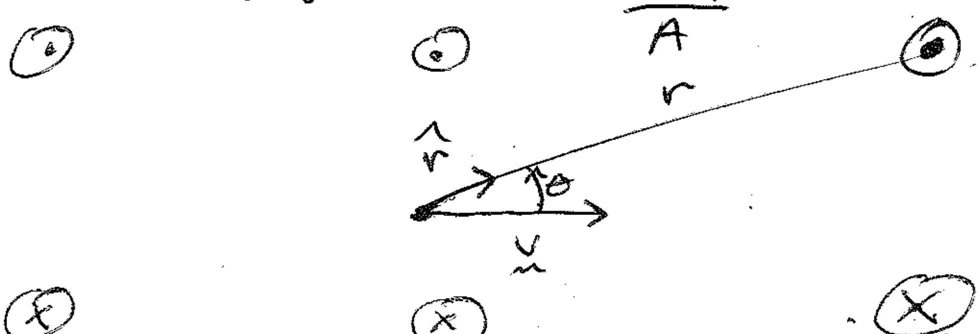
Generation of a Magnetic Field

Magnetic Field of a Moving Charge

Consider a charge q with a velocity $\underset{\sim}{v}$.
This moving charge produces a magnetic field.

$$\underset{\sim}{B} = \frac{\mu_0}{4\pi}\, q\, \frac{\underset{\sim}{v} \times \hat{r}}{r^2}$$

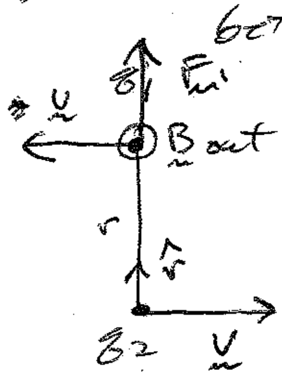
$$\mu_0 = 4\pi \times 10^{-7}\ \frac{\mathrm{T\,m}}{\mathrm{A}}$$



$$|\underset{\sim}{B}| = \frac{\mu_0}{4\pi}\, \frac{|q||v|}{r^2} \sin\theta$$

If have several moving charges must add the magnetic fields vectorally.

<u>example</u> Comparison of magnetic force between ~~a~~ moving ~~and stationary~~ charges



~~$\underset{\sim}{B}$ at P =~~

$$B = \frac{q_2 v}{r^2}\, \frac{\mu_0}{4\pi}$$

$$F_B = q_1 v B = \frac{q_1 q_2 v^2}{r^2}\, \frac{\mu_0}{4\pi}$$

electric force

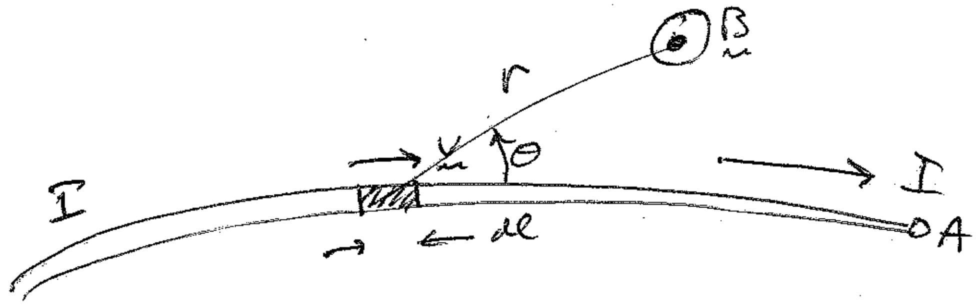
$$F_E = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

$$\frac{F_B}{F_E} = \frac{\cancel{q_1 q_2}\, v^2}{\cancel{r^2}} \frac{\mu_0}{\cancel{4\pi}} \frac{\cancel{4\pi\epsilon_0 r^2}}{\cancel{q_1 q_2}} = v^2 \mu_0 \epsilon_0$$

$$\frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3\times 10^8 \text{ m/s} = c = \text{velocity of light.}$$

$$\frac{F_B}{F_E} = \frac{v^2}{c^2}$$ For $\frac{v}{c} \ll 1$ electric force is much larger.

