Introduction (see Fitzpatrick)

The plasma state - basic parameters.

what is a plasma?

* a collection of charged particles that interact dominantly through the Coulomb and magnetic forces.

examples

Space : Interstellar gas
Stellar and solar coronae
Planetary magneto spheres
Ionosphere
magnetospheres of compact objects

Earth : Flames
Material processing reactors - chips
Light sources
fusion experiments
charged particles beams - accelerators
laser-plasma

⇒ most of the observable universe is in the plasma state.

* characteristics
  - ⇒ typically less dense than condensed matter
  - ⇒ typically hotter than room temperature.
    - ⇒ atoms must be ionized.
  - ⇒ like a gas
  - ⇒ mobile charge carriers
    - ⇒ good conductor

<u>Basic parameters and units (cgs-esu)</u>
(plasma formulary)

density: $n_{e,i}$ cm^{-3} e - electrons
i - ions

# of particles / cm3

charge: statcoulomb

$1 \text{ stat } C = 3 \times 10^9 \ C$

$e = 4.8032 \times 10^{-10} \ sC$

⇒ force

$$|\underset{\sim}{F}_{12}| = \frac{q_1 q_2}{r_{12}^2} \sim \text{dynes}$$

energy ~ ergs

Maxwell's Eqns (esu)

$$\nabla \cdot \underset{\sim}{B} = 0$$

$$\nabla \cdot \underset{\sim}{E} = 4\pi \rho \qquad \rho = \frac{\text{charge}}{\text{vol}}$$

$$\frac{1}{c}\frac{\partial \underset{\sim}{B}}{\partial t} + \nabla \times \underset{\sim}{E} = 0$$

$$\nabla \times \underset{\sim}{B} = \frac{4\pi}{c}\underset{\sim}{J} + \frac{1}{c}\frac{\partial \underset{\sim}{E}}{\partial t}$$

$$c = 3 \times 10^8 \, m/s$$

$\underset{\sim}{B}$ ~ magnetic field ~ gauss
1 tesla = 10^4 gauss

E ~ electric field ~ statvolts/cm

$E \sim B$ ~ same units

$$\underset{\sim}{F} = q\left(\underset{\sim}{E} + \frac{1}{c}\underset{\sim}{v} \times \underset{\sim}{B}\right)$$

$$e = 4.8032 \times 10^{-10} \text{ sC}$$

Potential: φ ~ statvolts
1 statvolt = $\frac{1}{3} \times 10^{-2}$ volts

Temperature e.g., energy per degree of freedom in ~~equil~~.

$$U = \frac{1}{2} k T \qquad \text{thermal equil.}$$

ergs

°K

Boltzmann's const. ergs/…

⟹ express T in units of energy.

$kT \longrightarrow T$ (ergs)

$U = \frac{1}{2} T$

⟹ usually measure T in eV (electron volts)

$1\,eV = 1.6022 \times 10^{-19}\,J = 1.6022 \times 10^{-12}$ ergs

$= 1.16 \times 10^{4}\ {}^{\circ}K$

$1\,eV \approx 10^{4}\ {}^{\circ}K$

Show table of typical plasma parameters for various plasmas.

(5)

APPROXIMATE MAGNITUDES IN SOME TYPICAL PLASMAS

Plasma Type	n cm^{-3}	T eV	ω_{pe} sec^{-1}	λ_D cm	$n\lambda_D{}^3$	ν_{ei} sec^{-1}
Interstellar gas	1	1	6×10^4	7×10^2	4×10^8	7×10^{-5}
Gaseous nebula	10^3	1	2×10^6	20	8×10^6	6×10^{-2}
Solar Corona	10^9	10^2	2×10^9	2×10^{-1}	8×10^6	60
Diffuse hot plasma	10^{12}	10^2	6×10^{10}	7×10^{-3}	4×10^5	40
Solar atmosphere, gas discharge	10^{14}	1	6×10^{11}	7×10^{-5}	40	2×10^9
Warm plasma	10^{14}	10	6×10^{11}	2×10^{-4}	8×10^2	10^7
Hot plasma	10^{14}	10^2	6×10^{11}	7×10^{-4}	4×10^4	4×10^6
Thermonuclear plasma	10^{15}	10^4	2×10^{12}	2×10^{-3}	8×10^6	5×10^4
Theta pinch	10^{16}	10^2	6×10^{12}	7×10^{-5}	4×10^3	3×10^8
Dense hot plasma	10^{18}	10^2	6×10^{13}	7×10^{-6}	4×10^2	2×10^{10}
Laser Plasma	10^{20}	10^2	6×10^{14}	7×10^{-7}	40	2×10^{12}

