Magnetic Fields

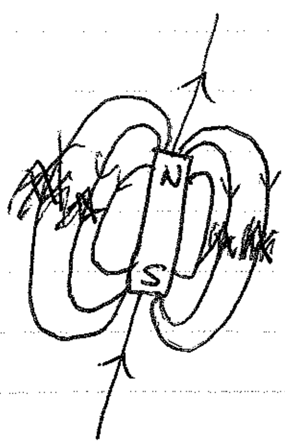
~~Magnetic materials~~

The existense of magnetic materials has been known for over 2000 years. Magnetic materials ~~attract each other~~ have "magnetic poles" called north and south poles. North poles of magnets repell each other while N/S pairs attract each other. Certain metals (iron) though unmagnetized are attracted by the magnet. Other metals such as aluminum or copper are not attracted by the magnetic material.

Magnetic forces clearly have a different character from electric forces. We introduced the concept of an electric field which surrounds charged particles.

To explore and understand magnetic forces we want to introduce the magnetic field $\underset{\sim}{B}$ (a vector) which surrounds magnetic materials. We will then explore (1) how this magnetic field acts on charged particles and (2) how this magnetic field is generated.

That such a field $\underset{\sim}{B}$ exists can be seen by looking at how iron filings distribute around a bar magnet.



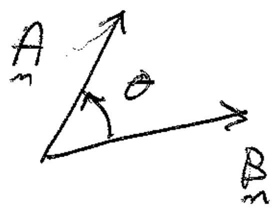
We will show ~~see~~ later that moving charged particles — currents also generate $\underset{\sim}{B}$.

Vector Product (cross product)

Suppose have vectors $\underset{\sim}{A}, \underset{\sim}{B}$

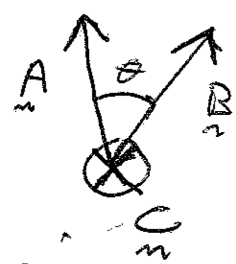
In physics define two types of products of vectors.

* dot product $c = \underset{\sim}{A} \cdot \underset{\sim}{B}$ in which c is a scalar $\quad c = AB\cos\theta$



* vector product

$$\underset{\sim}{C} = \underset{\sim}{A} \times \underset{\sim}{B}$$



Use right-hand rule to find direction $\underset{\sim}{C}$

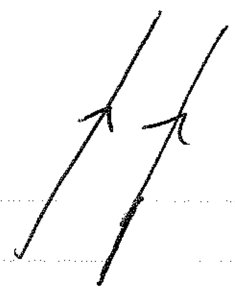
$$C = AB\sin\theta$$

C is perpendicular to $\underset{\sim}{A}, \underset{\sim}{B}$

In x, y, z coordinate system

$$\underset{\sim}{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\underset{\sim}{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_y B_z - B_y A_z) + \hat{j}(\quad) + \hat{k}(\quad)$$

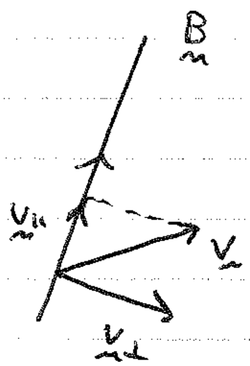


Magnetic Forces

How does a charged particle respond to a magnetic field? Observations

① A charged particle at rest has no force acting on it.

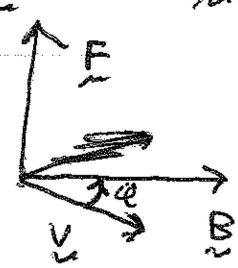
② Force is proportional to the component of $\underset{\sim}{v}$ which is perpendicular to $\underset{\sim}{B}$, $\underset{\sim}{v}_\perp$



③ Force is proportional to charge q of particle of interest. $|\underset{\sim}{F}| = |q||\underset{\sim}{v}_\perp||\underset{\sim}{B}|$

④ Force is $\perp$ to both $\underset{\sim}{v}_\perp$ and $\underset{\sim}{B}$.

