Example

resistivity of copper $\eta = 1.72\times10^{-8}\ \Omega m$

steel $\quad 20\times10^{-8}$

How long is a copper wire with $A = (1mm)^2$ which has $R = 1\Omega$.

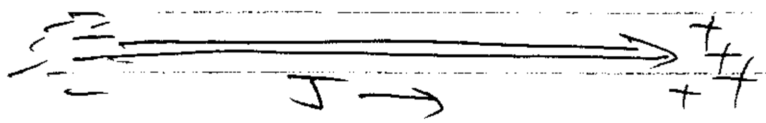
$$R = 1\Omega = \frac{1.72\times\cancel{10^{-8}}^{\,10^{-2}}\ \Omega m\ L}{\cancel{10^{-6}}\ m^2}$$

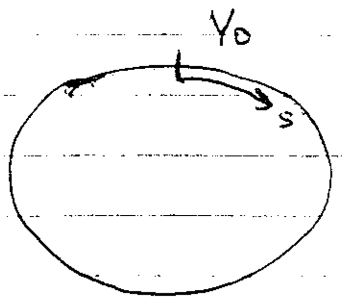
$$L_c = \frac{1}{.0172}\ m \sim 58\ m$$

$$L_s = \frac{1}{.2}\ m = 5m$$

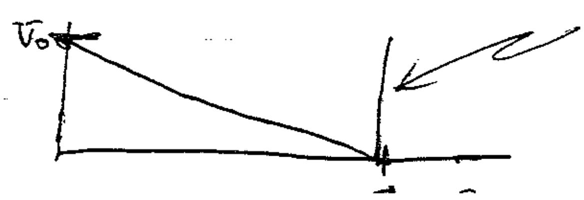
Electromotive Force and Circuits

An electric current always flows down a potential hill. To complete a circuit the current must close on itself or charge will accumulate





V can decrease around a closed loop



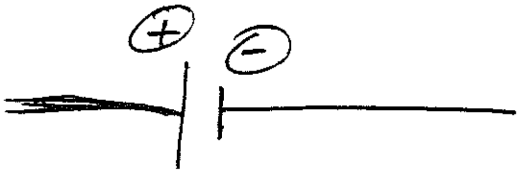
need some mechanism to raise the voltage back up.

source of voltage

An electromotive force ~~[illegible]~~ forces charge up hill to a higher potential (EMF)

⇒ acts against the electric field.

⇒ measured in volts



represents a source of emf.

⇒ example a roller coaster

⇒ the chain pulls the car up a long hill. (emf

⇒ gravity the ~~[illegible]~~ accelerates the car until reaches the start } current driven by potential drop.

⇒ cycle repeats

examples

A battery ⇒ chemical reaction forces charge up the potential

Electric Generator

etc

Van de Graaf Machine.

Kirchoff's Loop Rule

~~The sum of the~~

As you ~~to~~ move completely around a current loop you must arive back at the potential where you started.

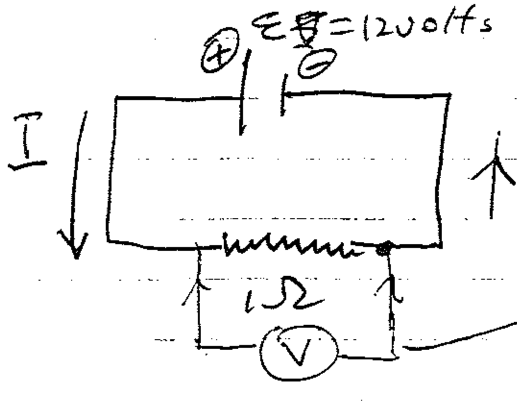
$\Rightarrow$ the sum of the potential drops around the loop must equal the sum of the EMFS.

example

Consider a circuit with an EMF of 12 volts and a resistor with a resistance of $1\,\Omega$.