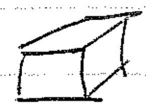
The diagram (facing) gives comparable information in graphical form.[22]

The plasma parameter

How important are individual Coulomb forces versus many particle interactions?

$\Rightarrow$ consider density n and temperature T

$\Rightarrow$ interparticle spacing

r_s {  One particle in cube of side r_s

$$n r_s^3 = 1$$

$$r_s = \frac{1}{n^{1/3}}$$

$\Rightarrow$ energy associated with adjacent particles - Coulomb

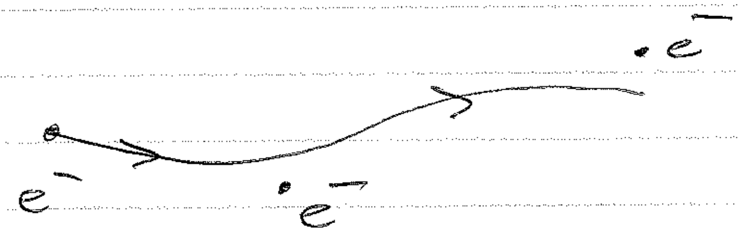
$$\bar{U}_c \sim \frac{e^2}{r_s}$$

$\Rightarrow$ typical thermal energy

$$U_T \sim \frac{3}{2} T$$

$$\frac{\bar{U}_c}{\bar{U}_T} \sim \frac{e^2}{r_s T} \equiv \Gamma$$ the plasma parameter

$\underline{\Gamma < 1}$ nearby particles are weakly correlated.

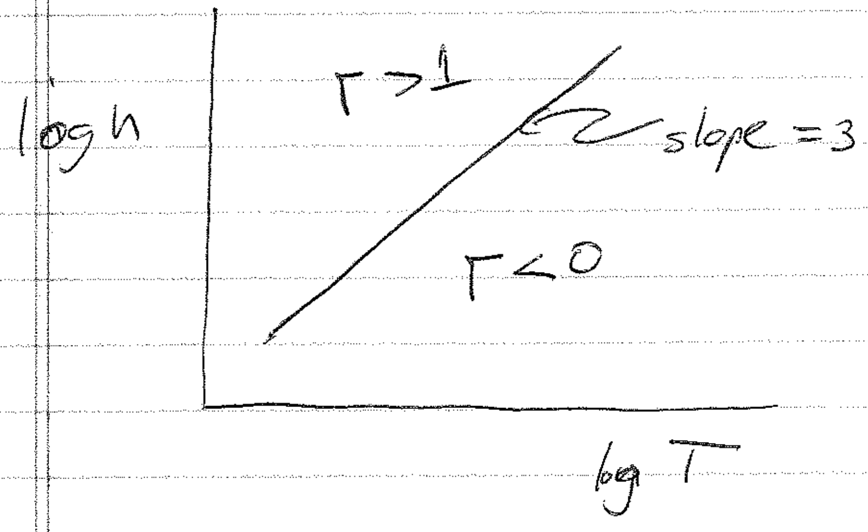


Charged particles move ballistically suffering only slight deflections due to individual interactions

$\underline{\Gamma > 1}$ nearby particles strongly correlated.

