<u>Ionization</u> (H.R. Griem, "Principles of Plasma Spectroscopy" Camb. Univ. Press)

Need to explore under what conditions a neutral gas can become ionized.
As supply energy to the gas, some of the neutral atoms will become ionized. How does this happen?

Two dominant processes

Electron impact ionization

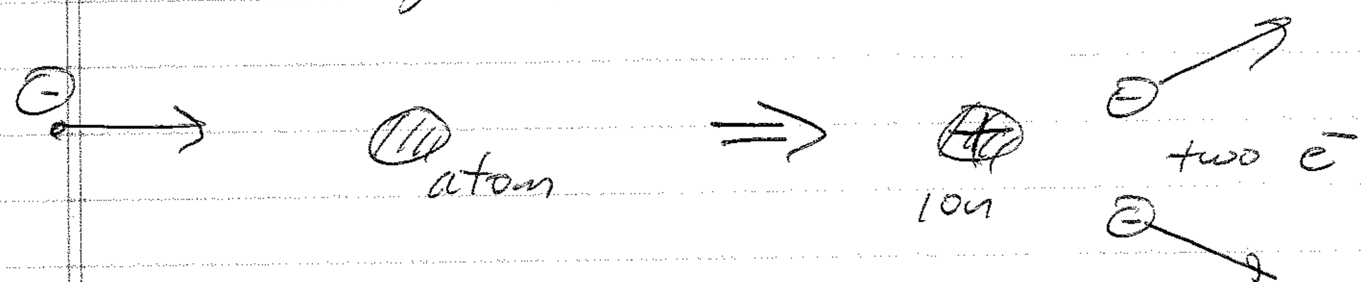
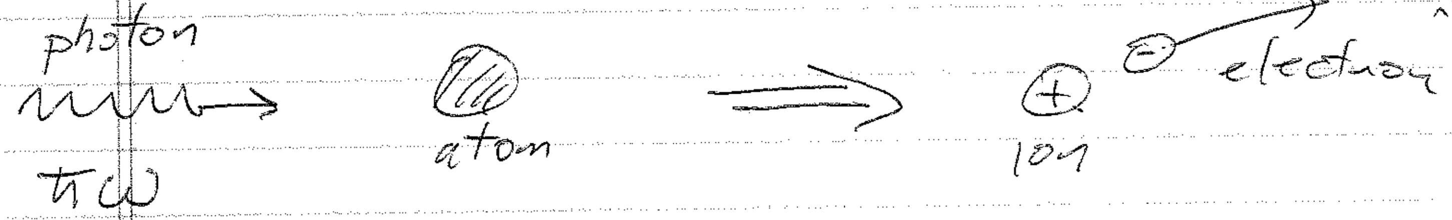
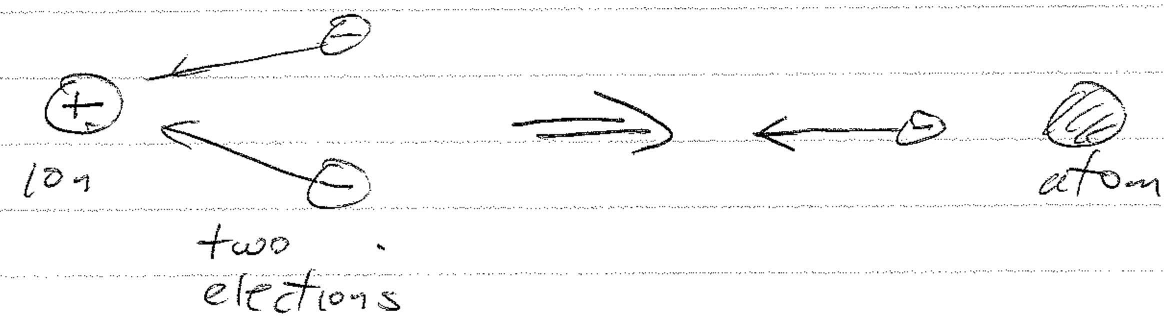


photo ionization

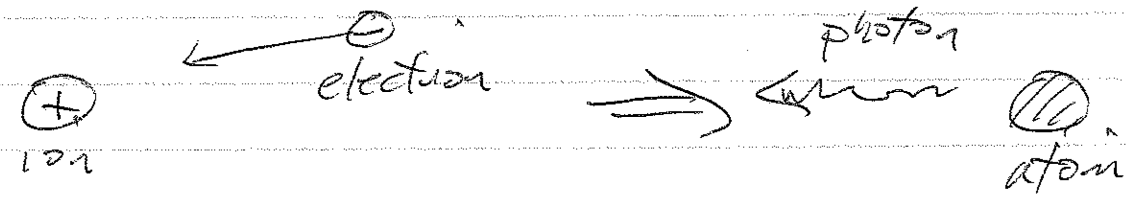


Inverse processes

Three body recombination



Radiative recombination



In thermal equilibrium these processes balance and determine the fraction of atoms that are ionized. Consider H for simplicity.

$$n_p = \text{density of protons}$$

$$n_H = \text{density of H (atoms)}$$

$$f = \frac{n_p}{n_p + n_H} = f(n_t, T)$$

$$n_t = n_p + n_H$$

(assume H_2 disassociated.)