$$\underset{\sim}{F} = q\,\underset{\sim}{v} \times \underset{\sim}{B}$$



$v_\perp = v \sin\varphi$

$$|\underset{\sim}{F}| = q v \sin\varphi B$$

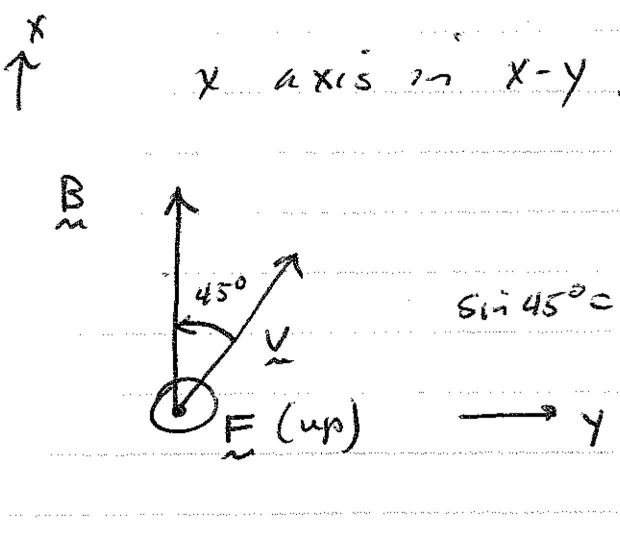
units $\quad B \sim \dfrac{N\,S}{C\,m} = \dfrac{N}{A\,m} = \text{tesla}$ in SI units

in CGS units

$$B \text{ in gauss} = 10^{-4}\ \text{Tesla}$$

example

A magnetic field of magnitude 1 T is in the x direction. A ~~charge~~ proton $q = 1.6\times10^{-19}\,C$ with a velocity of 10^6 m/s at an angle of 45° with respect to the x axis in x-y plane



$\sin 45^\circ = \frac{1}{\sqrt{2}}$

$$|F| = 1.6\times10^{-19}\,C\ 1T \times 10^6\,\text{m/s}\ \frac{1}{\sqrt{2}}$$

$$= 1.13\times10^{-13}\,N$$

$$F_z = -1.13\times10^{-13}\,N$$

$\hat{k} = \hat{\imath}\times\hat{\jmath}$ $\qquad F = -1.13\times10^{-13}\,N\ \hat{k}$

Motion of a charged particle in a magnetic field

$$m \frac{d\underset{\sim}{v}}{dt} = q\, \underset{\sim}{v} \times \underset{\sim}{B}$$

To find change in KE take dot product with $\underset{\sim}{v}$

$$m\, \underset{\sim}{v} \cdot \frac{d}{dt} \underset{\sim}{v} = m\left(v_x \frac{d}{dt} v_x + v_y \frac{d}{dt} v_y + v_z \frac{d}{dt} v_z\right) =$$

$$= \frac{d}{dt} \tfrac{1}{2} m \left(v_x^2 + v_y^2 + v_z^2\right)$$

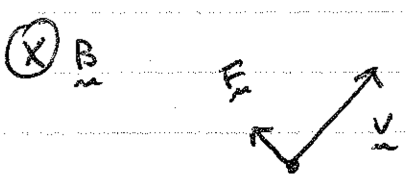
$$= \frac{d}{dt} \tfrac{1}{2} m v^2$$

$$\frac{d}{dt}\left(\tfrac{1}{2} m v^2\right) = q\, \underset{\sim}{v} \cdot (\underset{\sim}{v} \times \underset{\sim}{B}) = 0$$

$\underset{\sim}{v} \times \underset{\sim}{B}$ is $\perp$ to $\underset{\sim}{v}$ so $\underset{\sim}{v} \cdot (\underset{\sim}{v} \times B) = 0$

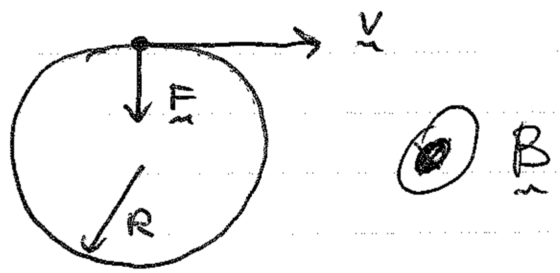
$$\Longrightarrow \quad \frac{d}{dt}\left(\tfrac{1}{2} m v^2\right) = 0$$

$\Longrightarrow$ the magnetic field does not change the kinetic energy of a charge.