$\Gamma > 2$ gas/liquid phase transition

$\Gamma > 180$ liquid crystal phase transition.



$$\Gamma = \frac{e^2 n^{1/3}}{T}$$

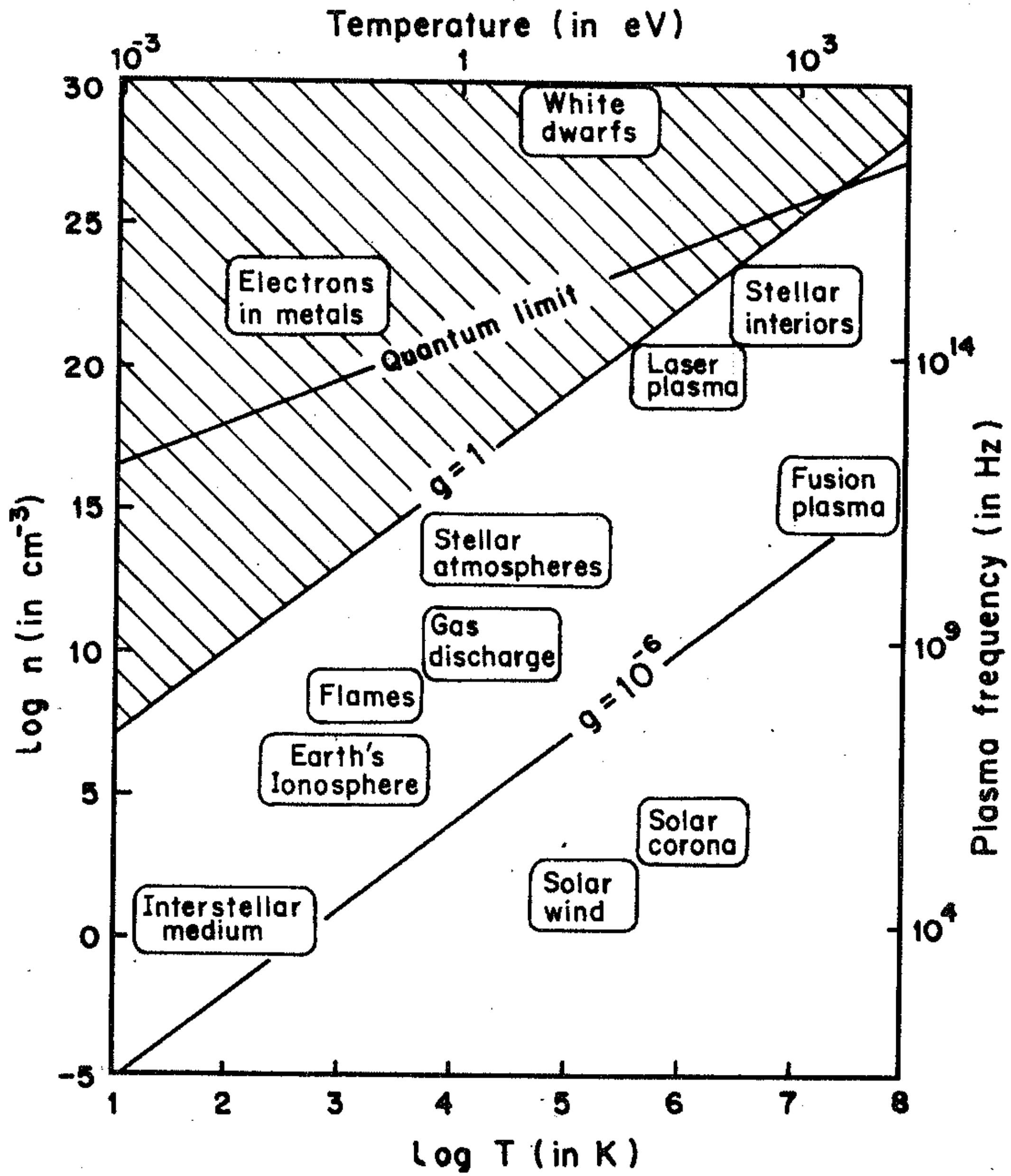


Figure 11.2 Different plasma systems indicated on a plot of the number density n of charged particles against the temperature T.

