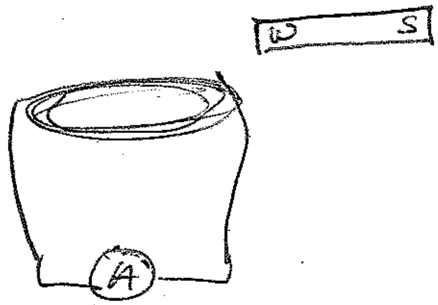
Electromagnetic Induction

Consider a coil of wire attached to an ~~ampmeter~~ ammeter which measures current.



Take a permanent magnetic move it toward coil

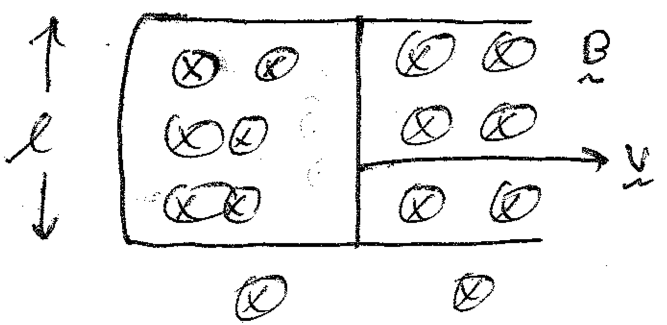
$\Longrightarrow$ see current

Move away see opposite current.

$\downarrow$ A current is is induced in the coil.

Motional EMF.

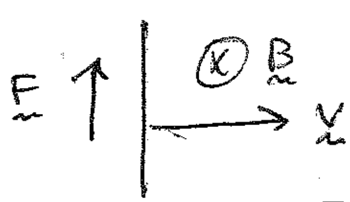
Consider a sliding conductor in a magnetic field as shown.



The sliding wire moves with a velocity v.

pretend ~~the~~ the charges are positive

$\Longrightarrow$ force on ~~electrons~~ free charges in wire



This force can move the charges up a potential,

$F = qvB$ $\Longrightarrow$ acts like an Emf

Work done on a charge q moving ~~across~~ along the conductor

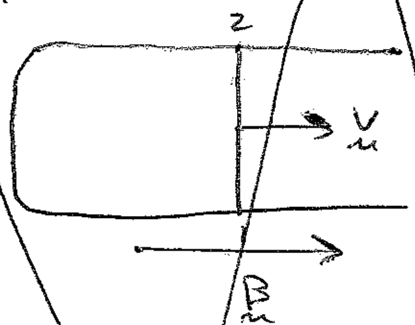
$$W = F\ell = qvB\ell$$

$$W = q\mathcal{E}$$

$$\boxed{\mathcal{E} = vB\ell}$$ motional electromotive force

discard

Can generalize this result to case where $\underset{\sim}{B}$ is not $\perp$ to motion.



In this case $\underset{\sim}{v} \times \underset{\sim}{B} = 0$ so no emf.

$\underset{\sim}{F} = q\,\underset{\sim}{v} \times \underset{\sim}{B}$ must lie along moving wire.

$$\mathcal{E} = \frac{1}{q}\int_1^2 \underset{\sim}{F} \cdot d\underset{\sim}{\ell} = \int_1^2 \underset{\sim}{v} \times \underset{\sim}{B} \cdot d\underset{\sim}{\ell}$$

<u>example</u>

$v = 1\,\mathrm{m/s}$
$\ell = .1\,\mathrm{m}$ $\qquad \mathcal{E} = .1$ volt
$B = 1\,\mathrm{T}$

If $R = 1\,\Omega$

$I = .1\,\mathrm{A}$

What's the power dissipated?

$$\mathcal{E} = vB\ell = IR$$

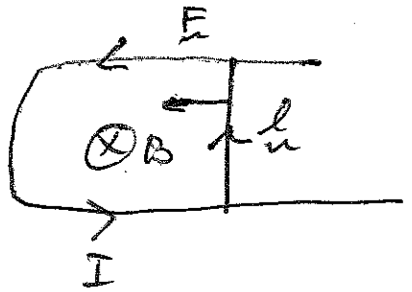
$$I = \frac{vB\ell}{R}$$

$$P = I^2R = \frac{v^2B^2\ell^2}{R^{\cancel{2}}}\cancel{R} = \frac{v^2B^2\ell^2}{R}$$

