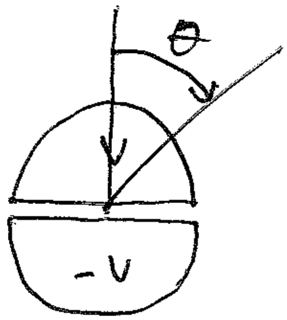
Hwk #10 Solutions Phys. 606

9.3



The radiation fields from the electric dipole are given by Eq. (9.19) in Jackson in terms of the dipole moment

$$\mathbf{p} = \int d\mathbf{x}' \, \mathbf{x}' \, \rho(\mathbf{x}')$$

$\Rightarrow$ by symmetry only p_z

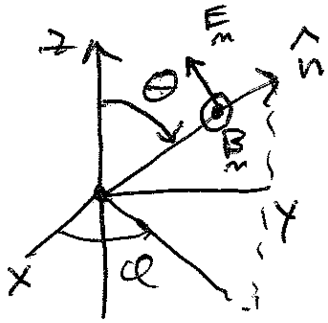
$$p_z = \int d\mathbf{x}' \, r\cos\theta \, \rho(\mathbf{x}')$$

Earlier calculated the fields from a static system. Since $\omega R/c \ll 1$, the fields near the conductors are like the static fields. Can calculate p_z from the static solution. The dipole moment comes from the $l=1$ component

~~Page 99 and 146 of notes~~

$$\Phi \simeq \frac{3}{2} V R^2 \frac{\cos\theta}{r^2} \equiv \frac{1}{4\pi\epsilon_0} \frac{p_z \cos\theta}{r^2}$$

$$\Rightarrow p_z = 6\pi\epsilon_0 V R^2$$



$$\mathbf{B} = \frac{\mu_0 k^2 c}{4\pi}\, \hat{n}\times \mathbf{p}\, \frac{e^{ikr-i\omega t}}{r}$$

$$\boxed{B_\varphi = -\frac{\mu_0 k^2 c}{4\pi}\, p_z\, \frac{\sin\theta \cos(kr-\omega t)}{r}}$$

$$\mathbf{E} = c\, \mathbf{B}\times\hat{n}$$

$$\boxed{E_\theta = c\, B_\varphi}$$

$$\mathbf{S} = \frac{1}{2}\frac{1}{\mu_0}\mathrm{Re}\, \mathbf{E}_0\times\mathbf{B}_0^*$$

$$= \frac{1}{2}\frac{c}{\mu_0}\left(\frac{\mu_0 k^2 c}{4\pi} p_z \sin\theta\right)^2 \frac{1}{r^2}\,\hat{n}$$

$$= \frac{1}{2}\, \frac{c^{\cancel{3}}\, \cancel{\mu_0}\, k^4}{\cancel{16}^{\,4}\, \cancel{\pi^2}}\, \frac{\cancel{36}^{\,9}\, \cancel{\pi^2}\, \epsilon_0^{\cancel{2}}\, V^2 R^4}{\cancel{c^2}}\, \frac{1}{r^2}\,\hat{n}\, \sin^2\theta$$

$$= \frac{9}{8}\, c\,(kR)^4\, \frac{V^2 \epsilon_0}{r^2}\,\hat{n}\,\sin^2\theta$$

$$\boxed{\frac{dP}{d\Omega} = \frac{9}{8}\, c\,(kR)^4\, \epsilon_0 V^2 \sin^2\theta}$$

$$P = \frac{9}{8}\, c\,(kR)^4\, \epsilon_0 V^2 \underbrace{\int_{-1}^{1} d\cos\theta\,(1-\cos^2\theta)}_{\underbrace{2-\frac{2}{3}}_{\frac{4}{3}}}\, 2\pi$$

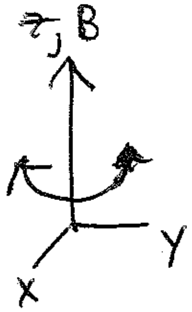
$$\boxed{P = 3\pi\, c\,(kR)^4\, \epsilon_0 V^2}$$

Charged particle of mass m, charge q

14.9

a) non-relativistic

Calculate energy radiated per unit time due to rotation $\perp$ to a uniform magnetic field.



