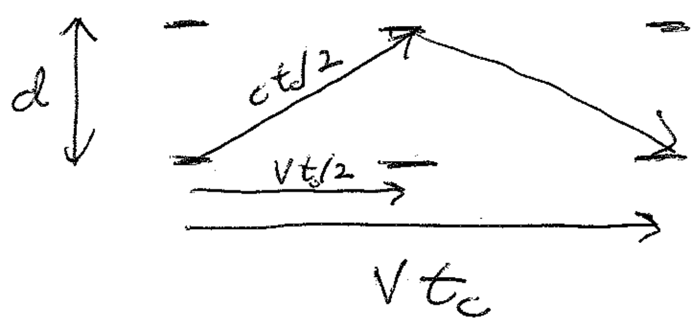
Hwk # 10 Solutions Physics 611

11.4

a) Clock made of flash tube and photocell. In clock frame "ticks" every $2d/c$ seconds $= t_c'$

Clock moves to the right in the lab frame S. Look at trajectory of light



$$d^2 + \frac{v^2 t_c^2}{4} = c^2 \frac{t_c^2}{4} \Rightarrow t_c = \frac{2d}{c(1-\beta^2)^{\frac{1}{2}}}$$

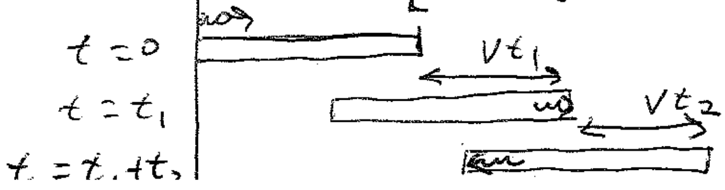
$$\beta = \frac{v}{c}$$

$$t_c = \frac{2d}{c}\gamma = \gamma t_c'$$

$\Rightarrow$ time dilation

b) Use length contraction result. In lab see the clock length d as contracted

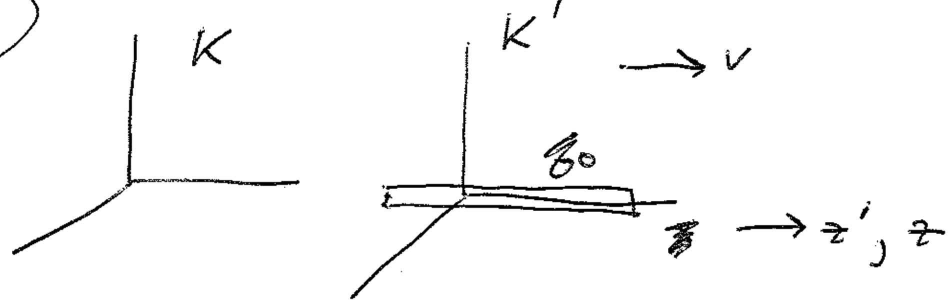
$$L = \frac{d}{\gamma}$$



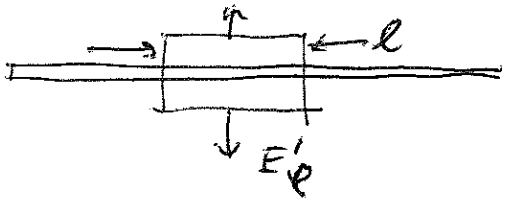
$$vt_1 + L = ct_1 \Rightarrow t_1 = \frac{L}{c(1-\beta)}$$

$$t_2 = \frac{L}{c(1+\beta)} \qquad t_c = \frac{2L}{c}\gamma^2$$

11.13



a) In rest frame of wire



$$\oint \mathbf{E}'\cdot d\mathbf{A}' = \frac{q_0}{\epsilon_0} l = 2\pi\rho' l E'_\rho$$

$$E'_\rho = \frac{q_0}{2\pi\epsilon_0 \rho'} \qquad E'_z = 0 \qquad \mathbf{B}' = 0$$

