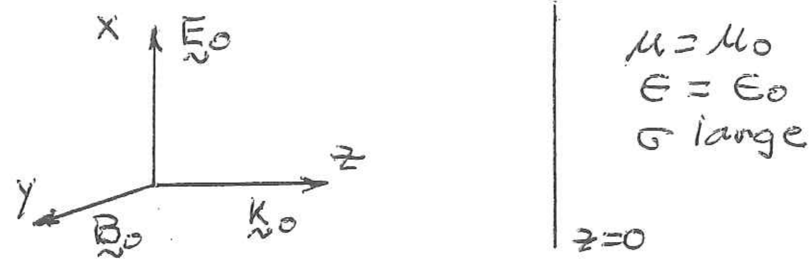


1. (45 points) An electromagnetic wave of frequency ω is normally incident from vacuum onto an infinite conducting surface located at $z = 0$ (see the figure above). Assume that $\mathrm{J} = \sigma \mathrm{E}$ inside of the conductor, where σ is a known constant.

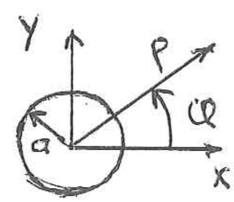
   (a) Sketch the reflected and transmitted waves similar to the sketch of the incident wave, indicating the direction of E and B and the wavevector **k** for each.

   (b) Starting from Maxwell's equations calculate the dispersion relation for the electromagnetic wave in the vacuum region and in the conductor. Assume large but finite conductivity σ to simplify the latter. $(k_o \delta << 1)$

   (c) What are the boundary conditions on the fields at the conducting surface? Be specific for the normal incidence conditions and polarization of the incident wave of the present problem.

   (d) Calculate the complex amplitude of the transmitted and reflected waves.

   (e) Calculate the Poynting vector S of the incident wave, reflected wave and transmitted wave at the boundary ($z = 0$). What fraction of the energy flux is transmitted into the conductor? What happens to this energy?



2. (55 points) An infinitely long cylindrical rod (radius a) of magnetically hard material has a uniform magnetization $\mathrm{M} = M_0 H(a - \rho)\hat{x}$ with H the Heaviside function, which has a value of unity for positive argument and a value of zero for negative argument. The axis of the rod is along z. The solution for B and

H can be written in terms of the magnetic potential $\phi_m(\rho, \phi)$ with ρ and ϕ the radial and angular variables in cylindrical coordinates.

(a) Sketch the magnetic field lines B in the $x - y$ plane.

(b) Derive an equation for ϕ_m and write expressions for H and B in terms of ϕ_m.
Hint: $dH(s)/ds = \delta(s)$ where δ is the Dirac delta function.

(c) What are the conditions that must be satisfied by B and H across the boundary at $\rho = a$?

(d) Write the solutions for ϕ_m for $\rho < a$ and $\rho > a$ and then complete the matching at $\rho = a$.
Hint: What are the basis functions for 2D cylindrical geometry?

(e) Calculate B everywhere.
Hint: See a potentially useful expression below.

(f) An identical rod is placed directly above ($\phi = \pi/2$) at a distance $\rho \gg a$. Estimate the energy per unit length associated with the second rod in the field of the first rod and calculate the force per unit length acting on this new rod. Is attractive or repulsive?
Hint: If you were unable to calculate the magnetic field from the first rod, write your answer in terms of an assumed magnetic field from the first rod.

Possibly useful formula in cylindrical geometry:

$$\nabla\psi = \frac{\partial\psi}{\partial\rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial\psi}{\partial\phi}\hat{\phi} + \frac{\partial\psi}{\partial z}\hat{z}$$