Equation Sheet

Maxwell's Eqns

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t}\mathbf{D} \qquad \mathbf{D} = \epsilon \mathbf{E} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

$$\nabla \times \mathbf{E} + \frac{\partial}{\partial t}\mathbf{B} = 0 \qquad \rho_{ind} = -\nabla \cdot \mathbf{P}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu \mathbf{H}$$

$$\nabla \cdot \mathbf{D} = \rho \qquad \mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{A} - \nabla \varphi$$

$\mathbf{S} = \mathbf{E} \times \mathbf{H}$ = Poynting flux $\qquad \mathbf{J} = \sigma \mathbf{E}$ conductor

$\mathbf{P} = \frac{1}{c^2}\mathbf{E} \times \mathbf{H}$ = momentum density $\qquad \delta = \sqrt{\frac{2}{\omega \mu \sigma}}$ = skin depth

wave eqn: $\left[\nabla^2 + \mu \epsilon \omega^2\right]\begin{pmatrix}\mathbf{E} \\ \mathbf{B}\end{pmatrix} = 0 \qquad K = \frac{1}{\delta}(1+i)$

Electrostatics

charge $\varphi(\mathbf{x}) = \dfrac{q}{4\pi\epsilon_0 |\mathbf{x}|} \qquad \mathbf{F} = q\mathbf{E}, \quad \mathbf{E} = -\nabla\varphi$

$u_E = \frac{1}{2}\epsilon |E|^2$

dipole: $\varphi(\mathbf{x}) = \dfrac{1}{4\pi\epsilon_0} \dfrac{\mathbf{p} \cdot \mathbf{x}}{|\mathbf{x}|^3}$

$\mathbf{P} = \int d\mathbf{x}\, \rho(\mathbf{x})\, \mathbf{x}$

$U_q = q\varphi, \quad U_p = -\mathbf{p} \cdot \mathbf{E}$

magnetostatics

$$d\mathbf{B} = \frac{\mu_0}{4\pi} I \frac{d\mathbf{l} \times \mathbf{x}}{|\mathbf{x}|^3} \qquad \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$d\mathbf{F} = I\, d\mathbf{l} \times \mathbf{B} \qquad \mathbf{A} = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \mathbf{x}}{|\mathbf{x}|^3} \text{ mag. dipole}$$

wire: $B = \dfrac{\mu_0 I}{2\pi\rho} \qquad \mathbf{m} = -\frac{I}{2}\oint d\mathbf{l} \times \mathbf{x}$

$u_B = \frac{1}{2\mu}|\mathbf{B}|^2, \quad U_m = -\mathbf{m} \cdot \mathbf{B} \qquad m = IA$

Math eqns

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

$$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 G(\underset{\sim}{x}, \underset{\sim}{x}') = -4\pi\,\delta(\underset{\sim}{x} - \underset{\sim}{x}') \,, \quad G = \frac{1}{|\underset{\sim}{x} - \underset{\sim}{x}'|}$$

Bessel eqn: $\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} + \left(1 - \frac{\nu^2}{x^2}\right) = 0$

$J_\nu(x) \sim x^\nu$ (small x) $\sim \frac{1}{x^{1/2}}\cos\left(x - \nu\frac{\pi}{2} - \frac{\pi}{4}\right)$ (large x)

$N_\nu(x) \sim x^{-\nu}$ (small x) $\sim \frac{1}{x^{1/2}}\sin\left(x - \nu\frac{\pi}{2} - \frac{\pi}{4}\right)$ (large x)

$\sim \ln x \; (\nu = 0)$

$H_\nu^{(1)} = J_\nu + i N_\nu \,, \quad H_\nu^{(2)} = J_\nu - i N_\nu$

Mod. Bessel eqn: $\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} - \left(1 + \frac{\nu^2}{x^2}\right) = 0$

$I_\nu(x) \sim x^\nu$ (small x) $\sim \frac{1}{x^{1/2}} e^{x}$ (large x)

$K_\nu(x) \sim x^{-\nu}$ (small x) $\sim \frac{1}{x^{1/2}} e^{-x}$ (large x)

$\sim \ln x \; (\nu = 0)$

$$\int_0^a d\rho\, \rho\, J_\nu^2\left(x_{\nu n}\frac{\rho}{a}\right) = \frac{a^2}{2} J_{\nu+1}^2(x_{\nu n})$$

Legendre Eqn: $\frac{d}{dx}(1 - x^2)\frac{d}{dx} + \left[\ell(\ell+1) - \frac{m^2}{1 - x^2}\right] = 0$

Waveguides

① TEM mode $\Rightarrow E_z, B_z = 0$

② TE mode $\Rightarrow E_z = 0, B_z \neq 0$

③ TM mode $\Rightarrow B_z = 0, E_z \neq 0$

BC's at surface: $\hat{n}\cdot\nabla B_z = 0$

$E_z = 0$