$\Rightarrow$ no current

In lab frame:

$$E_z = B_z = 0$$

$$\frac{1}{c}\mathbf{E}_\perp = \gamma\left[\frac{1}{c}\mathbf{E}'_\perp - \boldsymbol{\beta}\times\mathbf{B}'_\perp\right]$$

$$\mathbf{B}_\perp = \gamma\left[\mathbf{B}'_\perp + \boldsymbol{\beta}\times\mathbf{E}'_\perp\frac{1}{c}\right]$$

$$E_\rho = \gamma E'_\rho = \gamma\frac{q_0}{2\pi\epsilon_0\rho}$$

$$\mathbf{B}_\perp = \gamma\frac{1}{c}\beta\,\hat{z}\times\hat{\rho}\,E'_\rho = \frac{\gamma\beta}{c}\hat{\varphi}\,E'_\rho$$

$$\mathbf{B}_\perp = \frac{\gamma\beta}{c}\frac{q_0}{2\pi\epsilon_0\rho}\hat{\varphi}$$

$\Rightarrow$ note as $\beta\to 1$ $\quad B_\varphi \sim \frac{E_\rho}{c}$ $\Rightarrow$ light-like

Alternate apprach:

Calculate A'_z, Φ' and $\left(\frac{\Phi}{c}, \underset{\sim}{A}\right)$ is a 4-vector

$$E'_\rho = \frac{\lambda}{2\pi\epsilon_0\rho'} \Rightarrow \Phi' = -\frac{\lambda}{2\pi\epsilon_0}\ln(\rho')$$

$$A_z' = 0 \Rightarrow \text{no current}$$

$$\frac{1}{c}\Phi = \gamma\left[\frac{1}{c}\Phi' + \beta A_z'\right]$$

$$\Phi = \gamma\Phi' \qquad \boxed{E_\rho = \gamma E'_\rho}$$

$$A_z = \gamma\left(A_z' + \beta\frac{1}{c}\Phi'\right)$$

$$A_z = -\gamma\frac{\beta}{c}\frac{\lambda_0}{2\pi\epsilon_0}\ln\rho$$

$$\underset{\sim}{B} = \nabla\times\underset{\sim}{A} \Rightarrow B_\varphi = -\frac{\partial}{\partial\rho}A_z$$

$$\boxed{B_\varphi = \gamma\frac{\beta}{c}\frac{\lambda_0}{2\pi\epsilon_0}\frac{1}{\rho}} \quad \text{as before}$$

b) In K' frame, $\underset{\sim}{J}' = 0$

c) To find $\rho'(\rho')$ must have

$$\int_0^\infty d\rho' \, 2\pi\rho' \, \rho'(\rho') = q_0$$

$$\Rightarrow \boxed{\rho' = \frac{\delta(\rho')}{\pi\rho'} q_0}$$

$(c\rho, \underset{\sim}{J})$ form a 4-vector

$$c\rho = \gamma(c\rho' + \beta J_z')$$

$$\boxed{\rho = \gamma\rho'} \Rightarrow E_\rho = \gamma E_\rho'$$

$$J_z = \gamma(J_z' + \beta c\rho')$$

$$\boxed{J_z = \gamma\beta c \frac{\delta(\rho)}{\pi\rho} q_0}$$

$$\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 I = \mu_0 \gamma\beta c q_0$$

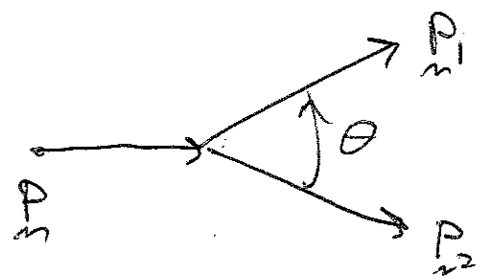
$$B_\varphi = \frac{\epsilon_0\mu_0 \gamma\beta c q_0}{2\pi\rho\epsilon_0} = \frac{\gamma\beta q_0}{c \, 2\pi\epsilon_0 \rho}$$

as in (a)

11.20

a) Lambda particle of $M = 1115$ Gev decays into a pi-meson with $m_2 = 140$ Mev and ~~a proton~~ nucleon of mass $m_1 = 939$ Mev

In the lab frame calculate the opening angle θ between $\underset{\sim}{P}_1$ and $\underset{\sim}{P}_2$.