The force acts to change the direction of $\underset{\sim}{v}$. The magnitude of $|\underset{\sim}{v}|$ and $|\underset{\sim}{F}|$ remain constant in a uniform magnetic field.

$\Rightarrow$ motion of the particle is a circle



circular motion

$$F = ma = m\left(\frac{V^2}{R}\right) = V q B$$

$$R = \frac{mV}{qB} = \text{radius of orbit}$$

$\Rightarrow$ radius increases with V

$$V = \omega R \qquad \omega = \frac{qB}{m} = \text{gyrofrequency}$$

~~cyclotron~~

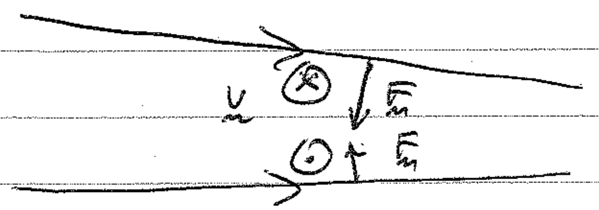
indep of V.

What does the motion of a particle moving along and across a uniform magnetic field look like?

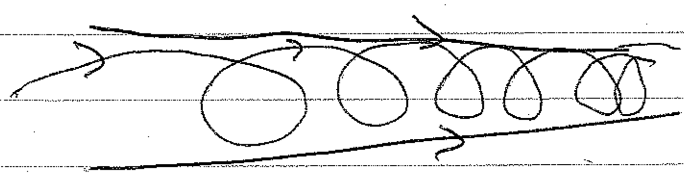
$\Rightarrow$ spiral

Trapping in magnetic bottles

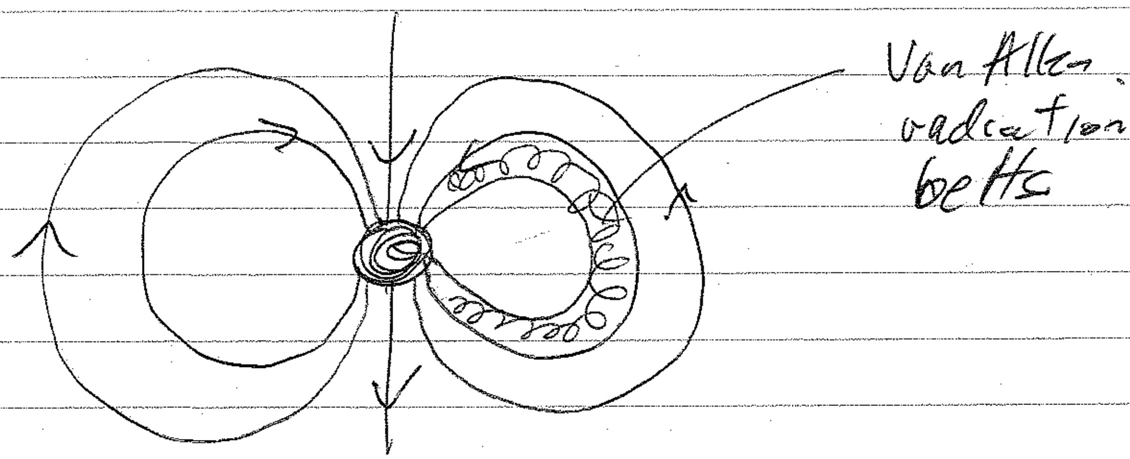
Consider the motion of particles in a converging magnetic field



to lowest order particle orbit is a circle but have a weak force which repells the charge from the high field region.