Magnetic Field of a Current Element



$$dQ = nA\,dl\,q$$

$$|\underset{\sim}{B}| = \frac{dQ\, v \sin\theta}{r^2} \cancel{4\pi} \frac{\mu_0}{4\pi}$$

$\underset{\sim}{dl}$ lies along wire

$$= \frac{\overbrace{nAq\,dl\,v}^{I} \sin\theta}{r^2} \cancel{4\pi} \frac{\mu_0}{4\pi}$$

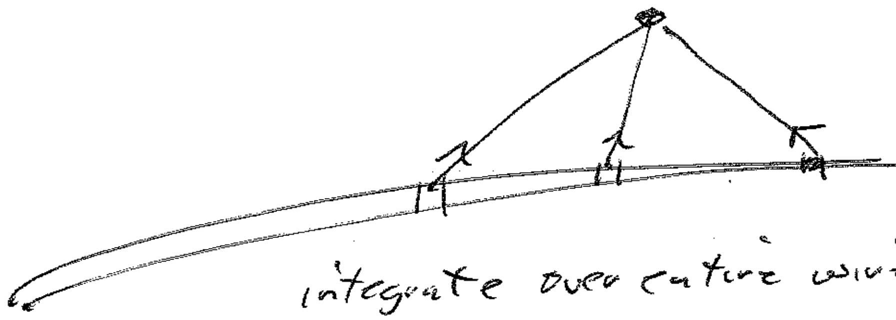
Biot Savart Law

$$= I \frac{dl \sin\theta}{r^2} \frac{\mu_0}{4\pi}$$

$$\underset{\sim}{B} = \frac{\mu_0}{4\pi} I \frac{\underset{\sim}{dl} \times \hat{r}}{r^2}$$

To find the magnetic field due the entire wire

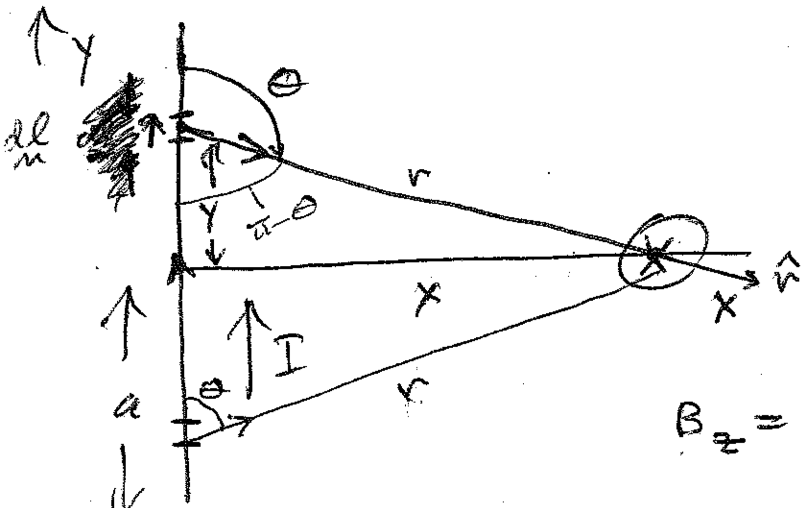
$$\mathbf{B} = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{I \, d\mathbf{l} \times \hat{r}}{r^2}$$



