\, \delta(\underset{\sim}{v}-\underset{\sim}{v}_i)\,\delta(\underset{\sim}{v}'-\underset{\sim}{v}_j)$$

$$a_{-k-\omega} = a^*_{k\omega}$$

$$\epsilon_{-k-\omega} = \epsilon^*_{k\omega}$$

$$\frac{1}{|\epsilon_{k\omega}|^2}\ \frac{\sin(\bar{\omega}_i T/2)}{\bar{\omega}_i T/2}\ \frac{\sin(\bar{\omega}_j T/2)}{\bar{\omega}_j T/2}\Big\rangle$$

$$\langle\,\rangle = \int \frac{d\underset{\sim}{x}_{i0}}{L^3} \int \frac{d\underset{\sim}{v}_i\, f(\underset{\sim}{v}_i)}{n} \int \frac{d\underset{\sim}{x}_{j0}}{L^3} \int \frac{d\underset{\sim}{v}_j\, f(\underset{\sim}{v}_j)}{n} \cdots\cdots$$

$\sum_{i,j} = \sum_i$ since otherwise $\langle e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}_{i0}}\rangle = 0$

$\sum_i \to N \Rightarrow$ no average over j

$$\int \frac{dx_{i0}}{L^3} \Rightarrow 1$$

$$= \underbrace{\frac{N}{L^6 n}}_{\frac{1}{L^3}} \int d\underset{\sim}{v}\, f(\underset{\sim}{v})\ \frac{1}{|\epsilon_{k\omega}|^2}\ \underbrace{\frac{\sin^2(\bar{\omega}T/2)}{\bar{\omega}^2 T^2/4}}_{\frac{2\pi}{T}\delta(\bar{\omega})}$$

$$\int d\bar{\omega}\, \sin^2 \frac{\bar{\omega}T/2}{\bar{\omega}^2 T^2/4} = \frac{2}{T}\pi$$

$$k^4 |a_{k\omega}|^2 = \frac{2\pi}{L^3 T}\int d\underset{\sim}{v}\, f(\underset{\sim}{v})\ \frac{\delta(\omega - \underset{\sim}{k}\cdot\underset{\sim}{v})}{|\epsilon_{k\omega}|^2}\quad 16\pi^2 e^2$$

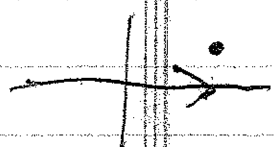
$$\left\langle \frac{|E_{k\omega}|^2}{8\pi} \right\rangle = \frac{2\pi}{L^3 T} \int d\mathbf{v}\, f(\mathbf{v})\, \frac{\delta(\omega - \mathbf{k}\cdot\mathbf{v})}{|\epsilon_{k\omega}|^2}\, \frac{2\pi e^2}{k^2}$$

$$\epsilon = 1 - \frac{4\pi e^2}{k^2 m}\int d\mathbf{v}\, \frac{\mathbf{k}\cdot \frac{\partial f}{\partial \mathbf{v}}}{\omega - \mathbf{k}\cdot\mathbf{v}}$$

$$= 1 - \frac{4\pi e^2}{T k^2}\int d\mathbf{v}\, \frac{\mathbf{k}\cdot\mathbf{v}\, f}{\omega-\mathbf{k}\cdot\mathbf{v}}$$

$$\frac{\partial f}{\partial \mathbf{v}} = -\frac{2\mathbf{v}}{v_t^2} f \qquad = 1 - \frac{k_{De}^2}{k^2}\int \frac{d\mathbf{v}\, f}{n_0}\,\frac{\mathbf{k}\cdot\mathbf{v}}{\omega-\mathbf{k}\cdot\mathbf{v}}$$

$$\mathrm{Im}\,\epsilon = -\frac{k_{De}^2}{k^2}\,\mathrm{Im}\int \frac{d\mathbf{v}\, f}{n_0}\,\frac{\omega}{\omega-\mathbf{k}\cdot\mathbf{v}}$$



$$= \frac{k_{De}^2}{k^2}\,\pi\omega \int \frac{d\mathbf{v}\, f}{n_0}\,\delta(\omega-\mathbf{k}\cdot\mathbf{v}).$$

$$\left\langle\frac{|E_{k\omega}|^2}{8\pi}\right\rangle = \frac{2\pi}{L^3 T}\,\frac{1}{|\epsilon_{k\omega}|^2}\,\frac{\mathrm{Im}\,\epsilon\ \cancel{n_0}}{\cancel{k^2}\ \frac{\cancel{k_{De}^2}}{\cancel{k^2}}\,\pi\omega}\ \frac{T_e}{2\,\pi}$$