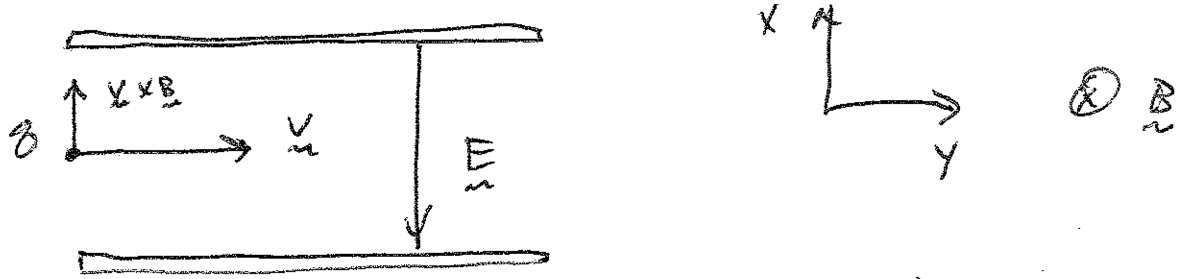


Earths magnetic field $\Rightarrow$ radiation belts



Motion in Crossed $\underset{\sim}{E}$ and $\underset{\sim}{B}$ Fields

Suppose have uniform magnetic field



$\Rightarrow$ can balance the forces $\quad (\underset{\sim}{v}\times\underset{\sim}{B})_x = v_y B_z - \cancelto{0}{v_z B_y}$

$$m\frac{dv_x}{dt} = qE_x + q v_y B_z = 0$$

adjust E_x so it cancels magnetic force

$$E_x = -v_y B_z$$

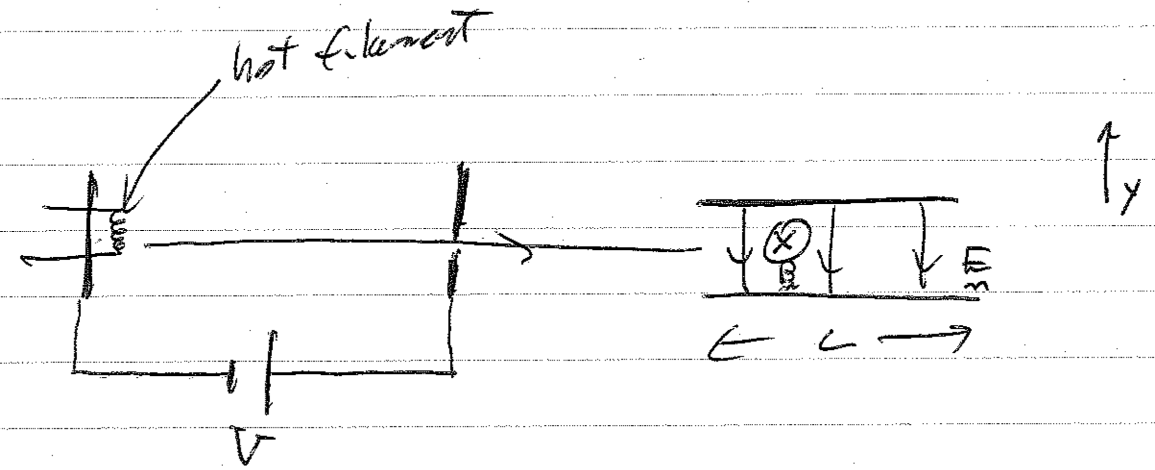
particle will move directly along y

in a straight line

$\Rightarrow$ technique for measuring the velocity of a charge.

$\Rightarrow$ knowing the E required to make the charge go in a straight line ~~given~~ with B known gives velocity.

⇒ can use this to measure the ratio of charge to mass of particles



⇒ shut off $\underset{\sim}{B}$ and measure displacement by $\underset{\sim}{E}$ field

$$\Delta y = \frac{1}{2} a t^2 = \frac{1}{2} \frac{eE}{m} \left(\frac{L}{v}\right)^2$$