integrate over entire wire

note that both r and $\hat{r}$ and $d\mathbf{l}$ may vary over length of wire

Magnetic Field of a Finite Length Wire



$\hat{\imath} \times \hat{\jmath} = \hat{k}$

$\hat{k}$ is out

B is in $-\hat{k}$ direction

$$B_z = -\frac{\mu_0 I}{4\pi} \int \frac{dl \sin\theta}{r^2}$$

$r^2 = x^2 + y^2$

$dl = dy$

$\sin(\pi - \theta) = \frac{x}{r} = \sin\theta$

$$B_z = -\frac{\mu_0 I}{4\pi} \int_{-a}^{a} dy \, \frac{x}{(x^2+y^2)^{3/2}}$$

$$\int dy \frac{1}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

$$B_z = -\frac{\mu_0 I}{4\pi} \frac{2a}{x(x^2+a^2)^{1/2}}$$ ~~large x~~

large x

2a

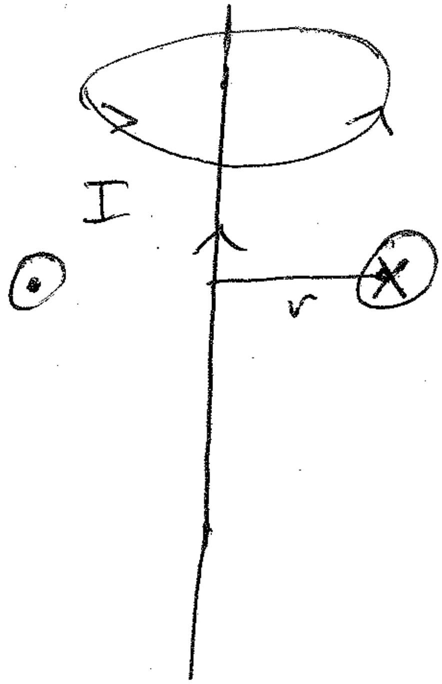
$$B_z = -\frac{\mu_0 I}{4\pi} \frac{2a}{x^2}$$ acts like a short current segment

small $x \ll a$

$$B_z = -\frac{\mu_0 I}{4\pi} \frac{2 \cancel{a}}{x \cancel{a}}$$

$$B_z = -\frac{\mu_0 I}{2\pi x}$$

Note that a has dropped out of problem. This is the magnetic field around an infinite wire



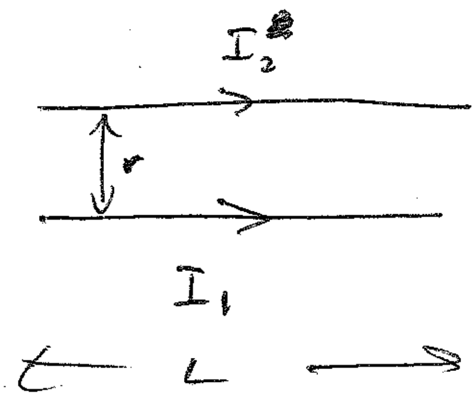
$$B = \frac{\mu_0 I}{2\pi r}$$

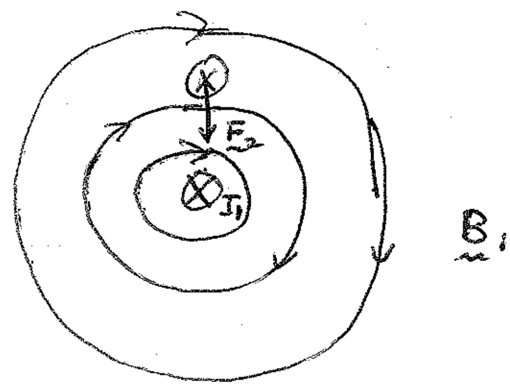
example What is B at 1cm from a 10A current.

$$B = \frac{\left(\cancel{4\pi}^{2} \times 10^{-7} \frac{Tm}{A}\right) 10A}{\cancel{2\pi}\ 1cm}$$

$$\approx 2 \times 10^{-4} T = 2 \text{ gauss}$$

Force Between parallel wires





magnetic field due to I_1

Want to calculate the force on I_2 due to the magnetic field of I_1

$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

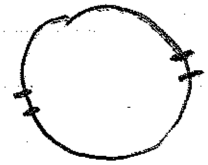
$$\underset{\sim}{F}_2 = I_2 \underset{\sim}{L}_2 \times \underset{\sim}{B}_1$$

