4. Consider an electromagnetic wave of frequency $\omega$ propagating along the positive $x$ direction in the laboratory reference frame with,

$$E = E_0 \hat{y} \cos(kx - \omega t).$$

An ideal plane conductor moves in the laboratory frame with a velocity $\mathbf{v} = -v\hat{x}$, which may be comparable to the velocity of light.

(a) Transform the space/time dependence of the wave in $S$ to the space/time coordinates of the frame $S'$ moving with the conductor (assume $x = x'$ at $t = t' = 0$). Note that the sign of $v$ is reversed from our usual convention. From the form of the wave in the $S'$ frame, define the local wavevector $k'$ and frequency $\omega'$ in the $S'$ frame. How do $k$ and $\omega$ transform under a Lorentz transformation? How does the phase of the wave $kx - \omega t$ transform? Why?

(b) Calculate the field of the right propagating wave in the $S'$ frame.

(c) The wave reflects from the ideal conductor. Evaluate the reflected wave in the $S'$ frame and then transform the reflected wave back to the $S$ frame. What happens to the wave under reflection?

(d) Calculate the force per unit area on the conductor as a result of the reflection.