Neglect resistance of the ~~wire~~ wire. (How make a resistor?)

counter clockwise



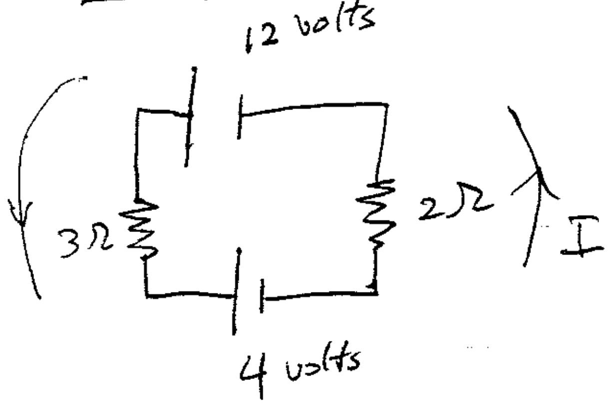
$$-IR + \mathcal{E} = 0$$

$$I = \frac{12\text{ volts}}{1\,\Omega}$$

$$= 12 \text{ amps}$$

$\mathcal{E} - IR = 0$

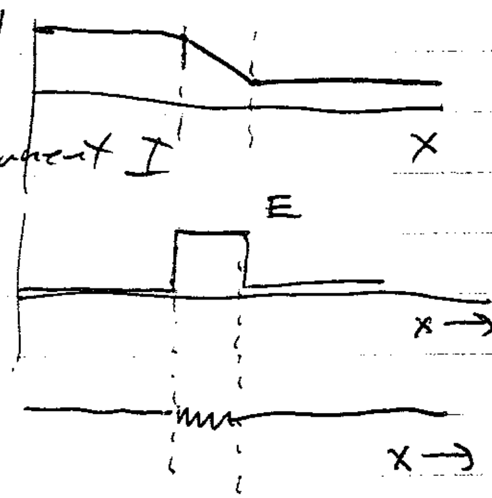
example



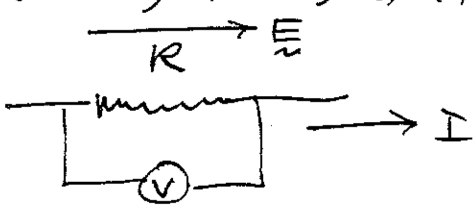
$$-I\,3\Omega - \underbrace{\cancel{4\text{ volts}} - 2\Omega I + \cancel{12\text{ volts}}}_{} = 0 \qquad 8\text{ volts}$$

$$\boxed{I = \frac{8}{5}\text{ amps}}$$

Energy Dissipation in a Resistor



Consider a resistor R with current I flowing through it.



$V = IR$

Consider a charge dQ which moves down the potential hill

$\Rightarrow$ receives energy,

$$dW = dQ\,V$$

from the electric field $\Rightarrow$ lost as heat during collisions in resistor

Rate of energy transfer to heat in resistor

$$\frac{dW}{dt} = P = \text{power} = \frac{dQ}{dt} V$$

$$= IV$$

$$\boxed{P = IV = I^2 R}$$

units $1\,\text{watt} = \frac{\cancel{\text{coul}}}{\text{sec}} \frac{\text{Joule}}{\cancel{\text{coul}}}$

$= \frac{\text{Joule}}{\text{sec}}$

$V = 120$ volts

$R = 10\,\Omega$ $I = 12$ amps

$P = \underline{1440}$ watt.

Source of Energy from EMF

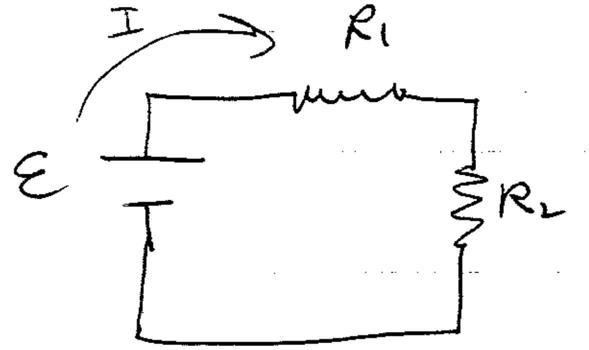
The emf raises the charge dQ up the potential $\mathcal{E}$

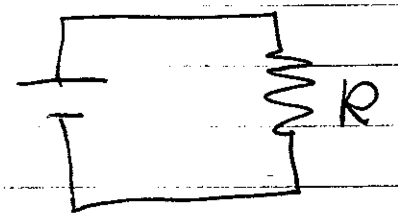
$\Rightarrow$ gives charge $dW = dQ\,\mathcal{E}$

$$\frac{dW}{dt} = I\mathcal{E}$$

$\mathcal{E} I$ = rate at which emf supplies energy to the charges moving through the EMF.