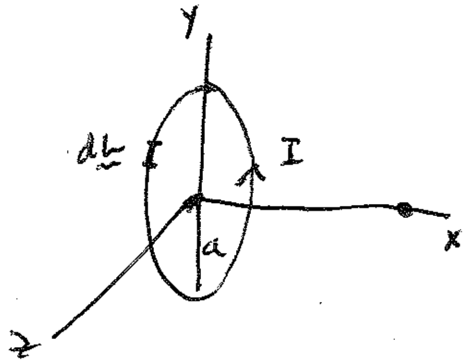
$$F_2 = I_2 L \frac{\mu_0 I_1}{2\pi r}$$

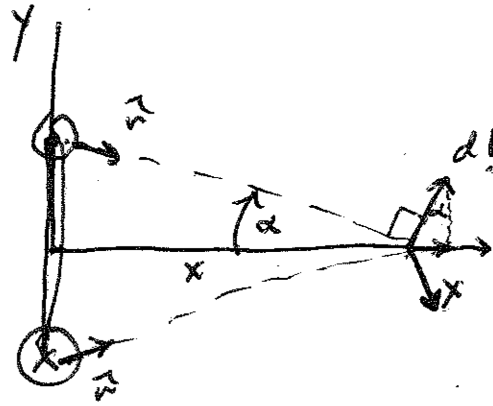
$$\boxed{F_2 = \frac{\mu_0}{2\pi r} L I_1 I_2}$$

$\Longrightarrow$ two wires carrying current in the same direction attract each other.

Magnetic Field of Circular Current Loop



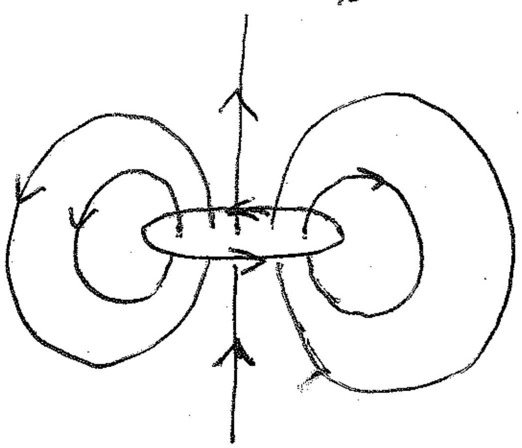




$$d\underset{\sim}{B} = I \frac{d\underset{\sim}{L} \times \hat{r}}{r^2} \frac{\mu_0}{4\pi}$$

Two current elements across from each other produce a net field which points in the $\hat{x}$ direction

$\Rightarrow$ $\underset{\sim}{B}$ lies along x.