Use the 4-vector relation

$$P = P_1 + P_2$$

$$P \cdot P = (P_1 + P_2) \cdot (P_1 + P_2)$$

$$= P_1 \cdot P_1 + P_2 \cdot P_2 + 2 P_1 \cdot P_2$$

Can evaluate the product of any pair of 4 vectors in any frame $\Rightarrow$ length is invariant

For the Λ $\quad P \cdot P = M^2 c^2$ etc.

$$P_1 \cdot P_2 = \frac{\varepsilon_1 \varepsilon_2}{c^2} - P_1 P_2 \cos\theta$$

$$\Rightarrow \boxed{M^2 c^4 = m_1^2 c^4 + m_2^2 c^4 + 2\varepsilon_1 \varepsilon_2 - 2 P_1 P_2 c^2 \cos\theta}$$

with P_1, P_2 the magnitudes of the 3 momenta

b) In lab frame the Λ has $\mathcal{E} = 10\,\text{GeV}$

$$\gamma = \frac{\mathcal{E}}{Mc^2} = \frac{10}{1.115} = 8.96$$

lifetime of Λ (in its rest frame) is

$\tau = 2.9 \times 10^{-10}\,s$. In the lab $t = \gamma\tau$

$$L = vt = \frac{\gamma v \tau M}{M} = \frac{p\tau c^2}{Mc^2}$$

$$\mathcal{E}^2 = p^2c^2 + M^2c^4$$

$$L = \sqrt{\frac{\mathcal{E}^2 - M^2c^4}{Mc^2}}\, c\tau = \sqrt{\frac{\mathcal{E}^2}{M^2c^4} - 1}\, c\tau$$

$$L = \sqrt{\gamma^2 - 1}\, c\tau$$

$$= \sqrt{8.96^2 - 1}\; 3\times10^8\,\frac{m}{s}\; 2.9\times10^{-10}\,s$$

$$= 7.75 \times 10^{-1}\,m$$

Calculate the range of angles θ from formula from a). For simplicity measure mass, momenta in energy units

$mc^2 \rightarrow m$ use $\mathcal{E}^2 = p^2 + m^2$

$pc \rightarrow p$

$$\cos\theta = \frac{m_1^2 + m_2^2 - M^2 + 2\varepsilon_1\varepsilon_2}{2\sqrt{\varepsilon_1^2 - m_1^2}\sqrt{\varepsilon_2^2 - m_2^2}}$$

$\Rightarrow$ also have energy cons. in lab frame

$$\varepsilon_1 + \varepsilon_2 = \varepsilon$$

Let f_1 = fraction of energy in ε_1

$1 - f_1$ = " " " " ε_2

$$\cos\theta = \frac{m_1^2 + m_2^2 - M^2 + 2\varepsilon^2 f_1(1-f_1)}{2\sqrt{\varepsilon^2 f_1^2 - m_1^2}\sqrt{\varepsilon^2(1-f_1)^2 - m_2^2}}$$

$\Rightarrow$ vary f_1 to find the range of θ

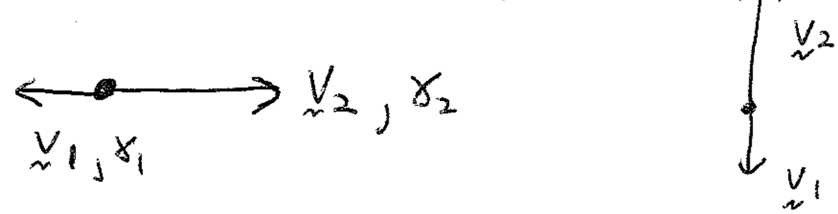
$$\Rightarrow \quad 0 < |\theta| < 5.03^\circ$$

from numerical solution

$\Rightarrow$ some more physics discussion

In the CM coordinate system (where Λ is at rest) can calculate the decay analytically.