Resistors in Series





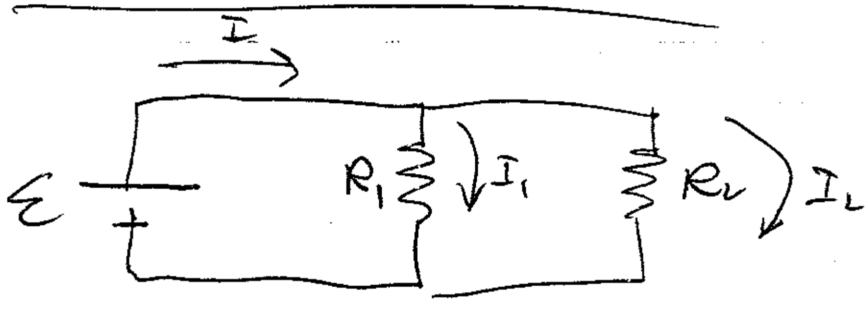
$$\mathcal{E} - I R_1 - I R_2 = 0$$

$$\mathcal{E} - I(R_1 + R_2) = 0$$

$$\boxed{R = R_1 + R_2}$$

$\Rightarrow$ currents through all resistors are the same

Resistors in Parallel



$\rightarrow$ voltage drop across R_1 and R_2 is equa

$$I_1 R_1 = I_2 R_2 \qquad I_1 = I_2 \frac{R_2}{R_1}$$

$$\cancel{\mathcal{E} = IR = (I_1 + I_2) R}$$

$$I = I_1 + I_2 = I_2 \frac{R_2}{R_1} + I$$

$$I_2 = \frac{I}{}$$

$$I = I_2\left(1 + \frac{R_2}{R_1}\right)$$

$$\mathcal{E} = I_2 R_2$$

$$I_2 = \frac{\mathcal{E}}{R_2}$$

$$I = \frac{\mathcal{E}}{R_2}\left(1 + \frac{R_2}{R_1}\right)$$

$$= \mathcal{E}\left(\frac{1}{R_2} + \frac{1}{R_1}\right) = \frac{\mathcal{E}}{R}$$

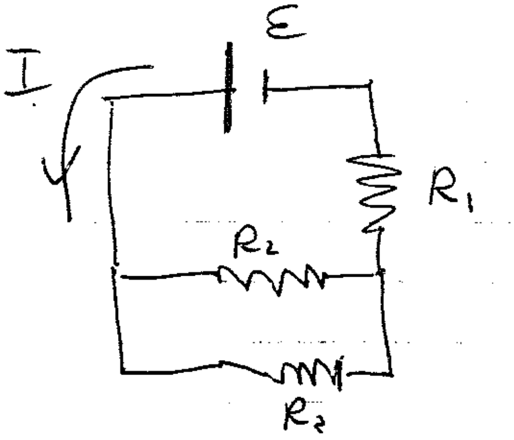
$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

~~Kirchoff's Rules~~

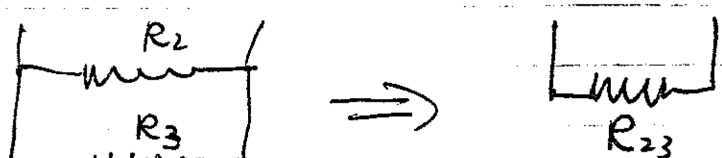
~~For circuits more~~

Mixed Circuits

Consider the circuit shown.