Larmor formula:

$$P = \frac{1}{6\pi} \frac{e^2}{c^3 \epsilon_0} \dot{v}^2$$

$$v_x \sim v_0 \cos \Omega t \qquad \Omega = \frac{qB}{m}$$

$$v_y \sim v_0 \sin \Omega t$$

$$\langle \dot{v}^2 \rangle = \langle \dot{v}_x^2 \rangle + \langle \dot{v}_y^2 \rangle = \Omega^2 v_0^2$$

$$P = \frac{1}{6\pi} \frac{q^2}{c^3 \epsilon_0} \frac{q^2 B^2}{m^2} v_0^2$$

$$\boxed{P = \frac{1}{6\pi\epsilon_0} \frac{q^4 B^2 v_0^2}{m^2 c^3}}$$

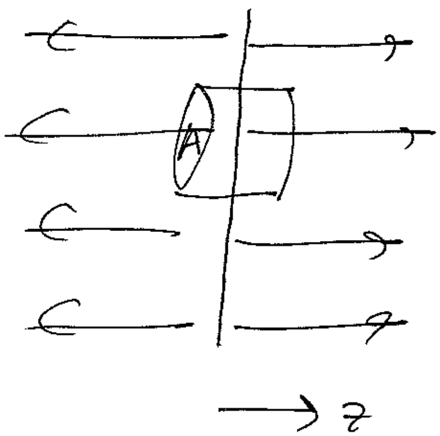
c) $$\frac{dT}{dt} = - \frac{2}{6\pi\epsilon_0} \frac{q^4 B^2}{m^3 c^3} T$$

Let $$\gamma = \frac{1}{3\pi\epsilon_0} \frac{q^4 B^2}{m^3 c^3}$$

$$\boxed{T = T_0 e^{-\gamma t}}$$

3 a) consider $d=0$ $\Rightarrow$ static $\frac{\text{charge}}{\text{area}}$ σ

$\nabla \cdot \underset{\sim}{E} = \frac{\rho}{\epsilon_0}$ Integrate over pillbox

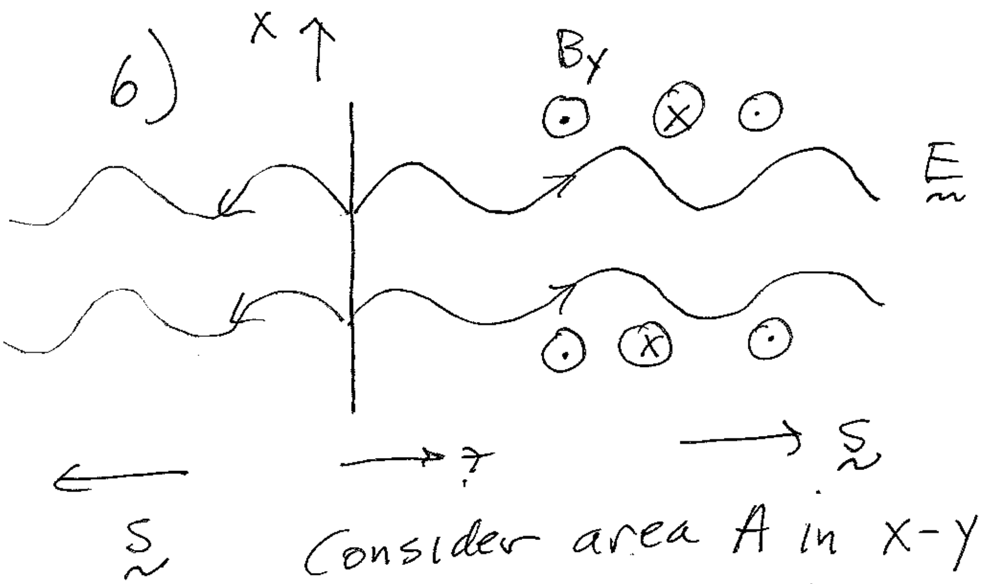


$$2 E_z A = \frac{\sigma A}{\epsilon_0}$$

$$\boxed{E_z = \frac{\sigma}{2\epsilon_0}}$$

$E_z > 0$ for $z > 0$
$E_z < 0$ for $z < 0$
for positive charge

b)



Consider area A in x-y plane
$SA = $ const but A const for plane system so S is const and E, B also const.