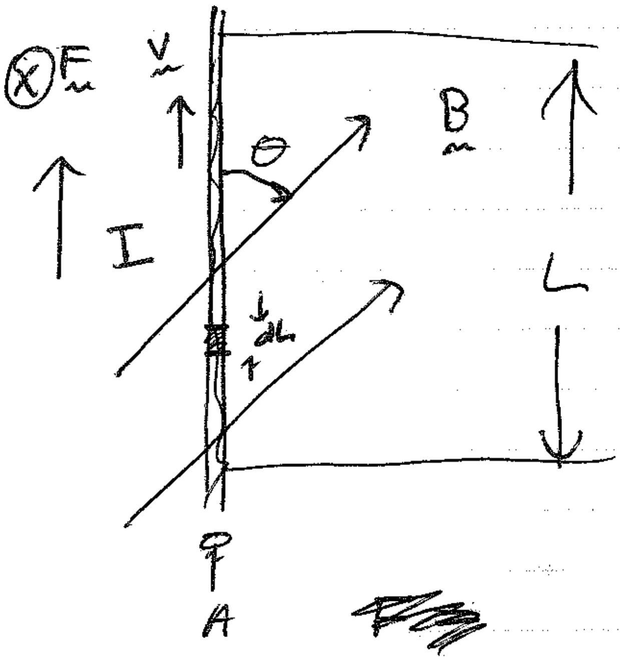
~~E = vB~~

Increase B until no deflection

$$E = vB$$

$$\boxed{\frac{e}{m} = \frac{2 \Delta y E}{B^2 L^2}}$$ JJ Thomson

Force on a current carrying wire



$$I = JA$$

$$J = nqV$$

$$dN = \#\text{ of charges in } dL$$

$$= AdLn$$

$$dQ = dNq$$

$$d\mathbf{F} = dQ\, \mathbf{v} \times \mathbf{B}$$

$$dF = dQ\, v B \sin\theta$$

$$= dNqV B\sin\theta$$

$$= A\,dl\, n q V B\sin\theta$$

$$= I\, dl\, B\sin\theta$$

$$F = I B \sin\theta \int dl$$

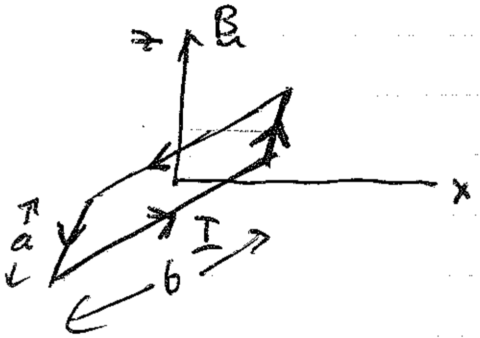
$$\boxed{F = I B \sin\theta\, L}$$

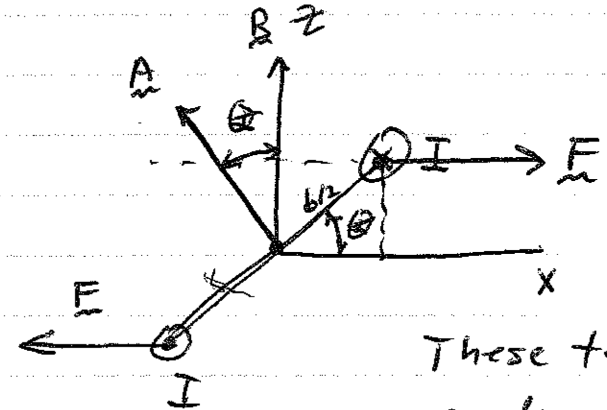
$$\mathbf{F} = I\, \mathbf{L} \times \mathbf{B}$$

$\mathbf{L}$ lies along wire in direction of current.

Note that the force is $\perp$ to $\underset{\sim}{B}$ and to the ~~[illegible]~~ wire.

Torque on a current loop





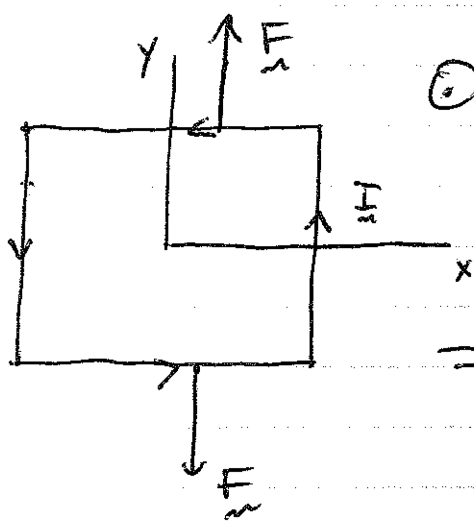
$|\underset{\sim}{F}| = BIa$

These two forces produce a torque around the y axis

$\tau = 2F\ell$

ℓ = moment arm

$= \frac{b}{2} \sin\theta$



$\Longrightarrow$ Forces cancel.