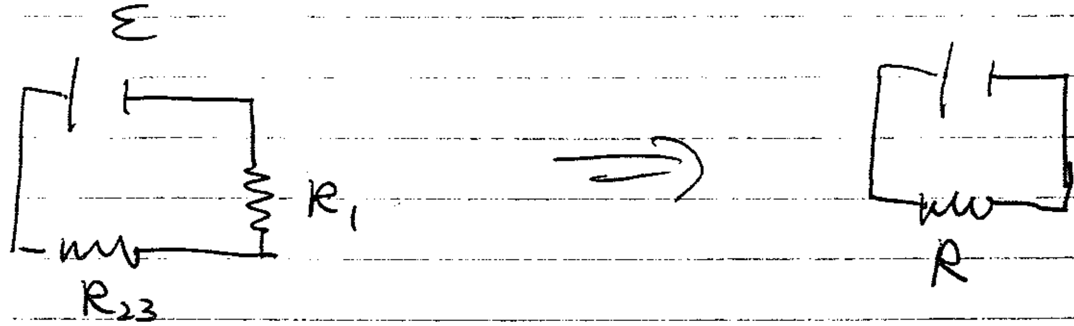


First consider



$$\frac{1}{} = \frac{1}{} + \frac{1}{}$$

We are left with



$$R = R_1 + R_{23}$$

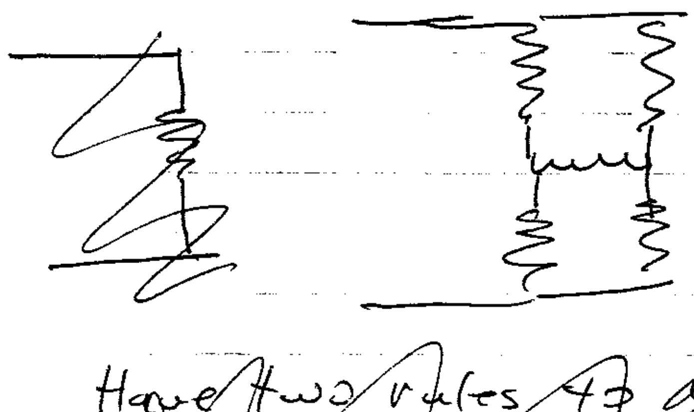
$$\boxed{R = R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

~~More Complex~~

Kirchoffs Rules

Some circuits can not be collapsed to parallel resistors or series resistors

Eg.



~~Have two rules to deal with such circuits.~~

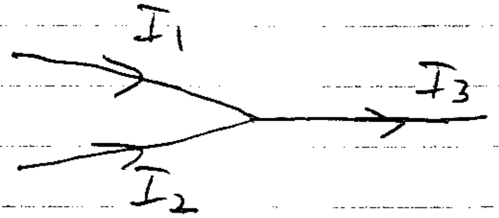
A **branch** point is a point where three or more conductors join together.

A **loop** is any closed path on the circuit.

Have two rules to deal with these complex circuits.

1. **point rule** — the sum of the currents **towards** any branch point is zero

$$\sum_i I_i = 0$$



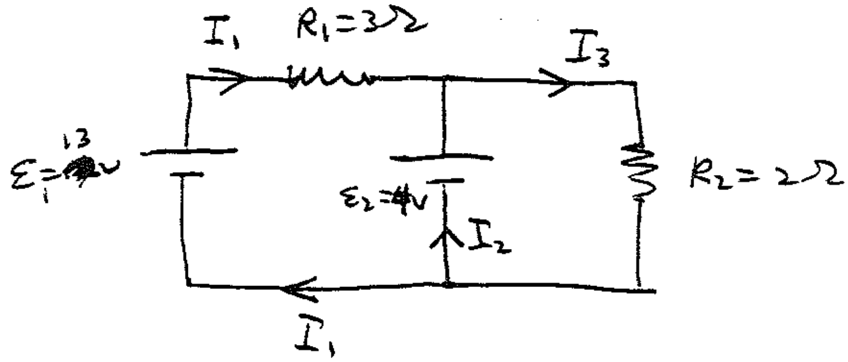
note that current toward B.P. are
I_1
I_2
$-I_3$

$$I_1 + I_2 - I_3 = 0$$

2. The sum of the potential differences around any loop must equal zero.

$\Rightarrow$ includes EMFs

example



pt rule

$$I_1 + I_2 - I_3 = 0$$

Loop rule

$$\mathcal{E}_1 - I_1 R_1 - \mathcal{E}_2 = 0 \implies \boxed{I_1 = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1}} \qquad I_1 = \frac{9v}{3\Omega} =$$

$$\mathcal{E}_2 - I_3 R_2 = 0$$

$$\boxed{I_3 = \frac{\mathcal{E}_2}{R_2}}$$

$$I_2 = I_3 - I_1 = -1A$$

$$\cancel{= \frac{\mathcal{E}_2}{R_2}}$$

$$I_3 = \frac{4v}{2\Omega} = 2A$$

what does this mean?