c) $\nabla \times \underset{\sim}{B} = \mu_0 \underset{\sim}{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \underset{\sim}{E}$

$\nabla \times \underset{\sim}{E} + \frac{\partial}{\partial t} \underset{\sim}{B} = 0$

$\nabla \times \dot{B} = \nabla \times (-\nabla \times \underset{\sim}{E}) = -\nabla(\cancel{\nabla \cdot \underset{\sim}{E}}) + \nabla^2 \underset{\sim}{E}$ ∂

$= \mu_0 \dot{J} + \frac{1}{c^2} \ddot{E}$

$$\nabla^2 \underset{\sim}{E} = \mu_0 \dot{\underset{\sim}{J}} + \frac{1}{c^2}\ddot{\underset{\sim}{E}} \qquad \underset{\sim}{J} \text{ in } x \text{ direction}$$

$$\nabla^2 E_x = \mu_0 \dot{J}_x + \frac{1}{c^2}\ddot{E}_x$$

$$\rho = \sigma\,\delta(z) \qquad J_x = \rho v_x = \sigma\,\delta(z)\,v_x$$

$$v_x = -i\omega d e^{-i\omega t}$$

$$\nabla^2 E_{x0} + \frac{\omega^2}{c^2}E_{x0} = \mu_0(-\omega^2)\,\sigma\,\delta(z)\,d$$

$$= -\omega^2\mu_0\sigma d\;\delta(z)$$

d) $z \neq 0$

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)E_{x0} = 0 \qquad \cancel{\Rightarrow E_{x0} = E_{x00}\,e^{-\frac{\omega}{c}|z|}}$$

$$E_{x0} \sim e^{\pm i\frac{\omega}{c}z} \quad \Rightarrow \text{want outgoing waves}$$

$$z>0 \qquad E_{x0} = E_{x00}\,e^{i\frac{\omega}{c}z} \qquad \text{since } E_x \sim e^{-i\omega t}$$

$$z<0 \qquad E_{x0} = E_{x00}\,e^{-i\frac{\omega}{c}z}$$

near $z=0$

$$\frac{\partial^2}{\partial z^2}E_{x0} = -\omega^2\mu_0\sigma d\,\delta(z)$$

$$\left.\frac{\partial E_{x0}}{\partial t}\right|_{z=-\epsilon}^{z=\epsilon} = -\omega^2\mu_0\sigma d$$

$$E_{x0}\big| = 0$$

$$\left(\frac{i\omega}{c} + \frac{i\omega}{c}\right)E_{x00} = -\omega^2\mu_0\sigma d$$

$$E_{x00} = i\omega c\mu_0\sigma d$$

$$E_x = i\omega c\mu_0\sigma d\; e^{i\frac{\omega}{c}z}\; e^{-i\omega t} \qquad z>0$$

$$\dot{\underset{\sim}{B}} + \nabla \times \underset{\sim}{E} = 0$$

$$-\cancel{i\omega} B_{y00} + \cancel{i\frac{\omega}{c}} E_{x00} = 0$$

$$B_{y00} = \frac{1}{c} E_{x00}$$

$$\boxed{B_y = \frac{1}{c} E_x}$$

$$S_z = \text{Re} \frac{1}{2} E_{x00} B_{y00}^* \frac{1}{\mu_0}$$

$$= \frac{\text{Re}}{2} \frac{1}{\mu_0} \left(i\omega \cancel{c} \cancel{\mu_0} \sigma d\right)\left(\frac{1}{\cancel{c}}\right)\left(-i\omega c \mu_0 \sigma d\right)$$

$$S_z = \frac{1}{2} \frac{\omega^2 \sigma^2 d^2}{4 \epsilon_0^2} \frac{c \mu_0 \epsilon_0 4}{c^2}$$

$$= 2 \frac{\omega^2 d^2}{c^2} E_z^2 \epsilon_0 c$$

$$\boxed{S_z = 2 k^2 d^2 \left(c \epsilon_0 E_z^2\right)}$$

e) $|z| \ll R$ and $\rho \ll R$

Also require $kR = \frac{\omega}{c} R \gg 1$

so converts to EM wave where the earlier approximation is valid

f) If the plate oscillates in the z direction

$\Rightarrow$ no radiation field