~~Radiative Processes in Astrophysics~~

H.R. Griem "Principles of Plasma Spectroscopy" CUP

Thermal Equilibrium (Saha Equation)

Let $P(N_e, N)$ be the probability that there will be N_e electrons in a gas of N total particles.

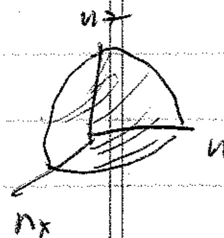
$$P = \frac{z_e^{N_e}}{N_e!}\,\frac{z_p^{N_p}}{N_p!}\,\frac{z_H^{N_H}}{N_H!} \qquad N_e + N_H = N_t$$

partition function:

$$z_e = \sum_E e^{-\frac{E}{T}} \Longrightarrow$$

note this is the classical limit of FD valid for $n a_B^3 \ll 1$

$$n^2 = n_x^2 + n_y^2 + n_z^2$$



$$E_n = \frac{\hbar^2}{2m}\left(\frac{n^2\pi^2}{L^2}\right) \qquad \text{quantized states}$$

$$= \frac{g_e}{8}\int_0^\infty 4\pi n^2\, dn\; e^{-\frac{\hbar^2}{2m}\frac{n^2\pi^2}{L^2}\frac{1}{T}}$$

because only want $n_x, n_y, n_z > 0$

$$= \frac{g_e}{2}\pi\left(\frac{2mL^2T}{\pi^2\hbar^2}\right)^{3/2}\underbrace{\int_0^\infty dp\, p^2 e^{-p^2}}_{\frac{\sqrt{\pi}}{4}}$$

$$z_e = g_e\left(\frac{m_e L^2 T}{2\pi\hbar^2}\right)^{\frac{3}{2}}$$

$$z_p = g_p\left(\frac{m_p L^2 T}{2\pi\hbar^2}\right)^{3/2}$$

$$z_H = g_H\left(\frac{m_p L^2 T}{2\pi\hbar^2}\right)^{3/2} e^{+\frac{E_H}{T}}$$

only ground state. when fraction of ionization is of order unity

$E_H \equiv 13.6\,eV \gg T$

$\Rightarrow$ will be in ground state

Maximize with respect to N_e with

$$N_e = N_p \quad , \quad N_H = N - N_e$$

$$\ln n! \simeq n \ln n - n$$

$$\frac{\partial}{\partial N_e} \ln P = \ln z_e + \ln z_p - \ln z_H$$

$$- \ln N_e - \ln N_e + \ln(N - N_e) = 0$$

$$\frac{z_e z_p}{z_H} = \frac{N_e^2}{N - N_e}$$

$$\left.\begin{aligned} g_e = g_p = 2 \\ g_H = 4 \end{aligned}\right\} \text{spin degeneracy}$$

$$\left(\frac{m_e L^2 T}{2\pi \hbar^2}\right)^{3/2} e^{-\frac{E_H}{T}} = \frac{N_e^2}{N - N_e}$$

Want to get rid of $\hbar$

$$= \frac{n_e^2}{n - n_e} \, L^3$$

$\Rightarrow$ Bohr radius

$$a_B = \frac{\hbar^2}{e^2 m}$$

$\Rightarrow L^3 =$ volume

$$n_e = \frac{N_e}{L^3}$$

$$E_H = \frac{e^2}{2a_B} = 13.6\,\text{eV}$$

$$\hbar^2 = m e^2 a_B = m \, 2 a_B^2 E_H$$

$$\hbar = (2m E_H)^{\frac{1}{2}} a_B$$

$$\frac{1}{n}\left(\frac{\cancel{m_e}\, T}{2\pi\, 2\cancel{m}\, E_H a_B^2}\right)^{\frac{3}{2}} e^{-\frac{E_H}{T}} = \left(\frac{n_e^2}{n-n_e}\right)\frac{1}{n}$$

$$\frac{n_e}{n} = \delta_e = \text{fraction ionization.}$$

$$\boxed{\frac{1}{n a_B^3}\left(\frac{T}{4\pi E_H}\right)^{3/2} e^{-\frac{E_H}{T}} = \frac{\delta_e^2}{1-\delta_e}}$$

Saha Eqn.