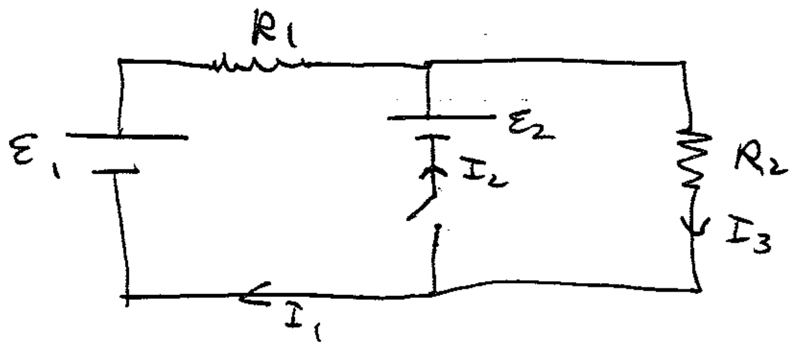
voltage drop across R_1?

$$I_1 R_1 = 9 \text{ volts}$$

voltage across R_2

$$I_3 R_2 = 4 \text{ volts}$$

Add switch on I_2 wire



$\Rightarrow I_2 = 0$

$\Rightarrow I_1 = I_3$

$$\mathcal{E}_1 - I_1R_1 - I_3R_2 = 0$$

$$\mathcal{E}_1 - I_1R_1 - I_1R_2 = 0$$

$$I_1 = \frac{\mathcal{E}_1}{R_1+R_2} = \frac{13}{5}\,A.$$

what is pot drop across R_1? $\quad R_2$?

$I_1R_1 = \frac{39}{5}\,V \qquad \frac{26}{5}\,V$

across switch?

$$\Delta V + \mathcal{E}_2 + I_1R_1 - \mathcal{E}_1 = 0$$

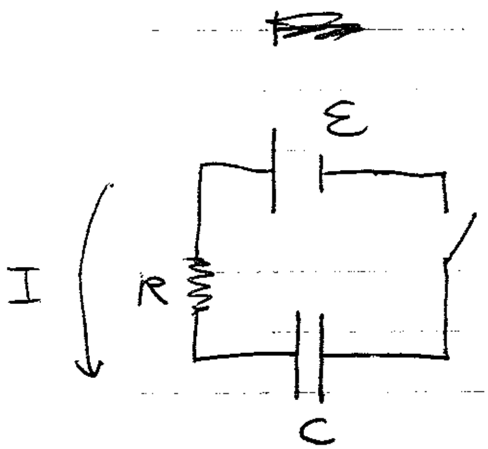
$$\Delta V + 4V + \frac{39}{5}V - 13V = 0$$

$$\Delta V - 9V + 7\tfrac{4}{5}V = 0$$

$$\Delta V = 1\tfrac{1}{5}V$$

RC Circuit

Consider a circuit with an emf $\mathcal{E}$ a resistor R and a capacitor C.



At $t=0$ we throw the switch. A current will flow. Charge will build up on the capacitor until the potential drop across the capacitor is equal to $\mathcal{E}$. To show this, add up the potential ~~changes~~ jumps around the loop.

$$V_c = \frac{Q}{C}$$

$$\mathcal{E} - IR - \frac{Q}{C} = 0$$

$$\mathcal{E} - R\frac{dQ}{dt} - \frac{Q}{C} = 0$$

$$\frac{dQ}{dt} + \frac{1}{RC}Q = \frac{\mathcal{E}}{R}$$

Let $\tau = RC$

$$\frac{dQ}{dt} + \frac{1}{\tau} Q = \frac{\mathcal{E} C}{RC} = \frac{\mathcal{E} C}{\tau}$$

τ = time constant for RC circuit

$\Longrightarrow$ characteristic time required to build up charge on the capacitor.

$\Longrightarrow$ larger resistance to current flow $\Longrightarrow$ longer time

$\Longrightarrow$ larger capacitance longer time.