$d\underset{\sim}{L}$ and $\hat{r}$ are $\perp$

$$dB_x = dB \sin\alpha$$

$$dB_x = I \frac{dL}{r^2} \frac{\mu_0}{4\pi} \sin\alpha$$

$$r^2 = x^2 + a^2 \qquad \sin\alpha = \frac{a}{(x^2+a^2)^{1/2}}$$

$$B_x = \frac{\mu_0}{4\pi} \frac{1}{r^2} \frac{a}{r} I \int dL$$

$$= \frac{\mu_0}{2} \frac{a^2}{(x^2+a^2)^{3/2}} I$$

$$\boxed{B_x = \frac{\mu_0 \mu}{\pi\, 2 (x^2+a^2)^{3/2}}}$$

$$\mu = I \pi a^2$$

large x

$$B_x = \frac{\mu_0 \mu}{2\pi x^3}$$

same as electric dipole

~~All magnetic dipoles produce~~

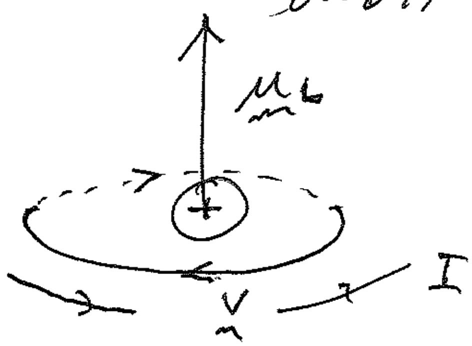
Magnetic materials

magnetic dipoles produce magnetic fields

$$B = \frac{\mu_0}{2\pi} \frac{\mu}{r^3}$$

materials ~~have~~. can have pre-existing magnetic moments

⇒ associated with classical orbit



⇒ electrons have an intrinsic dipole moment associated with their spin. μ_s

~~Typically spins are randomly distributed~~

. The magnetic moments typically cancel pairwise as electrons fill their orbitals in an atom.

⇒ ~~materials w~~ atoms with odd # of electrons have magnetic moment.

⇒ random directions in a material

A magnetic field can cause the magnetic moments to line up.

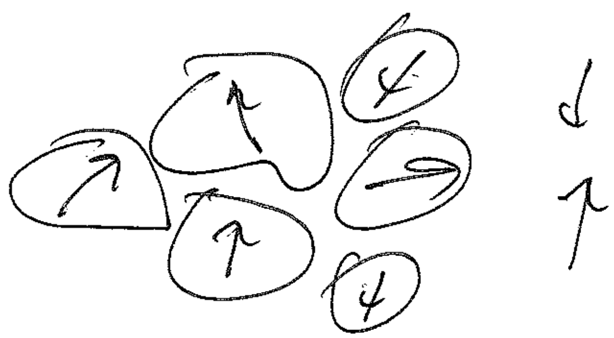
$$U = -\underset{\sim}{\mu} \cdot \underset{\sim}{B}$$

$\underset{\sim}{\mu}$ and $\underset{\sim}{B}$ align.

$\Longrightarrow$ can cause B to increase

in Ferromagnetic materials dramatic increase (iron, nickel, cobalt)

The magnetic moments are pre-aligned in domains.

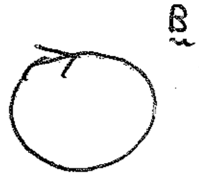


0.1 mm

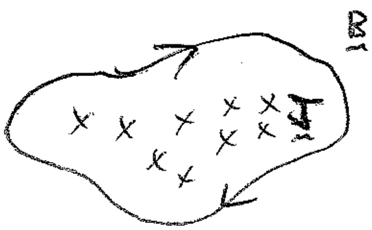
domains align to produce very strong magnetic field.

Amperes Law

Consider a magnetic field



What current produces this field →

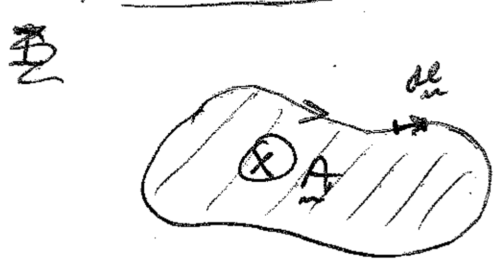


Some distribution of current produces this field.

Want a relation between $\underset{\sim}{B}$ and the current flow

$$\boxed{\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 I}$$

Amperes Law.



I = total current flowing through the area enclosed by the path.

What about the sign of I.