$\Rightarrow V_{lab} \gtrsim c$



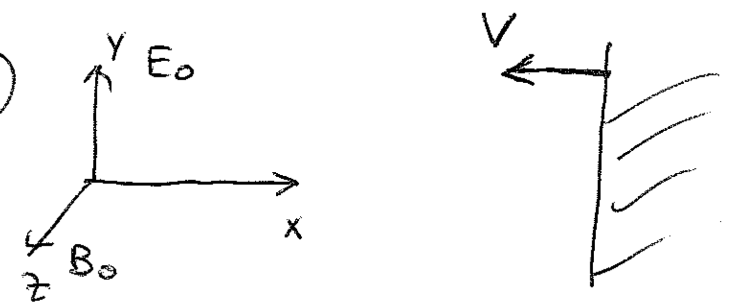
In CM frame $\gamma_2 \sim 1.22$ and $\frac{V_1}{c} \sim \frac{m_2}{m_1} \ll 1$

$\Rightarrow$ small mass has highest ...

The decay in the CM frame can occur in any direction with respect to the Λ motion in the lab frame $\underset{\sim}{V}_{lab}$. If the decay is along $\underset{\sim}{V}_{lab}$ (left picture),

and since both γ_1, γ_2 are small compared with $\gamma_\Lambda = 8.96$, both will be going in the same direction in the lab frame and $\theta = 0$. If γ_Λ were small compared with γ_2, then m_2 could be moving opposite to the original Λ in the lab frame. When the particle decay is as the diagram to the right the θ will be nonzero.

(4)



$$\frac{E_0}{c} = B_0$$

Transform wave to frame of conductor

$$\frac{1}{c} E_y' = \gamma\left(\frac{1}{c} E_y^0 + (\underset{\sim}{\beta} \times B)_y\right)$$

$$= \gamma\left(\frac{1}{c} E_y^0 - \beta_x B_z^0\right) = \frac{\gamma}{c} E_y^0 (1 + \beta)$$

$$B_z' = \frac{1}{c} E_y'$$

transform space time coordinates

$$\cos(kx-\omega t) = \cos\left[k(x'-\beta ct')\gamma - \gamma\omega\left(t'-\frac{\beta}{c}x'\right)\right]$$

$$x = \gamma(x'+\beta_x ct') = \gamma(x'-\beta ct') \quad \text{since } \beta_x = -\beta$$

$$ct = \gamma(ct'+\beta_x x') = \gamma(ct'-\beta x')$$

$$\to \cos\left[\gamma\left(k+\beta\frac{\omega}{c}\right)x' - \gamma(\omega+kc\beta)t'\right]$$

$$\equiv \cos(k'x'-\omega't')$$

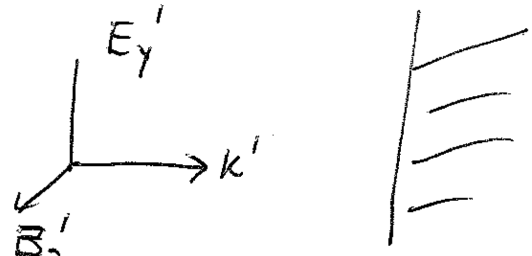
$$\left.\begin{aligned} k' &= \gamma\left(k+\beta\frac{\omega}{c}\right) \\ &= \gamma(1+\beta)k \\ \frac{\omega'}{c} &= \gamma\left(\frac{\omega}{c}+\beta k\right) \\ &= \gamma\frac{\omega}{c}(1+\beta) \end{aligned}\right\} \left(\frac{\omega}{c}, \frac{k}{m}\right) \text{ are a 4-vector.}$$