Where does the energy come from

$\Rightarrow$ current flowing through wire so must be a force

$$\underset{\sim}{F} = I\underset{\sim}{\ell} \times \underset{\sim}{B} \quad \Longrightarrow \text{inhibits motion of wire.}$$



to keep the wire moving we must apply the force.

$$F = I\ell B$$

we are doing work at the rate

$$P = Fv = I\ell B v$$

$$= \left(\frac{vB\ell}{R}\right)\ell B v = \frac{B^2v^2\ell^2}{R}$$

$\Longrightarrow$ power dissipated in the resistor is supplied by us as we push the wire across the magnetic field.

8Hgot here

$$\mathcal{E} = vBl$$

What is vl?

$\Rightarrow$ rate of change of the area of the loop.

$$vl = \frac{dA}{dt}$$

$$\mathcal{E} = B\frac{dA}{dt} = \frac{d}{dt}(BA)$$

$A = lx$

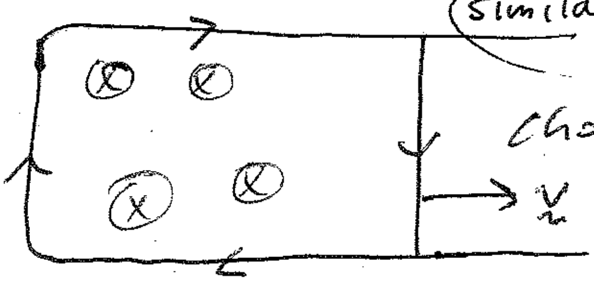
$\frac{dA}{dt} = l\frac{dx}{dt} = lv$

BA = flux of the magnetic field through the loop.

Define $\Phi = \int \underset{\sim}{B} \cdot d\underset{\sim}{A}$

valid even if $\underset{\sim}{B}$ is not a constant over loop.

similar to electric field flux $\int \underset{\sim}{E} \cdot dA$



choose loop as shown

$d\underset{\sim}{A}$ is into the page (use RH rule)

$\Phi = \int \underset{\sim}{B} \cdot d\underset{\sim}{A}$ = flux through loop.

$$\boxed{\mathcal{E} = -\frac{d}{dt}\Phi}$$

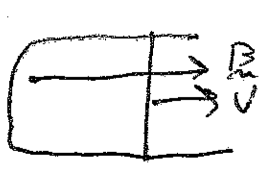
Faradeys Law

$\mathcal{E}$ in this case is negative

$\Rightarrow$ EMF is opposite to the sense of the path around the loop.

$\Rightarrow$ reverse direction of motion of wire $\Rightarrow$ $\mathcal{E}$ reverses

$\Rightarrow$ valid even if $\underset{\sim}{B}$ is not $\perp$ to the loop.



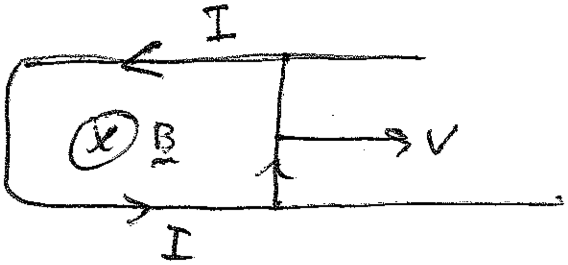
$\int \underset{\sim}{B} \cdot d\underset{\sim}{A} = 0$

Emf is generated only if the magnetic flux through the loop changes with time

Lenz's law

How do you tell in which direction have the induced EMF?

As move the wire to the right the magnetic flux linking the loop increases.

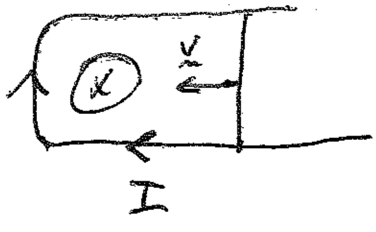


Current flows as shown.

The direction of the magnetic field produced by the current inside of the loop is outward.