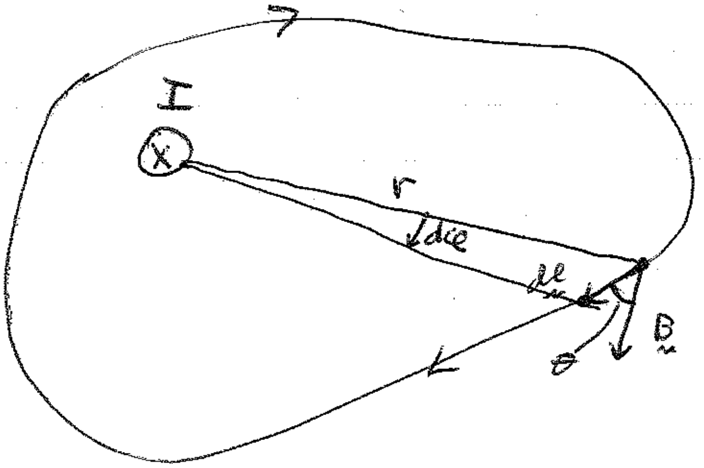
~~$I = \int d\underset{\sim}{A} \cdot \underset{\sim}{J}$~~

~~The direction of $d\underset{\sim}{A}$ is given by the right hand rule~~

Use right hand rule to determine the direction $\underset{\sim}{A}$. I is positive in direction of $\underset{\sim}{A}$ and negative if in opposite direction.

Proof for single wire

Consider a path around a current carrying wire



$$B = \frac{\mu_0 I}{2\pi r}$$

$$B \cdot d\ell = B\,d\ell\cos\theta$$

~~$r\,d\varphi = d\ell\cos\theta$~~

$$\tan(d\varphi) = \frac{d\ell\cos\theta}{r}$$

$$d\varphi \simeq \frac{d\ell\cos\theta}{r}$$

$$\oint B \cdot d\ell = \oint B\,d\ell\cos\theta$$

$$= \oint \frac{\mu_0 I}{2\pi r} r\,d\varphi$$

$$= \int_0^{2\pi} \frac{\mu_0 I}{2\pi}\,d\varphi$$

$$= \frac{\mu_0 I}{2\pi}\int_0^{2\pi} d\varphi$$

$$= \frac{\mu_0 I}{2\pi} 2\pi = \mu_0 I$$

$\Rightarrow$ valid for any path ~~with encloses~~ that circles I.

~~$\Rightarrow$ can superimpose arbitrary~~

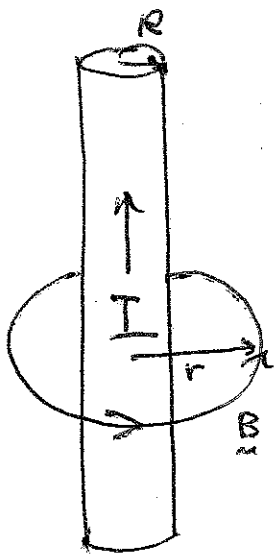
$\Rightarrow$ When is Amperes law useful?

$\Rightarrow$ symmetry

Field Inside Finite Sized Conductor

Consider an infinite (~~long~~) conductor of radius R carrying a current I. Calculate $\underset{\sim}{B}$ everywhere assuming that the current density in the wire is uniform.

$$J = \frac{I}{\pi R^2} = \frac{\text{current}}{\text{unit area}}$$



First consider $r > R$

Choose a path on which B is a constant

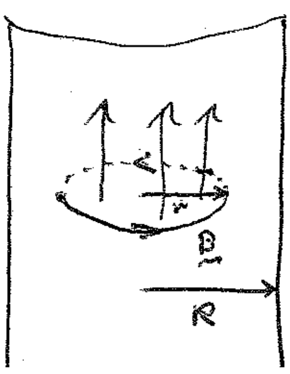
$\Rightarrow$ circle of radius r.