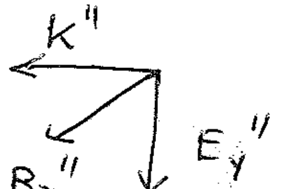
the phase $\varphi = kx - \omega t$

$= -k\cdot x$ is a Lorentz scalar

b) see expressions for E_y', B_z'

c)





reflected wave

$E_y' = -E_y''$ at conducting surface

$\Rightarrow E_y^{total} = 0$

$k_x'' = -k_x'$

Refected wave

$$E_y'' = -\gamma E_y^0 (1+\beta) \cos(k'x' + \omega' t')$$

$$B_z'' = \frac{\gamma}{c} E_y^0 (1+\beta) \cos(\quad)$$

Back to lab frame

$$\frac{1}{c} E_y = \gamma \left(\frac{E_y''}{c} - (\underset{\sim}{\beta} \times \underset{\sim}{B}'')_y \right)$$

$$= \gamma \left(\frac{E_y''}{c} + \beta_x B_z'' \right)$$

$$= \gamma \left(\frac{E_y''}{c} - \beta B_z'' \right)$$

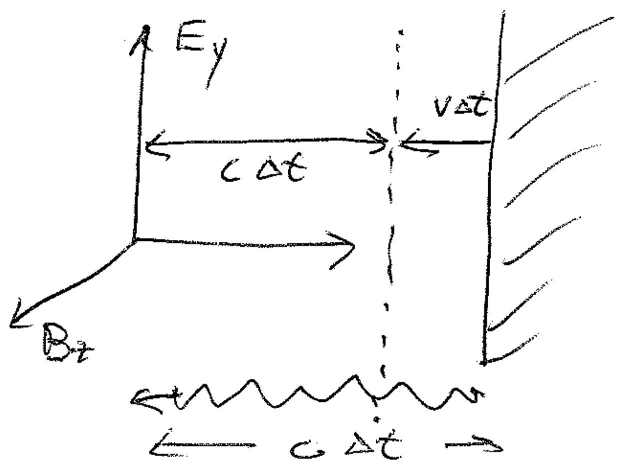
$$= -\frac{\gamma^2}{c} E_y^0 (1+\beta)^2$$

$$E_y = -\gamma^2 E_y^0 (1+\beta)^2 = -E_y^0 \frac{(1+\beta)^2}{1-\beta^2}$$

$$B_y = \frac{1}{c} E_y \qquad = -E_y^0 \frac{1+\beta}{1-\beta}$$

a) calculate the force per unit area as a result of the relection

calculate the momentum of the EM waves striking the plate in a time Δt and the momentum carried off by the reflected waves.



$$\Delta P_{x\,incident} = (c+v)\Delta t\,\frac{1}{2}\epsilon_0 E_0^2 \frac{1}{c}$$

reflected light occupies a region of size $\Delta t(c-v)$

$$\Delta P_{x\,reflect} = -\cancel{(c\Delta t)}\,\frac{1}{2}\epsilon_0 E_y^2 \frac{1}{c}(c-v)\Delta t$$

$$\frac{\Delta P}{\Delta t} = F = \frac{1}{2}\epsilon_0 E_0^2\left[1+\beta + (1-\beta)\gamma^4(1+\beta)^4\right]$$

$$= \frac{1}{2}\epsilon_0 E_0^2\left[1+\beta + \underbrace{\frac{(1+\beta)^4(1-\beta)}{(1-\beta^2)^2}}_{\frac{(1+\beta)^2}{1-\beta}}\right.$$

$$\boxed{F = \epsilon_0 E_0^2\left(\frac{1+\beta}{1-\beta}\right)}$$