$$T \ll E_H \Rightarrow \delta_e \to 0$$

$$T \gg E_H \Rightarrow \delta_e = 1$$

Note that $n a_B^3$ = # of particles in a Bohr sphere is very small

e.g., $\delta_e = \frac{1}{2}$

$$\left(\frac{T}{4\pi E_H}\right)^{3/2} e^{-\frac{E_H}{T}} = \frac{1}{2} n a_B^3 \ll 1$$

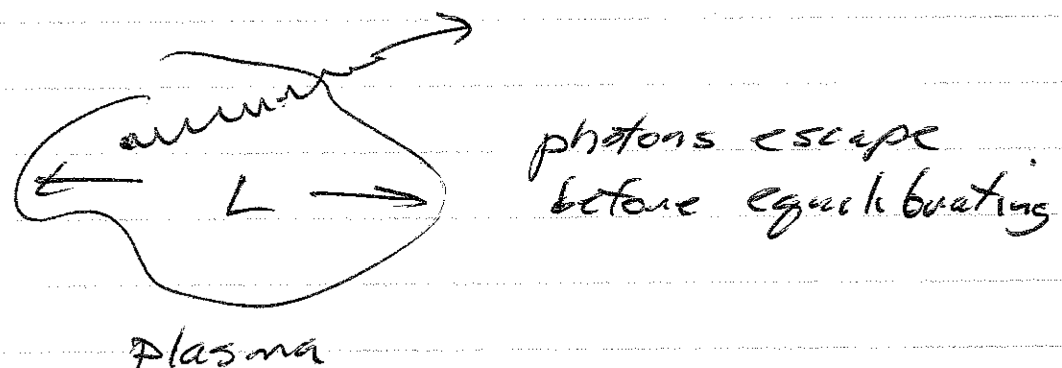
$$\Rightarrow E_H/T \gg 1$$

$$T \ll E_H$$

$\to$ free electrons have many available states

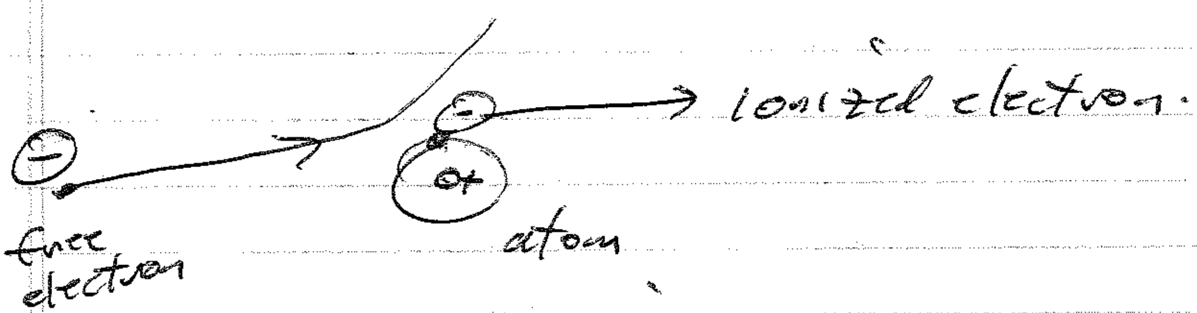
R-G Ch. 10

The Saha calculation requires that the photons have a black body spectrum. This is typically not the case in most plasmas of interest since the photon mean-free-path is too long



Reaction cross sections

Consider collisional ionization



Rate of ionization

$$\langle \sigma n v \rangle_v = \frac{\#\text{ of ionizations}}{\text{sec}}$$

σ = cross section ~ area

σ depends on v

$\langle\ \rangle_v$ = average over electron velocity

$$\langle \sigma n v \rangle = \int d^3v\, f_e(\underset{\sim}{x}, \underset{\sim}{v})\, \sigma v$$

rate of ionization

$$\dot{n}_e = \nu_I n_H \cancel{\simeq n_H \langle \sigma n_e v \rangle}$$

$$= n_H \int d^3v\, \sigma_I v f_e$$

ionization
~~Thomson~~ cross section

$$\sigma_I = 4\pi a_B^2 \left(\frac{E_H}{E}\right)^2 \left(\frac{E}{E_H} - 1\right)$$

$E > E_H$ or electron doesn't have enough energy to ionize

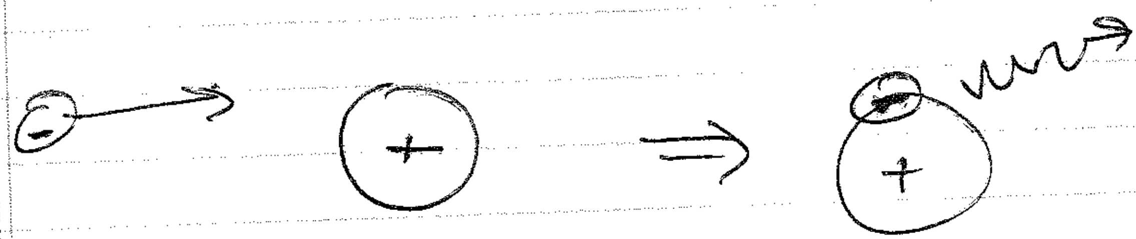
$\sigma_I \sim \frac{1}{E}$ for large E

$\Rightarrow$ electron passes the atom too fast to give ~~much~~ enough energy to ionize so cross section goes down.

