Physics 606 | Midterm #1 | Spring 2020

Dr. Drake

1. (50 points) Consider an infinite, hollow cylindrical conductor of radius "a" which is sliced at $z = \pm L$. The segment $|z| < L$ is maintained at a potential V. The remainder of the cylinder is grounded. In the following consider only the region with $\rho < a$. Laplace's equation in cylindrical coordinates is

$$\nabla^2\Phi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial\Phi}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2} = 0.$$

   (a) Sketch the electric field E in the vicinity of the ring of potential V.

   (b) Consider the limit in which $a << L$. Sketch the electric field in the region $\rho < a$ in this limit and sketch E_z as a function of z for $\rho = 0$. What is the scale length over which E_z becomes small outside of the region of potential V. Estimate the maximum value of E_z for $\rho = 0$.

   (c) Derive an exact expression for the potential Φ in the region $\rho < a$ for arbitrary L/a. Your answer will consist of an integral over a wavevector k. You do not need to evaluate this integral.
   Hint: See the various equations at the end of the exam.

2. (50 points) Consider a infinitely long hollow grounded square conductor whose interior is defined by $x \in (0, a)$ and $y \in (0, a)$. A charge q is positioned at the center of the cavity at $z = 0$. Assume that the charge is constrained to remain in this location. The Poisson equation for the potential inside of the cylinder is given by

$$\nabla^2\Phi = -\frac{\varrho}{\epsilon_0}$$

where $\varrho = q\delta(x - a/2)\delta(y - a/2)\delta(z)$ is the charge density.

   (a) Evaluate the potential $\Phi(x, y, z)$ inside the cavity. What is the behavior of the potential for large, positive z?

   (b) A dielectric with $\epsilon > \epsilon_0$ is added to the region inside of the cavity where $z > z_0 > 0$. What is the direction of the force on the particle before the dielectric is inserted? After? Why?

(c) The potential inside of the cavity was observed to change by an amount $\delta\Phi(x, y, z)$ when the dielectric was added. Write an explicit expression for the function form of this potential change for all values of z. Is this potential continuous across $z = 0$? Across $z = z_0$? Outline how you would go about obtaining an expression for the potential $\delta\Phi$. You do not have to do the calculation.

(d) Once you obtained the correction $\delta\Phi$, how would you calculate the force on the charge q?

Bessel's equation is given by

$$\rho \frac{d}{d\rho} \rho \frac{dJ_\nu(k\rho)}{d\rho} + (k^2\rho^2 - \nu^2) J_\nu(k\rho) = 0.$$

The modified Bessel equation is given by

$$\rho \frac{d}{d\rho} \rho \frac{dQ_\nu(k\rho)}{d\rho} - (k^2\rho^2 + \nu^2) Q_\nu(k\rho) = 0.$$