Formulas Exam 1 - electrostatics

Basic eqns - no dielectrics

$\nabla \cdot \underset{\sim}{E} = \frac{\rho}{\epsilon_0}$, $\nabla \times \underset{\sim}{E} = 0$, $\underset{\sim}{E} = -\nabla \Phi$

$\int_S d\underset{\sim}{s}' \cdot \underset{\sim}{E} = \frac{1}{\epsilon_0} \int_V d\underset{\sim}{x}' \rho(\underset{\sim}{x}')$

$\Phi = \frac{1}{\epsilon_0} \int d\underset{\sim}{x}' \frac{\rho(\underset{\sim}{x}')}{|\underset{\sim}{x} - \underset{\sim}{x}'|}$, $\underset{\sim}{F} = q \underset{\sim}{E}$

$\nabla^2 G(\underset{\sim}{x}, \underset{\sim}{x}') = -4\pi \, \delta(\underset{\sim}{x} - \underset{\sim}{x}')$, $G(\underset{\sim}{x}, \underset{\sim}{x}') = \frac{1}{|\underset{\sim}{x} - \underset{\sim}{x}'|}$

Electric field

$\underset{\sim}{E} = \frac{q}{4\pi\epsilon_0} \frac{\underset{\sim}{x}}{|\underset{\sim}{x}|^3}$ point charge

$E = \frac{\sigma}{2\epsilon_0}$ surface charge

$E_n = \frac{\sigma}{\epsilon_0}$ conductor

$\underset{\sim}{E} = \frac{1}{4\pi\epsilon_0} \left(3 \underset{\sim}{p} \cdot \hat{\underset{\sim}{x}} \, \hat{\underset{\sim}{x}} - \underset{\sim}{p} \right) \frac{1}{|\underset{\sim}{x}|^3}$, $\underset{\sim}{p} = \int dx' \underset{\sim}{x}' \rho(\underset{\sim}{x}')$

dipole

$\Phi = \frac{1}{4\pi\epsilon_0} \frac{\underset{\sim}{p} \cdot \underset{\sim}{x}}{|\underset{\sim}{x}|^3}$

Energy

$W = \frac{1}{2} \int d\underset{\sim}{x}' \rho(\underset{\sim}{x}') \Phi(\underset{\sim}{x}')$

$W = \frac{1}{2} \epsilon_0 \int d\underset{\sim}{x}' |\underset{\sim}{E}|^2$

$W = - \underset{\sim}{p} \cdot \underset{\sim}{E}$

Basic eqns - dielectric

$\underset{\sim}{D} = \epsilon_0 \underset{\sim}{E} + \underset{\sim}{P} = \epsilon \underset{\sim}{E}$, $\underset{\sim}{P} = \epsilon_0 \chi_e \underset{\sim}{E}$, $\nabla \cdot \underset{\sim}{D} = \rho$

$\epsilon = \epsilon_0 (1 + \chi_e)$ $\rho_p = - \nabla \cdot \underset{\sim}{P}$, $W = \frac{1}{2} \int dx' \epsilon |\underset{\sim}{E}|^2$

Math eqns

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

$$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Bessel eqn: $\left[x^2\frac{d^2}{dx^2} + x\frac{d}{dx} + (x^2 - \nu^2)\right] y_\nu = 0$

$$J_\nu(x) \sim x^\nu \text{ (small } x) \sim \frac{1}{x^{1/2}}\cos\left(x - \nu\frac{\pi}{2} - \frac{\pi}{4}\right) \text{ large } x$$

$$N_\nu(x) \sim x^{-\nu} \text{ (small } x) \sim \frac{1}{x^{1/2}}\sin\left(x - \nu\frac{\pi}{2} - \frac{\pi}{4}\right) \text{ large } x$$
$$\sim \ln(x) \ (\nu = 0)$$

$$H_\nu^{(1)}(x) = J_\nu + i N_\nu, \quad H_\nu^{(2)}(x) = J_\nu - i N_\nu$$

Mod. Bessel Eqn: $\left[x^2\frac{d^2}{dx^2} + x\frac{d}{dx} - \left(1 + \frac{\nu^2}{x^2}\right)\right] y_\nu = 0$

$$I_\nu(x) \sim x^\nu \text{ (small } x) \sim \frac{1}{x^{1/2}} e^{x} \text{ (large } x)$$

$$K_\nu(x) \sim x^{-\nu} \text{ (small } x) \sim \frac{1}{x^{1/2}} e^{-x} \text{ (large } x)$$
$$\sim \ln(x) (\nu = 0)$$

$$\int_0^a d\rho\, \rho\, J_\nu\left(x_{\nu n}\frac{\rho}{a}\right) J_\nu\left(x_{\nu m}\frac{\rho}{a}\right) = \frac{a^2}{2}\delta_{mn} J_{\nu+1}^2(x_{\nu n})$$

Legendre eqn: $\frac{d}{dx}(1-x^2)\frac{d}{dx} + \left[\ell(\ell+1) - \frac{m^2}{1-x^2}\right] = 0$

$$\int d\Omega\, |Y_\ell^m|^2 = 1, \quad Y_\ell^m = \left[\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}\right]^{1/2} P_\ell^m(\cos\theta)\, e^{im\varphi}$$