This magnetic field tends to ~~cancel the~~ reduce $\underset{\sim}{B}$ $\Rightarrow$ tends to try to prevent the flux inside the loop from increasing.

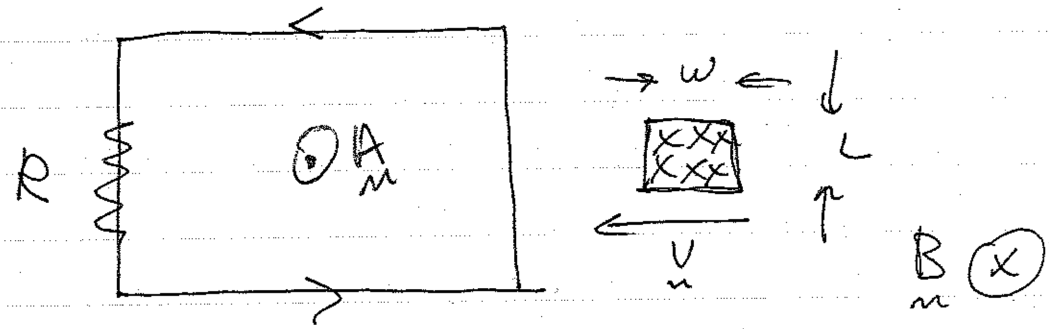
$\Rightarrow$ the currents flows so as to oppose the change in flux.



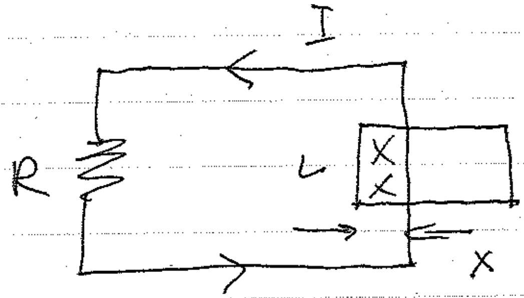
In this case the flux linking loop decreases

$\Rightarrow$ current flows to increase field in loop

Example



What happens as the magnet cross the current loop?



$$\Phi = -xLB$$

note: $\Phi < 0$. why?

$$\frac{d\Phi}{dt} = -\dot{x}LB$$

$$= -VLB$$

$$\mathcal{E} = VLB$$

$$VLB = IR$$

$$I = \frac{VLB}{R} \qquad F = ILB = \frac{VL^2B^2}{R}$$

Force on wire? $\quad \underset{\sim}{F} = I\,\underset{\sim}{L} \times \underset{\sim}{B}$

$\Longrightarrow$ calculating magnetic flux linking a loop is important.

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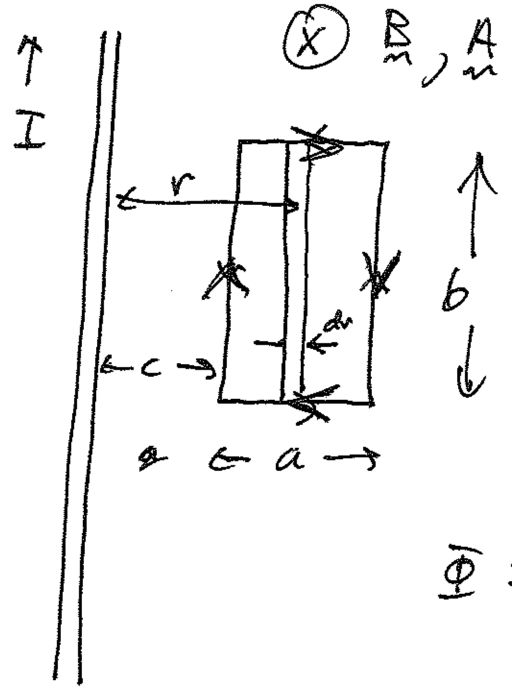
Calculating magnetic flux in a non-uniform magnetic field



$$B = \frac{\mu_0 I}{2\pi r}$$

$$d\Phi = B\, b\, dr$$

$$= b \frac{\mu_0 I}{2\pi} \frac{dr}{r}$$

$$\Phi = \int_c^{c+a} d\Phi$$

$$\Phi = \frac{\mu_0 I}{2\pi} b \ln\left(\frac{a+c}{c}\right)$$

Direction of current when loop moves away from wire?