$\underset{\sim}{B}$ parallel to $d\underset{\sim}{l}$

$$\oint \underset{\sim}{B}\cdot d\underset{\sim}{l} = B\, 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{as before}$$

Now take $r < R$



Plot B versus r

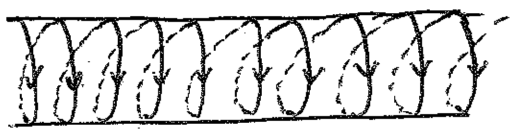
$$\oint \underset{\sim}{B}\cdot d\underset{\sim}{l} = B\, 2\pi r = \mu_0 I_r$$

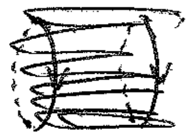
$$I_r = J \pi r^2 = \frac{I}{\pi R^2}\pi r^2 = I\frac{r^2}{R^2}$$

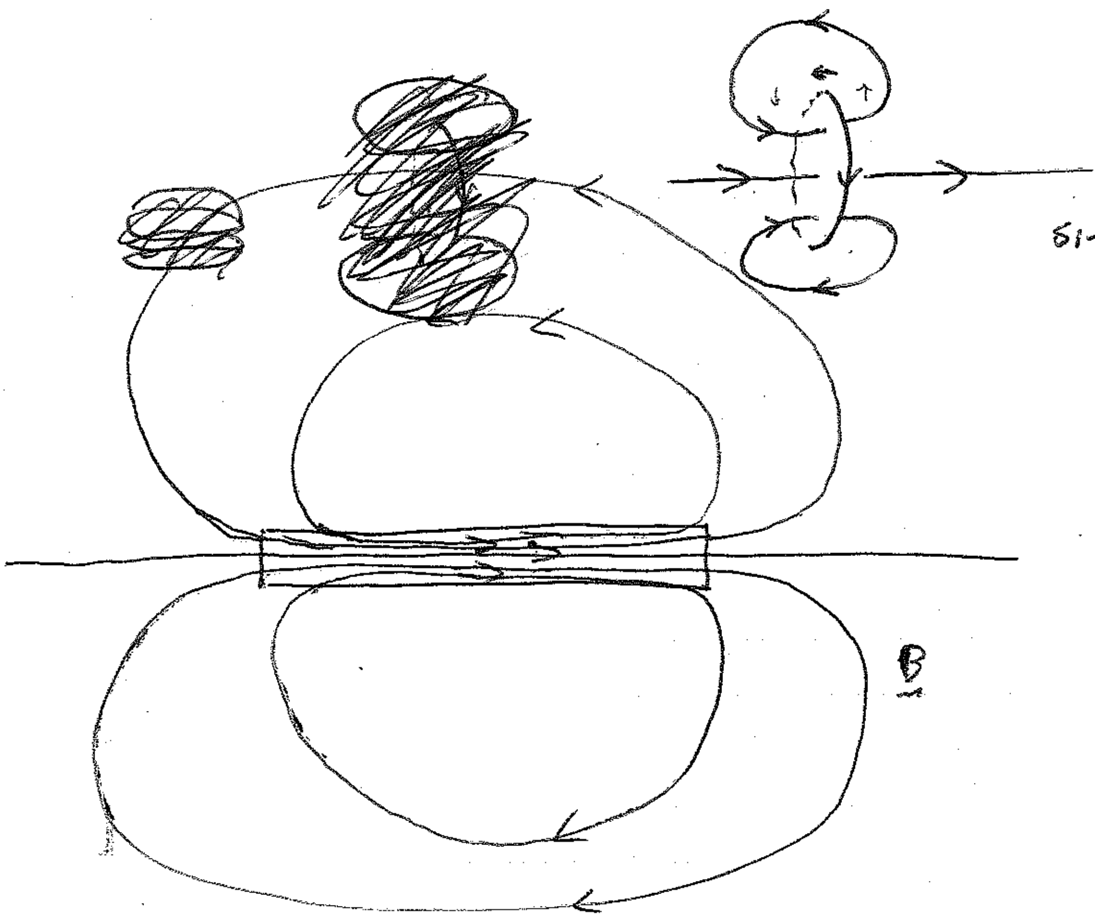
$$B = \frac{\mu_0 I \frac{r^{\cancel{2}}}{R^2}}{2\pi \cancel{r}} = \frac{\mu_0 I}{2\pi R^2}\, r$$

Magnetic Field in a Solenoid.