$\dot{n}_e = n_H n_e \langle \sigma_I v \rangle \qquad \langle \sigma_I v \rangle = \langle \sigma_I v \rangle (T_e)$

Now need to include recombination

$\Rightarrow$ dominant process is radiative recombination. Since three body recombination has low probability unless the density is very high



Electron is captured by ~~the~~ proton and emits a photon.

$$\frac{dn_e}{dt} = + n_H n_e \langle \sigma_I v \rangle - n_p n_e \langle \sigma_R v \rangle$$

σ_R = radiative recombination cross section.

$$\sigma_R = \sigma_R(T_e)$$

$$\frac{dn_e}{dt} = n_e \left[n_H \langle \sigma_I v \rangle - n_p \langle \sigma_R v \rangle \right]$$

in equilibrium

$$\frac{n_p}{n_H} = \frac{\langle \sigma_I v \rangle}{\langle \sigma_R v \rangle} = \text{function only of } T_e$$

$\Rightarrow$ Coronal equilibrium

25

bottom line

$\Rightarrow$ H becomes ionized around 1 eV

$\Rightarrow$ show figure

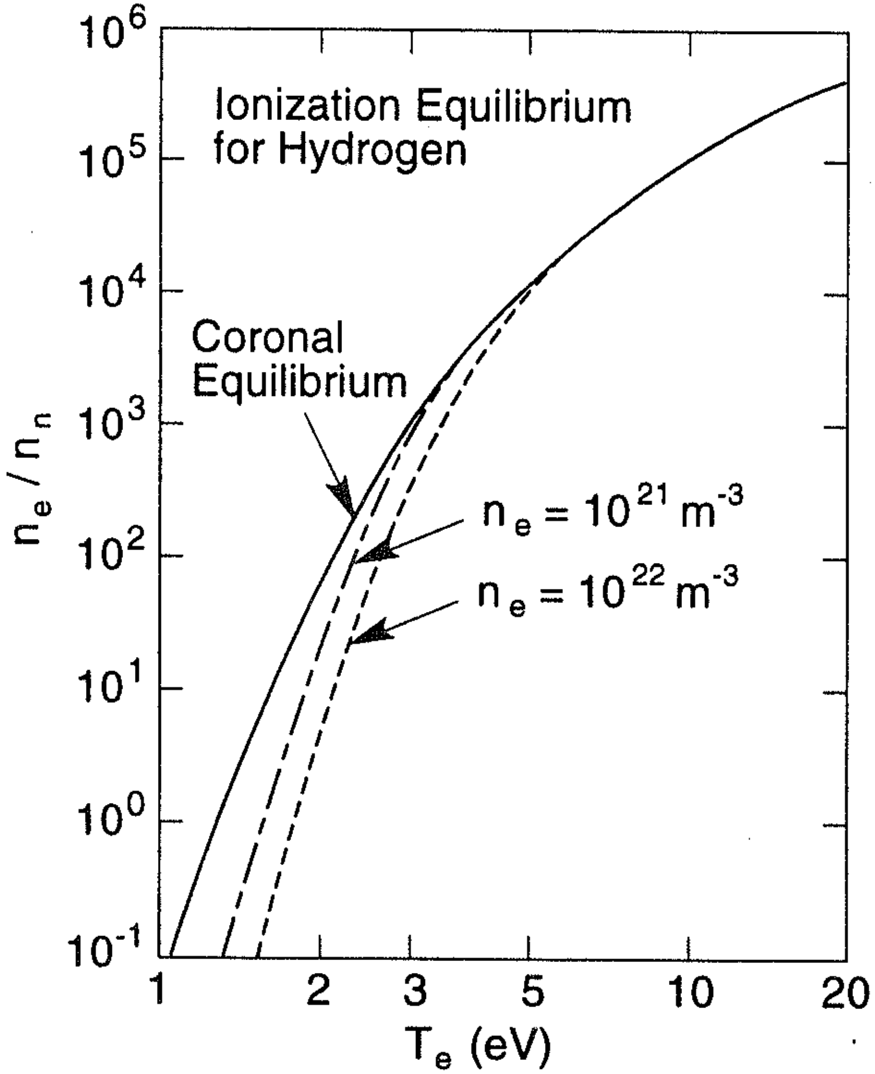


Figure 10.5. Ionization equilibrium for hydrogen in the coronal equilibrium model, and at higher electron densities with three-body recombination included.