Suppose moves with velocity v so

$$\dot{c} = v$$

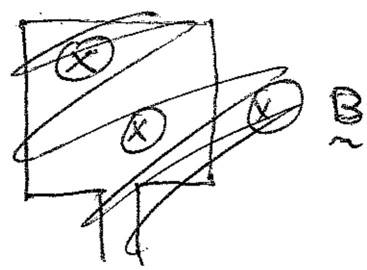
$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\mu_0 I}{2\pi} b \left(\frac{\dot{c}}{a+c} - \frac{\dot{c}}{c} \right)$$

$$= \frac{\mu_0 I}{2\pi} b v \left(\frac{1}{c} - \frac{1}{a+c} \right)$$

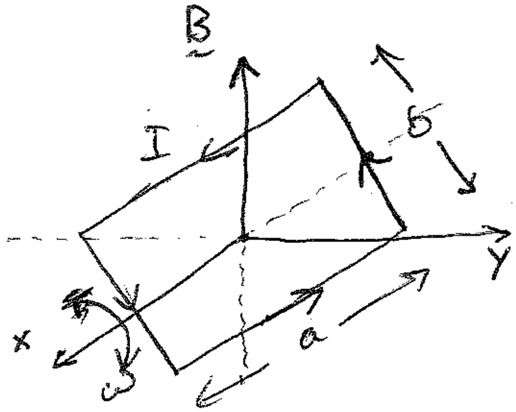
$$= \frac{\mu_0 I}{2\pi} v \frac{ab}{c(a+c)}$$

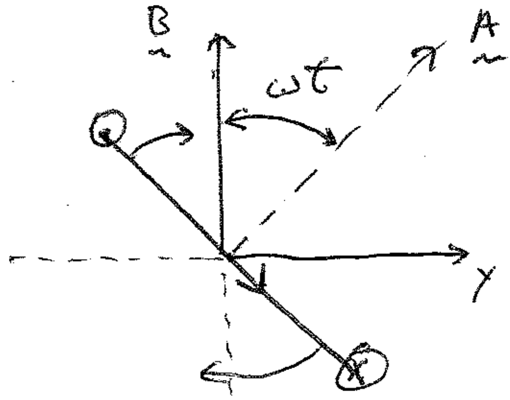
Example

Rotating wire loop in a magnetic field



Let R = resistance





Choose direction of current with $\underset{\sim}{A}$ given by right hand rule.

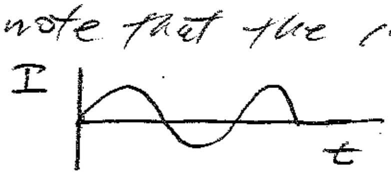
$$\Phi = \int \underset{\sim}{B} \cdot d\underset{\sim}{A} = BA\cos\omega t$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = +BA\omega\sin\omega t \qquad A = ab$$

$$I = \frac{\mathcal{E}}{R} = \frac{BA\omega}{R}\sin\omega t$$

Note that the current is a maximum when the plane of the loop is parallel to $\underset{\sim}{B}$.

$\Rightarrow$ note that the current oscillates in time



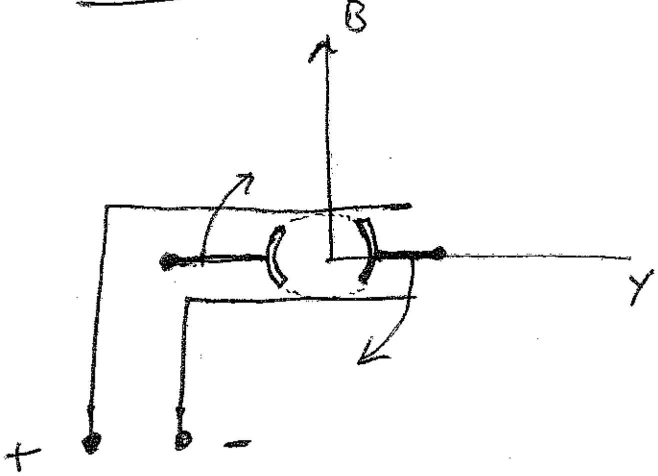
direction of current is to oppose change in magnetic flux.

