Physics 611 Homework #8 Spring '22
Dr. Drake

1. Jackson 8.3a: don't bother with the δ correction to L.

2. A waveguide consists of a vacuum enclosed by a cylindrical conductor of radius "a" and infinite extent along z. Assume that the conductivity of the metal wall is infinite. The guide is excited with an antenna of frequency ω. The following questions relate to the lowest order TM ($B_z = 0$) mode of the guide.

   (a) Sketch the electric and magnetic field lines for this mode. What are the non-zero compoinents of $\mathbf{E}$ and $\mathbf{B}$?

   (b) Starting with Maxwell's equations derive an equation for the mode. What boundary conditions must be applied at the conducting surface?

   (c) Solve the equation derived in (b) and calculate the velocity at which energy propagates down the guide. What is the limiting form of the velocity when ω is large? Small? Interpret both of these limits. What is the lowest frequency for which energy propagates down the waveguide?

3. A resonance cavity consists of the hollow space between two metallic cylinders of radius a, b with $b > a$ and length L. The cylinders are capped on either end.

   (a) Find the resonance frequency of the lowest order TE mode with $m = 0$. Hint: Your solution will involve derivatives of the Bessel and Neumann functions at the two radial boundaries.

   (b) Sketch the fields.

   (c) Estimate the Q of this mode.