Define - Debye length

$$\lambda_D^2 = \frac{T}{4\pi n e^2} \Rightarrow T = 4\pi n e^2 \lambda_D^2$$

$$\Gamma = \frac{\cancel{e^2} n^{1/3}}{4\pi n \cancel{e^2} \lambda_D^2} = \frac{1}{4\pi} \frac{1}{(n\lambda_D^3)^{2/3}}$$

$\Gamma < 1 \quad n\lambda_D^3 > 1$

$\Gamma > 1 \quad n\lambda_D^3 < 1$

Show plot 8'

Maxwell - Boltzmann Distribution

In thermal equilibrium (or in many quasi-static processes) finding a particle with energy E is proportional to

$$\sim C\, e^{-(E/T)} \qquad C \sim \text{const.}$$

Take E to be given by kinetic energy and potential ϕ

$$E = \frac{1}{2} m (v_x^2 + v_y^2 + v_z^2) + e\varphi$$

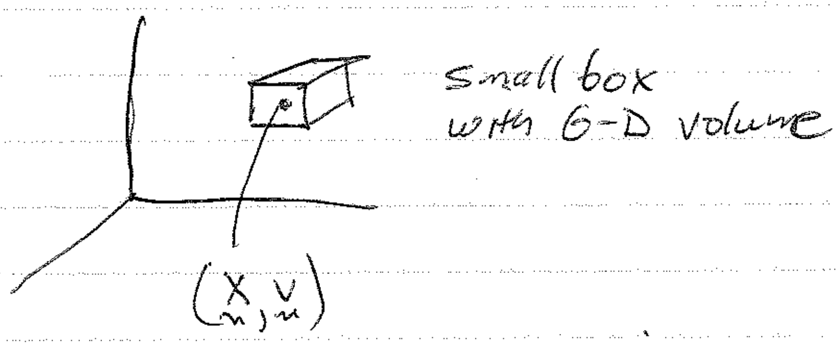
Particle Distribution Function

Describe the distribution of particle velocities in a plasma by the particle distribution function.

$$f_{e,i}(\underset{\sim}{x}, \underset{\sim}{v}, t) \longleftarrow \text{arguments}$$

x, y, z $\quad$ v_x, v_y, v_z

6-dimensional space $\underset{\sim}{x}, \underset{\sim}{v}$



The number of particles in the infinitesimal box of volume $d\underset{\sim}{x}\, d\underset{\sim}{v} \equiv d^3x\, d^3v$

$$\equiv dx\,dy\,dz\,dv_x\,dv_y\,dv_z$$

is dN

$$dN = f(\underset{\sim}{x}, \underset{\sim}{v})\, d^3x\, d^3v$$

Integrating over all of phase space, obtain total # of particles

$$N_{e,i} = \int d^3x \underbrace{\int d^3v \; f_{e,i}(\underset{\sim}{x}, \underset{\sim}{v})}_{n_{e,i}(\underset{\sim}{x}) \;=\; \text{local density}}$$

$$N_{e,i} = \int d^3x \; n_{e,i}(\underset{\sim}{x})$$

In thermal equil.

$$f = \frac{n_0}{(2\pi T/m)^{3/2}} e^{-\left[\frac{1}{2}mv^2 + q\varphi\right]\frac{1}{T}}$$

$$\int_{-\infty}^{\infty} dp \frac{1}{\sqrt{\pi}} e^{-p^2} = \underline{1}$$

local density

$$n = \int d^3v \, f = n_0 e^{-\frac{q\varphi}{T}}$$

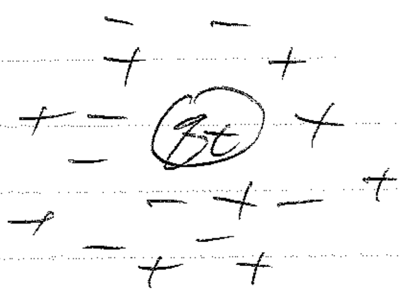
Debye Shielding

Consider an electron-ion plasma which is charge neutral

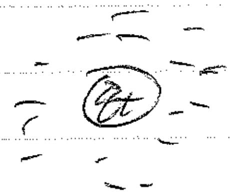
$$q_e n_e \simeq q_i n_i$$

How does the plasma respond to a test charge q_t?

$\Rightarrow$ will tell us about how the charges respond to each other.