This an A.C. generation

$\Rightarrow$ produces an oscillatory EMF

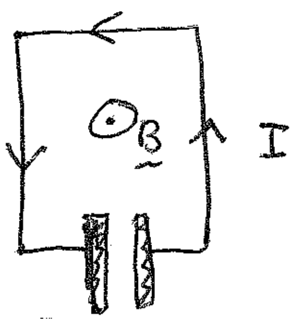
How can you produce a D.C. generator?

D.C. Generator



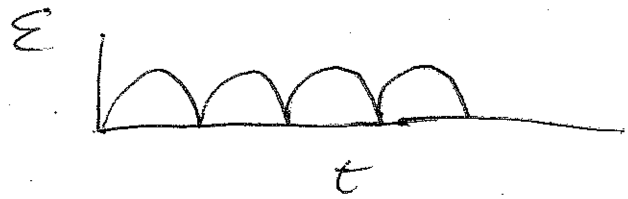
EMF is always positive

top view



~~Current tries to maintain magnetic flux upward.~~

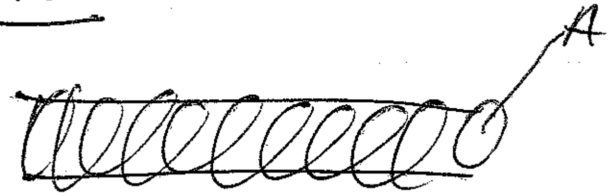
As loop rotates flux upward through loop decreases. EMF is down to try to maintain upward flux



Induced Electric Field — time varying $\underset{\sim}{B}$ field.

All examples ~~...~~ so far have involved constant magnetic fields in which loop is varying. Suppose have time dependent $\underset{\sim}{B}$.

example



$$B = \mu_0 n I$$

$$\Phi = BA = \mu_0 n I A$$

Consider a solenoid with a single loop wrapped around it.

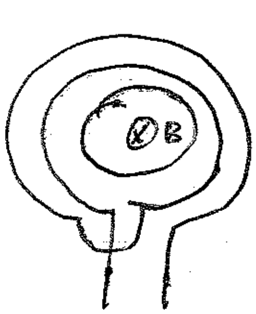
suppose that we increase the current I in the solenoid so that $\underset{\sim}{B}$ increases. What is direction of induced Emf $\mathcal{E}$ in the loop?

$$\mathcal{E} = -\frac{d}{dt}\mu_0 n I A$$

$$= -\mu_0 n A \frac{dI}{dt}$$

any change of flux either by moving the loop or by increasing B link the loop produces an EMF. Direction from Lenz Law

Suppose have two loops of wire.



total induced emf

$$\mathcal{E} = -2nA\frac{dI}{dt}\mu_0$$

for N wraps of wire

$$\mathcal{E} = -N\left(nA\frac{dI}{dt}\right)\mu_0$$

$\Longrightarrow$ this is a transformer

For a given potential drop across solenoid (can adjust potential across secondary coil by adjusting the # of windings)

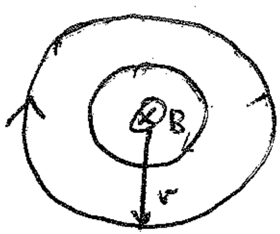
In this problem there is no $\underset{\sim}{V} \times \underset{\sim}{B}$ force so what is producing the EMF

$\Rightarrow$ <u>induced electric field.</u>

$$\mathcal{E} = \oint \underset{\sim}{E} \cdot d\underset{\sim}{l} = -\frac{d\Phi}{dt}$$

Note that this is not an electrostatic field. Why? Induced $\underset{=}{E}$ is not a conservative field.

Consider solenoid again. Consider a path of radius r around the solenoid. The does not need to be a conducting wire present.