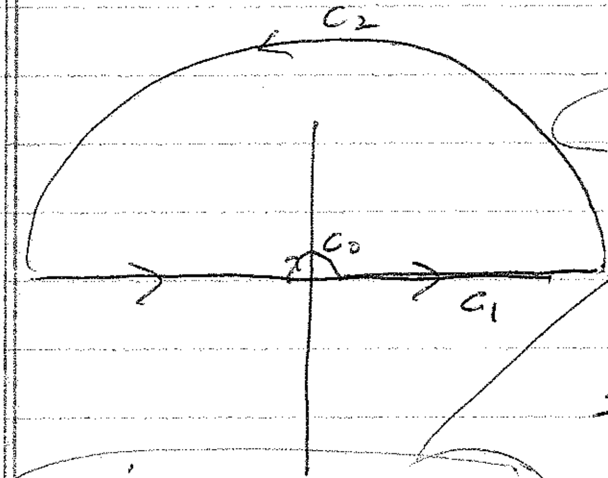
$$\boxed{L^3\left\langle\frac{|E_{k\omega}|^2}{8\pi}\right\rangle = -\frac{2\pi}{T}\,\frac{1}{\pi}\,\mathrm{Im}\left(\frac{1}{\epsilon_{k\omega}}\,\frac{1}{\omega}\right)\frac{T_e}{2}}$$

Want to sum over ω at a given K since k defines a degree of freedom.

~~Improved an Exact Calculation~~

$$\left\langle L^3 \sum_\omega \frac{|E_{k\omega}|^2}{8\pi} \right\rangle = -\frac{1}{\pi}\,\frac{T_e}{2}\int \frac{d\omega}{\omega}\;\mathrm{Im}\,\frac{1}{\epsilon_{k\omega}}$$

$$= -\frac{1}{\pi}\,\frac{T_e}{2}\,\mathrm{Im}\,P\int \frac{d\omega}{\omega}\,\frac{1}{\epsilon_{k\omega}}$$



Note: $\frac{1}{\omega}\,\mathrm{Im}\,\frac{1}{\epsilon_{k\omega}}$ not singular at $\omega = 0$.

$$\oint_C \frac{d\omega}{\omega}\left(\frac{1}{\epsilon} - 1\right) = 0$$

$$\left(\frac{1}{\epsilon} - 1\right) \longrightarrow 0 \text{ as } \omega \to \infty$$

$$\Rightarrow \int_{C_2} (\;) = 0$$

$$\lim_{\omega \to 0} \epsilon_{k\omega} = 1 + \frac{k_D^2}{k^2}$$

$$\int_{C_1} \frac{d\omega}{\omega}\left(\frac{1}{\epsilon} - 1\right) = -\int_{C_0} \frac{d\omega}{\omega}\left(\frac{1}{\epsilon_{k\omega}} - 1\right)$$

$$= +i\pi\left(\frac{1}{\epsilon_{k0}} - 1\right)$$

$$= i\pi\left(\frac{1}{1+\frac{k_D^2}{k^2}} - 1\right)$$

$$= -i\pi\,\frac{1}{1 + k^2/k_D^2}$$

$$\lim_{\omega \to \text{large}} \epsilon_{k\omega} = 1 - \frac{\omega_{pe}^2}{\omega^2}$$

$$\boxed{\left\langle L^3 \sum_\omega \frac{|E_{k\omega}|^2}{8\pi} \right\rangle = \frac{T_e}{2}\,\frac{1}{1 + k^2/k_D^2}}$$

skip 2 Particle Correlation Function

$$\mathcal{J}(\underset{\sim}{x}, \underset{\sim}{v}, t) = \sum_i \delta(\underset{\sim}{x} - \underset{\sim}{x}_i)\,\delta(\underset{\sim}{v} - \underset{\sim}{v}_i)$$

$$\int d\underset{\sim}{x}\, d\underset{\sim}{v}\, \mathcal{J} = N = \text{total \# of particles}$$

$$\langle \mathcal{J} \rangle = f(\underset{\sim}{v})$$

$$\langle \mathcal{J}(x, v, t)\, \mathcal{J}(x', v', t') \rangle = f(\underset{\sim}{v})\, f(\underset{\sim}{v}') + \sum_i \delta(\underset{\sim}{x} - \underset{\sim}{x}_i(t))\, \delta(\underset{\sim}{x}' - x_i(t'))\, \delta(\underset{\sim}{v} - \underset{\sim}{v}_i)\, (\delta(\underset{\sim}{v}' - \underset{\sim}{v}_i))$$

$$\langle\,\rangle_v = \frac{\int dv_i\, f(v_i)(\,)}{\int dv_i\, f(v_i)}$$

~~$x_i(t') = x + v(t - t')$~~ ~~$x_i(t')$~~

$$\langle\,\rangle_x = \frac{\int dx_i}{V}$$

$$x_i(t') = x_i(t) + v_i(t' - t)$$

$$x_i(t') = x' \qquad x_i(t) = x' - v_i(t' - t)$$

$$\langle \mathcal{J}(x, v, t)\, \mathcal{J}(x', v', t') \rangle = f(v)\, f(v') + N \frac{1}{V}\, \delta[x - x' + v(t' - t)]\, \underline{f(v)\, \delta(v - v')}$$

$$n$$

$$\langle \delta \mathcal{J}(x, v, t)\, \delta \mathcal{J}(x', v', t') \rangle = f(v)\, \delta(v - v')\, \delta[x - x' + v(t' - t)]$$