A solenoid consists of wire wrapped around a cylinder. Let n = # of wraps per unit length with current I in the wire

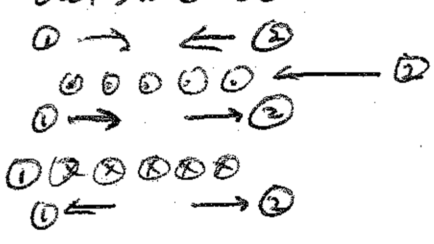


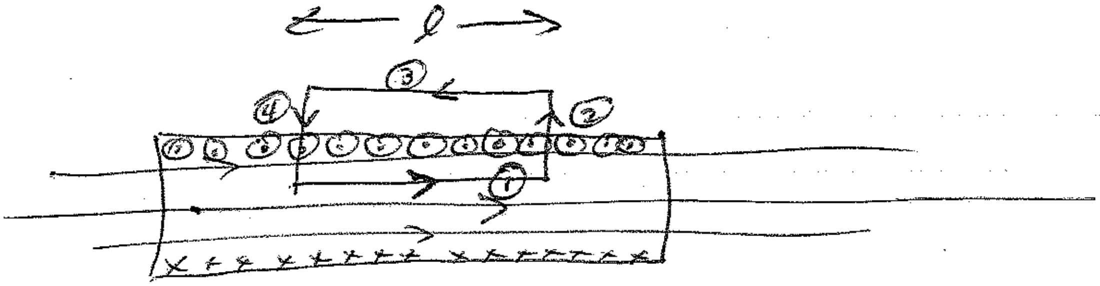




$\underline{B}$ is large inside solenoid and small just outside

Field is small outside because ~~currents~~ fields add inside and subtract outside





$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{tot}$$

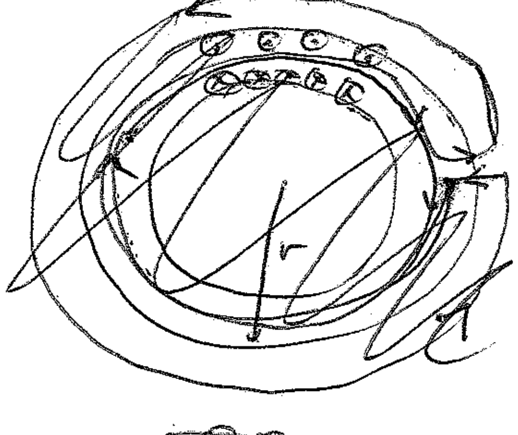
$$\underbrace{\int_{1} \mathbf{B}\cdot d\mathbf{l}}_{Bl} + \underbrace{\cancel{\int_{2} \mathbf{B}\cdot d\mathbf{l}}}_{\mathbf{B}\cdot d\mathbf{l}=0} + \underbrace{\cancel{\int_{3} \mathbf{B}\cdot d\mathbf{l}}}_{\mathbf{B}\approx 0} + \underbrace{\cancel{\int_{4} \mathbf{B}\cdot d\mathbf{l}}}_{\mathbf{B}\cdot d\mathbf{l}=0} = \mu_0 (nl) I$$

$$B\cancel{l} = \mu_0 n \cancel{l} I$$

$$\boxed{B = \mu_0 n I}$$

Toroidal Solenoid

N turns total

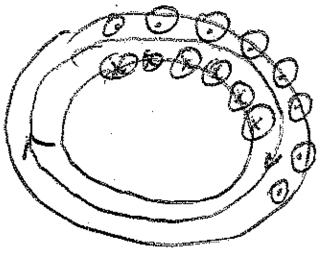


$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{tot}$$

$$\int_{inside} \mathbf{B}\cdot d\mathbf{l} = \mu_0 N I$$

$$B\, 2\pi r = \mu_0 I N$$

$$\boxed{B = \frac{\mu_0 I N}{2\pi r}}$$



notice that B is larger ~~on the inside of the torus.~~ at smaller radii