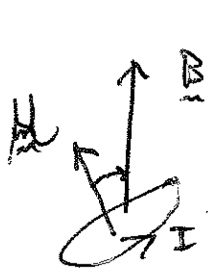
Direction of $\underset{\sim}{A}$ is given by right hand rule: $\perp$ to surface

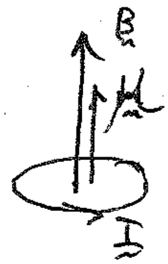
$$\tau = \cancel{2}\, BIa \frac{b}{\cancel{2}} \sin\theta = BIab \sin\theta = BIA \sin\theta$$

$\mu = IA$ = magnetic moment of the loop.
= product of area and current

$$\tau = B\mu \sin\theta \qquad \boxed{\underset{\sim}{\tau} = \underset{\sim}{\mu} \times \underset{\sim}{B}}$$

The loop wants to move so that μ and B are aligned.





Expression for μ = area x current is valid for any loop of current.

$\Rightarrow$ to find torque on loop

calculate μ

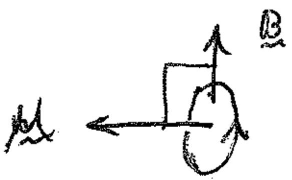
$$\tau = \mu \times B$$

example

Consider a loop of area $1\,cm^2$ with a 20 turn coil of wire carrying a current of 5A. 1 T magnetic field.

$$\mu = (10^{-4})\,m^2\,(5\times 20)\,A$$

$$= 10^{-2}\,m^2 A \qquad T = \frac{N}{Am}$$



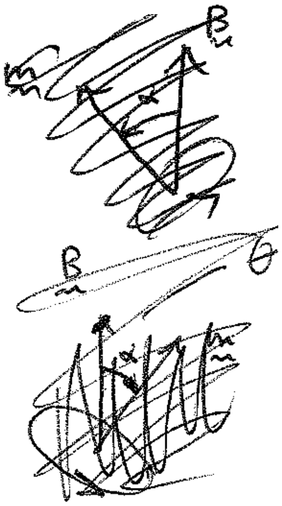
$$\tau = 10^{-2}\,m^2 A\;1T = 10^{-2}\,Nm$$

Energy of Current Loop in a Magnetic Field.

Consider a loop with magnetic moment μ and moment of inertia I_m

$$I_m \frac{d\omega}{dt} = \tau$$

$$I_m \frac{d^2\theta}{dt^2} = -\mu B \sin\theta \qquad \omega = \frac{d\theta}{dt}$$

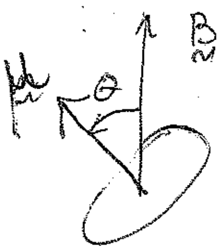


$$I_m \frac{d\theta}{dt}\frac{d^2\theta}{dt^2} = I_m \omega \frac{d\omega}{dt} = -\mu B \frac{d\theta}{dt}\sin\theta$$

$$= \mu B \frac{d}{dt}(\cos\theta)$$

$$\frac{d}{dt}\,\tfrac{1}{2}I_m\omega^2 = \frac{d}{dt}(\mu B\cos\theta)$$

$$\frac{d}{dt}\left(\tfrac{1}{2}I_m\omega^2 - \mu B\cos\theta\right) = 0$$



$$U = -mB\cos\theta$$

$$U = -\mu \cdot B$$

U = potential energy of a magnetic moment in a magnetic field.

~~At=0~~

At $t=0$, the loop is at rest with $\theta = \frac{\pi}{2}$.

Torque $\tau = \mu \times B$ causes

loop to rotate along $\underset{\sim}{B}$. What is the angular rotation $\omega = d\theta/dt$ when $\theta = 0$?

$$W = \frac{1}{2} I_m \omega^2 + U = \text{const}$$

at $t=0$, $\theta = \frac{\pi}{2}$

$$W = -\underset{\sim}{\mu} \cdot \underset{\sim}{B} = 0$$

when $\theta = 0$,

$$0 = \frac{1}{2} I_m \omega^2 - \mu B$$

$$\omega = \left(\frac{2\mu B}{I_m}\right)^{\frac{1}{2}}$$

Simple Electric Motor