(over)

Solution

$$Q = \frac{\mathcal{E}\tau \cancel{C}}{R\cancel{C}}\left(1 - e^{-t/\tau}\right)$$

For $t \gg \tau \Longrightarrow e^{-t/\tau} \ll 1$

$$Q \simeq \frac{\mathcal{E}\tau}{R\cancel{C}} = \mathcal{E} C$$

$\Longrightarrow$ potential across capacitor $= \mathcal{E}$

$$I = \frac{dQ}{dt} = \frac{\mathcal{E}\tau}{R}\left(\frac{d}{dt}1 - \frac{d}{dt} e^{-\frac{t}{\tau}}\right)$$

$$= \frac{\mathcal{E}\tau}{R}\left(\frac{1}{\tau} e^{-\frac{t}{\tau}}\right) = \frac{\mathcal{E}}{R} e^{-t/\tau}$$

$t \ll \tau \quad I = \frac{\mathcal{E}}{R} \qquad t \gg \tau \quad I \to 0.$

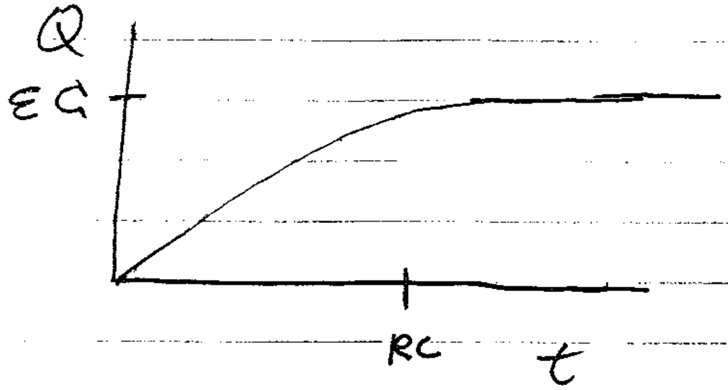
$$\frac{dQ}{dt} = \frac{\varepsilon C - Q}{\tau}$$

$$\int_0^Q \frac{dQ}{\varepsilon C - Q} = \int_0^t \frac{dt}{\tau}$$

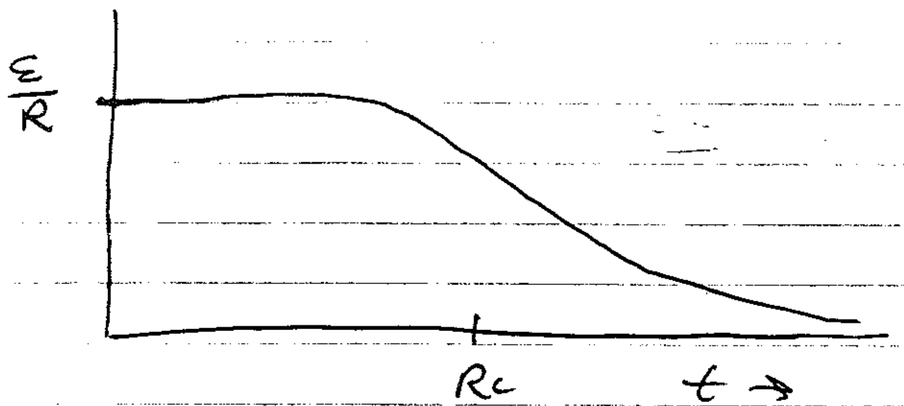
$$+ \ln(\varepsilon C - Q)\Big|_0^Q = -\frac{t}{\tau}$$

$$\frac{\varepsilon C - Q}{\cancel{\varepsilon C}} = e^{-\frac{t}{\tau}}(\varepsilon C)$$

$$Q = \varepsilon C\left(1 - e^{-\frac{t}{\tau}}\right)$$

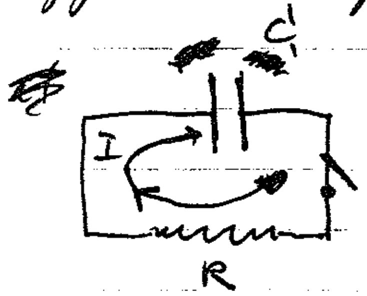


$$I = \frac{dQ}{dt}$$



Discharging a Capacitor

Suppose a capacitor C has a charge Q_0.



The capacitor is connected in series with a resistor. At $t=0$ the switch is closed.

$$-IR - \frac{Q}{C} = 0$$

$$\frac{dQ}{dt} + \frac{Q}{\tau} = 0$$

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\int_0^t \frac{dt}{\tau} = -\frac{t}{\tau}$$

$$\ln \frac{Q}{Q_0} = -\frac{t}{\tau} \qquad \frac{Q}{Q_0} = e^{-\frac{t}{\tau}}$$