Negative charges attract - positive repell.



cloud of negative charge

$\Rightarrow$ shields q_t

$\Rightarrow$ characteristic scale λ_D

Want to quantify this $\Rightarrow \underset{\sim}{E} = -\nabla\varphi$

$$\nabla \cdot \underset{\sim}{E} = -\nabla^2 \varphi = 4\pi \rho$$

$$\rho = n_e q_e + n_i q_i + q_t\, \delta^3(\underset{\sim}{x} - \underset{\sim}{x}_t)$$

$$n_e q_e = n_{eo} q_e\, e^{-q_e \varphi / T} \qquad \varphi > 0 \quad n_e \uparrow$$

$$n_i q_i = n_{io} q_i\, e^{-q_i \varphi / T} \qquad n_i \downarrow$$

Take $\Gamma << 1$ (weakly coupled)

$$\left|\frac{q\varphi}{T}\right| << 1$$

$$-\nabla^2\varphi = 4\pi q_i n_{io}\left(1 - \frac{q_i\varphi}{T}\right) + 4\pi q_e n_{eo}\left(1 - \frac{q_e\varphi}{T}\right) + 4\pi q_t\, \delta^3 (\underset{\sim}{x} - \underset{\sim}{x}_t)$$

charge neutrality

$$q_i n_{io} = q_e n_{eo} \quad \text{\} initial state}$$

$$-\nabla^2\varphi = -\sum_{i,e} \frac{4\pi q_{e,i}^2 n_{e,io}\varphi}{T} + 4\pi q_t\, \delta^3(\underset{\sim}{x} - \underset{\sim}{x}_t)$$

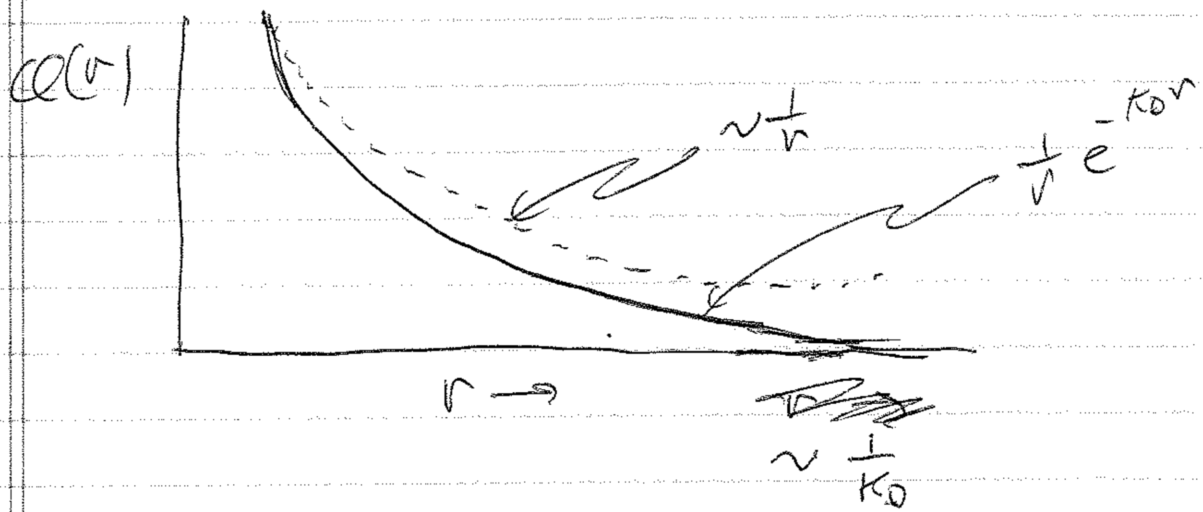
$$k_D^2 = \sum_{e,i} \frac{4\pi q_{e,i}^2 n_{e,io}}{T}$$

$$-\nabla^2\varphi + k_D^2\varphi = 4\pi q_t\, \delta^3(\underset{\sim}{x} - \underset{\sim}{x}_t)$$

take q_t at origin $\Rightarrow$ spherically symmetric

$$-\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial\varphi}{\partial r} + k_D^2\varphi = 4\pi q_t \frac{2\delta(r)}{4\pi r^2}$$

Solution $\varphi(r) = q_t \dfrac{e^{-k_D r}}{r}$



$\Rightarrow$ the test charge is shielded out in a distance

$$r \sim \frac{1}{k_D} \equiv \lambda_D$$

since $n \lambda_D^3 = \dfrac{1}{\Gamma^{3/2}} \gg 1$

$\Rightarrow$ large number of shielding electrons.