$\underset{\sim}{E}$ lies along loop and must have const. magnitude by symmetry.

$$\oint \underset{\sim}{E} \cdot d\underset{\sim}{l} = E\, 2\pi r = -\frac{d\Phi}{dt}$$

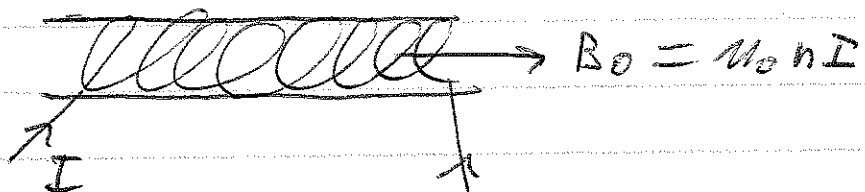
$$E = -\frac{1}{2\pi r}\frac{d\Phi}{dt}$$

If $\underset{\sim}{B}$ increases then E is in counter clockwise direction.

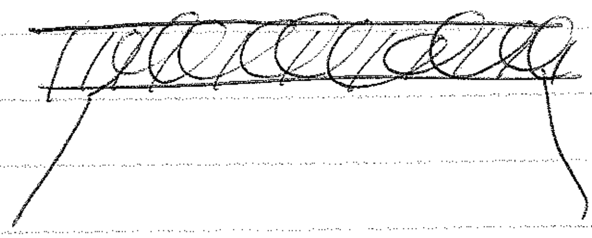
Magnetic materials and Magnetization

What happens when a material (metal, plastic, wood) is placed in a magnetic field?

⇒ consider for example a solenoid



⇒ add a core to this coil



$B_m = \cancel{\mu_0 K_m n I} \quad K_m B_0$

K_m = permeability constant

~~three~~ (3) classes of materials

(1) diamagnetic $\cancel{K_m < 1}$

$K_m - 1 \sim -(10^{-3} - 10^{-6})$

⇒ magnetic field reduced.

Why is $\underset{\sim}{B}$ changed.

The imposed magnetic field, $\underset{\sim}{B_0}$, causes magnetic dipoles in the material to align with the magnetic field

$$U = -\underset{\sim}{\mu} \cdot \underset{\sim}{B_0}$$

The dipoles tend to align with $\underset{\sim}{B_0}$ or in some cases magnetic dipoles are created. Can add up individual dipoles

$$\underset{\sim}{\mu} = \sum_i \underset{\sim}{\mu_i}$$

Define the dipole moment per unit volume

$$\underset{\sim}{M} = \frac{\underset{\sim}{\mu}}{V} = \text{magnetization}$$

For the solenoid

$$\underset{\sim}{B} = \underset{\sim}{B_0} + \mu_0 \underset{\sim}{M} = K_m \underset{\sim}{B_0}$$

$$\underset{\sim}{M} = \frac{(K_m - 1)\underset{\sim}{B_0}}{\mu_0}$$

K_m = permeability constant

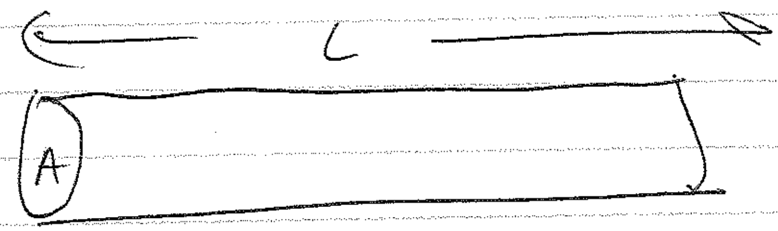
Three classes of materials

1) diamagnetic

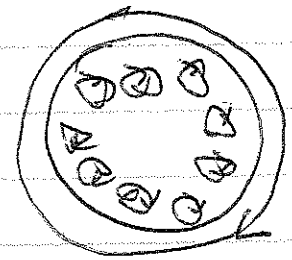
$K_m \approx 1 - (10^{-3} - 10^{-6})$

$\Rightarrow$ B reduced!

