Physics 611 Midterm #1 Spring '22

Dr. Drake

1. (35 points) Consider an infinite conducting plate in the x-y plane at $z = 0$ which has been sliced at $x = \pm a$. The segment $|x| < a$ is maintained at a potential V. The remainder of the conductor is grounded. In the following consider only the region on or above the plate ($z \geq 0$).

   (a) Sketch the electric field $\mathbf{E}$ in the $x - z$ plane. Estimate the magnitude of the electric field just above the center of the strip ($x \sim 0$). Estimate the surface charge density σ in the same region. Over what scale length does the electric field fall off in the z direction?

   (b) Calculate the potential ϕ in the region $z > 0$. Your answer will consist of an integral over the wavevector k. You do not need to evaluate this integral.

   (c) Evaluate the electric field E_z along the symmetry line ($x = 0$) for all z. Hint: you will now have a closed form expression.

   (d) Evaluate electric field and the surface charge density on the conductor at $x = 0$. Are the results consistent with the estimates in part (a)?

2. (35 points) Consider a cylindrical conducting can of radius a with a closed bottom at $z = 0$ and which extends to ∞ in the positive z direction. The can is grounded. A charge q is placed along the axis of the can a distance b from the bottom. Poisson's equation for the potential Φ in cylindrical coordinates is given by

$$\nabla^2\Phi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial\Phi}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2} = -\frac{\varrho}{\epsilon_0},$$

   where ϱ is the charge density.

   (a) Sketch the electric field lines for $b \ll a$ and $b \gg a$..

   (b) What is the direction of the force on the charge q? Estimate this force when $b \ll a$.

   (c) Write an expression for the charge density ϱ associated with q in cylindrical coordinates and show that the volume integral yields q.

(d) Calculate the potential Φ inside of the can.
Hint: See the various equations at the end of the exam.

3. (30 points) Consider a long, cylindrical dielectric rod of dielectric constant ϵ and radius a that is inserted into a initially uniform electric field $\mathbf{E}_0$ in the z direction. The axis of the rod lies along the z axis.

   (a) Calculate the electric field $\mathbf{E}_\epsilon$ inside the rod.
   Hint: This is a very short calculation (only a couple of lines).

   (b) The rod is sliced with the cut perpendicular to the axis and the two ends are separated by a small distance d with $d \ll a$. Calculate the electric field in the gap between the two segments of the rod. Is the electric field in the gap larger or smaller than inside the rod. Ignore end effects associated with the finite value of d/a.
   Hint: If you were unable to calculate $\mathbf{E}_\epsilon$ from part (a), just write your answer in terms of $\mathbf{E}_\epsilon$.

   (c) Make a sketch of the gap region showing the electric field and the induced surface charge density. You don't have to calculate the surface charge density.

   (d) Calculate the energy associated with the electric field in the gap region, the energy change compared with no gap and the force between the two segments.
   Hint: What should the force be if $\epsilon = \epsilon_0$?

Bessel's equation is given by

$$\rho \frac{d}{d\rho} \rho \frac{dJ_\nu(k\rho)}{d\rho} + (k^2\rho^2 - \nu^2) J_\nu(k\rho) = 0$$

and its normalization integral is given by

$$\int_0^a d\rho \rho J_\nu^2(x_{\nu n}\rho/a) = \frac{a^2}{2} J_{\nu+1}^2(x_{\nu n}).$$

Possibly useful information

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$