Physics 373 | Sample Midterm #2 | Spring '16
Dr. Drake

1. (a) Find the solutions of the following equation,

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - \alpha y = 0.$$

Show that the solutions are linearly independent. What are the solutions when $\alpha = -1$?

(b) The Spherical Bessel Equation is given by

$$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + [x^2 - n(n+1)]y = 0.$$

What is the form of the differential equation very close to the singularity at $x = 0$? What is the lowest order behavior of the solutions near $x = 0$? Show that these two solutions are linearly independent.

(c) Take the Laplace transform of $sin(\omega t)$. If $\omega = \omega_0 + i\gamma$ is complex such that ω_0 and γ are real, what is the requirement on p for the Laplace transform to be valid?

(d) Legendre's equation is given by

$$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + [\alpha(\alpha+1)]y = 0.$$

What is the lowest order behavior of the solutions of this equation near $t = x + 1 = 0$?

2. A differential equation is given by

$$(1 + x^2)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0.$$

Calculate the series solutions of this equation around $x = 0$. What is the radius of convergence of this series?

3. Consider the differential equation

$$\frac{d^2y}{dt^2} + y = e^{-t},$$

where the source on the right side of the equation is turned on at $t = 0$ and with the initial conditions $y(0) = 0$ and $dy(0)/dt = 0$.

(a) Calculate the homogeneous and particular solutions of the equation. Construct a linear combination of these to satisfy the initial conditions at $t = 0$.

(b) Take the Laplace transform of the equation to obtain an equation for $Y(p)$. Write down the inverse transform for $y(t)$ and evaluate the integral in the complex p plane to obtain $y(t)$. It should agree with your earlier solution based on the particular and homogeneous solutions.