For the solenoid



$\mu = MAL$



$\Rightarrow$ internal currents cancel leaving surface currents

$M\cancel{A}L = I\cancel{A}$

$M = \frac{I}{L}$ = current / length

$B = \mu_0 \underbrace{nI}_{\text{current/length}} + \mu_0 M \equiv K_m B_0$

K_m = permeability constant

Three classes of materials

① Diamagnetic

$K_m \sim 1 - (10^{-3} - 10^{-6})$

B reduced

② paramagnetic

$K_m - 1 \sim 10^{-3} - 10^{-6}$

$\Rightarrow$ magnetic field slightly increased

③ ferromagnetic

$K_m \sim 10^3 - 10^4$

$\Rightarrow$ strong increase of $\underline{B}$.

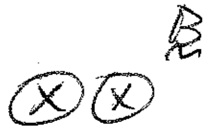
① Diamagnetic

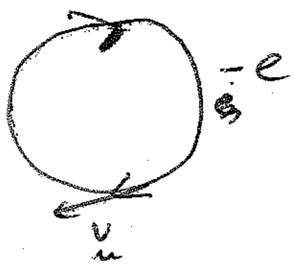
$K_m \lesssim 1$

lead $\Rightarrow K_m \sim 1 - 1.8 \times 10^{-5}$

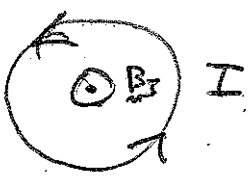
$\Rightarrow$ B slightly reduced

Physics





Electrons spin in the material around the magnetic field



Current produces a magnetic field out which opposes imposed field.

(2) Paramagnetic

$K_m \gtrsim 1$

$O^2 \quad K_m = 1 + .19 \times 10^{-5}$

B increases slightly.

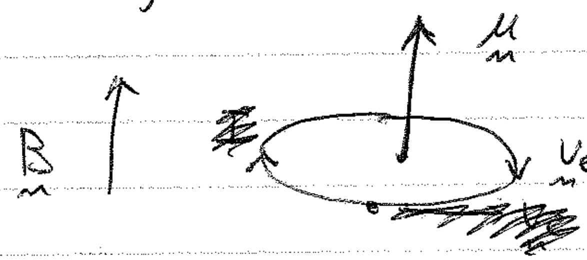
Physics

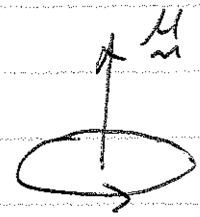
material has pre-existing dipole moments

a) electron orbits

b) intrinsic magnetic moments

a) Electron orbits





$\underset{\sim}{\mu}$ aligns with $\underset{\sim}{B}$

field adds to $\underset{\sim}{B}$

$$\mu = IA = \pi r^2 \left(\frac{ev}{2\pi r}\right) = \frac{evr}{2}\frac{m}{m}$$

$$= \frac{e}{2m} L$$

L = angular momentum

$$\underset{\sim}{\mu} = -\frac{e}{2m}\underset{\sim}{L} \quad \Rightarrow \quad L \text{ is quantized.}$$

$$\hbar = \frac{h}{2\pi}$$

$$h = 6.63\times10^{-34}\ \mathrm{Js}$$

= Planck's constant

$$\mu_B = \hbar\frac{e}{2m}$$

$$= 9.27\times10^{-24}\ \mathrm{Am^2}$$

= intrinsic unit of μ

= Bohr magneton

b) Intrinsic magnetic moments

$\Rightarrow$ fundamental particles have L, μ

electron $\quad L = \frac{1}{2}\hbar$

$$\mu_e = -\frac{e}{m}L = -\frac{e}{m}\frac{\hbar}{2}$$

③ Ferromagnetic materials

$K_m \gg 1$

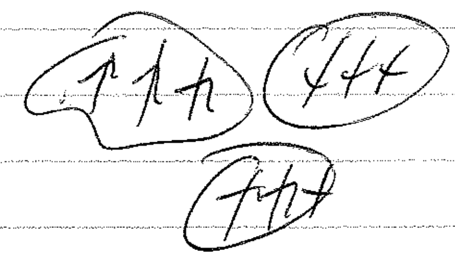
$K_m \sim 10^3 - 10^4$

$\Rightarrow$ B strongly increased

Iron, cobalt, nickel

$\Rightarrow$ pre-existing magnetic moments are aligned in domains

$\Rightarrow$ magnetic moments of all atoms in a domain are aligned.



~~Hysteresis~~