$\Rightarrow$ distribution method (f) is ok.

$\Rightarrow$ does not work if $n\lambda_D^3 \sim 1$

Plasma Waves

A plasma can support many types of collective oscillations

e.g. – sound waves as in a normal gas

Can also support unusual waves involving only the motion of electrons

⟹ plasma waves

⟹ longitudinal wave

Let

$$\underline{E} = \mathrm{Re}\left(\hat{x} E_0 e^{-i\omega t}\right)$$

⟹ electron motion along $\underline{E}$

$$\underline{v} = \mathrm{Re}\left(\hat{x}\, v_0 e^{-i\omega t}\right)$$

⟹ neglect ion motion since are too heavy to respond.

Newtons law

$$-i\omega m_e v_0 = -e E_0$$

$$\underline{J} = \mathrm{Re}\left[-n_0 e v_0 e^{-i\omega t}\hat{x}\right]$$

$$\hat{x}\cdot\underbrace{\cancel{(\nabla\times \underset{\sim}{B})}}_{0} = \frac{4\pi}{c}J_x + \frac{1}{c}\frac{\partial}{\partial t}E_x$$

$$0 = \frac{4\pi}{\cancel{c}}(\cancel{+}n_0 e v_0) \cancel{+} i\frac{\omega}{\cancel{c}}E_0$$

$$0 = 4\pi n_0 e\frac{(eE_0)}{i\omega m_e} + i\omega E_0$$

ions $\frac{\omega_{pi}^2}{\omega^2}$

$$E_0\left(1 - \frac{\omega_{pe}^2}{\omega^2}\right) = 0$$

$$\omega_{pe}^2 = \frac{4\pi n e^2}{m_e} \Rightarrow \text{plasma frequency.}$$

note that $\omega_{pi}^2 = \frac{4\pi n e^2}{m_i} \ll \omega_{pe}^2$

$\Rightarrow$ ok to neglect ions.

$$\epsilon \equiv 1 - \frac{\omega_{pe}^2}{\omega^2}$$ = dielectric for high freq. waves

$\Rightarrow \epsilon = 0$

$$\boxed{\omega = \omega_{pe}}$$

no B_0 $\Rightarrow$ $\nabla\times \underset{\sim}{E} = 0$

$\Rightarrow$ electrostatic wave.

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$$\underset{\sim}{E} = -\nabla \varphi$$

$$\Rightarrow \frac{\partial}{\partial x} \neq 0$$

$$\underset{\sim}{E} = \mathrm{Re}\left[\hat{x}\, E_{00}\, e^{ikx - i\omega t}\right]$$

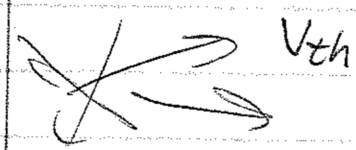
require oscillation in x be small compared to wavelength.

$$k X_{osc} \ll 1$$

$$X_{osc} \sim \frac{V_0}{\omega}$$

$$\frac{k V_0}{\omega_{pe}} << 1 \Rightarrow$$ amplitude not too large.

What about thermal motion.



$$\frac{1}{2} m V_{th}^2 \equiv T$$

want $\frac{V_{th}}{\omega} \sim$ thermal electron displacement small

$$\frac{k V_{th}}{\omega} \sim \frac{k V_{th}}{\omega_{pe}} \sim k\left(\frac{2 V_{th}^2 m_e}{2(4\pi n e^2)}\right)^{\frac{1}{2}}$$

$K = \frac{2\pi}{\lambda}$

$\lambda \gg \lambda_D$ neglect thermal $\sim k \lambda_D \sqrt{2} << 1$