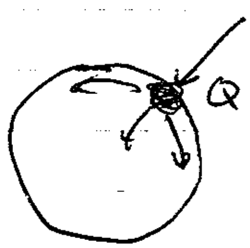
Current and The Motion of Charge

In our study of electrostatics we were primarily interested charge distributions which were stationary in time. ~~We assumed that~~ We are ~~intested~~ interested in investigating the motion of charge.

We say that charges in motion produce a current. How do we produce such a current. Suppose we take a conductor and place charge on it in some local region. The charge will ~~b~~ move due to the



local electric field $\underset{\sim}{E}$ and spread out until $\underset{\sim}{E} = 0$ inside the conductor. The current is a transient which flows only when $\underset{\sim}{E} \neq 0$.

Once $\underline{E} = 0$ the current dies away. why?

In a conductor the electrons can move but they are constantly bumping into the particles which make ~~[illegible]~~ up the ~~[illegible]~~ crystal lattice of the metals. As the bump into these stationary particles they lose their energy $\Rightarrow$ ~~the current dies~~

basically friction can not be neglected.

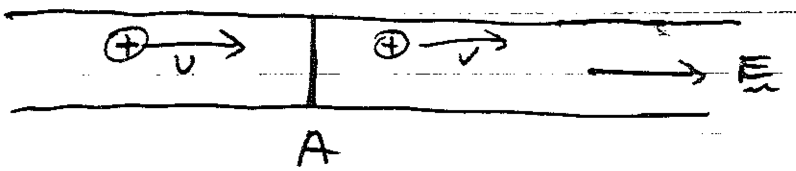
Thus, the current dies away when $\underline{E} = 0$.

$\Rightarrow$ To maintain a current must maintain the electric field.

$\Rightarrow$ For now we will assume there is some mechanism for maintaining $\underline{E}$ in a conductor so that charges remain in motion.

~~Consider a conductor that~~

Consider a thin wire made of a conductor along which have an electric field

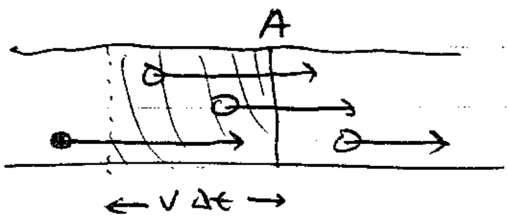


Suppose that the charges have a velocity v along $\underset{\sim}{E}$.

$$I = \text{current} = \frac{\Delta Q}{\Delta t}$$

ΔQ = total charge which crosses the area A during a time Δt

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$



units ampere = $\frac{\text{coul}}{\text{sec}}$

I is a scalar.

Speak of current flowing in a wire ⇒ don't need to specify the direction in space

All the moving charge in the shaded volume crosses A during Δt.

$$\Delta Q = \underbrace{\overbrace{A v \Delta t}^{\Delta V}\, n}_{\Delta N = \#\text{ of charges}} q$$

with

$$n = \frac{\#\text{ of moving charges}}{\text{unit volume}}$$

$$q = \text{individual charge}$$

$$I = \frac{qAvn\,\Delta t}{\Delta t} = qAvn$$

~~units I = Amperes = 1 coul/sec ⇒ current is a scalar~~

Define the current per unit area J as

$$J = \frac{I}{A} = qnv$$

Can define the current density as a vector

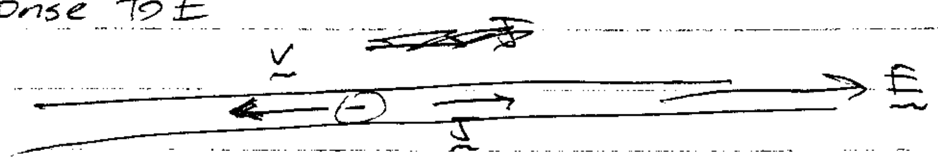
$$\underset{\sim}{J} = qn\,\underset{\sim}{v}$$

where $\underset{\sim}{v}$ is typically in the direction of $\underset{\sim}{E}$.

~~Resistivity~~

~~Consider~~

In a metallic conductor the electrons move in response to E



$$\underset{\sim}{J} = nq\underset{\sim}{v} = -ne\underset{\sim}{v}$$

$\Rightarrow$ $\underset{\sim}{J}$ is the same direction as $\underset{\sim}{E}$ even though $\underset{\sim}{v}$ opposite direction.

Drift velocity of electrons

In Cu $n = 8.49 \times 10^{28} / m^3$

$A = 1\,mm^2$

$I = 1A$

$J = nev$

$$v = \frac{1A/mm^2}{1.6\times10^{-19}\,C \; 8.5\times10^{28}/m^3}$$

$$= \frac{\cancel{C}}{s}\,\frac{1 \qquad m^{\cancel{3}}}{10^{-6}\,\cancel{m^2}\;1.6\times10^{-19}\,\cancel{C}\;8.5\times10^{28}}$$

$$= \frac{10^{-3}}{(8.5)(1.6)}\,\frac{m}{s} = 7.4\times10^{-5}\,\frac{m}{s}$$

Resistivity

How do electrons respond to $\underset{\sim}{E}$ in a conductor?

$$m\frac{d\underset{\sim}{v}}{dt} = -e\underset{\sim}{E} - m\nu_e\underset{\sim}{v}$$

ν_e = rate at which electron collide with the background lattice of the metal

$\Longrightarrow$ similar to frictional force of an object moving through air.

The velocity increases until the force of the electric field balances the drag.

$$m\nu_e\underset{\sim}{v} = -e\underset{\sim}{E}$$

$$\underset{\sim}{v} = -\frac{e\underset{\sim}{E}}{m\nu_e}$$

$$\underset{\sim}{J} = -ne\underset{\sim}{v} = \frac{ne^2\underset{\sim}{E}}{m\nu_e}$$

$$\frac{m\nu_e}{ne^2} = \eta = \text{resistivity}$$

note that this is intrinsic to the material

$$\boxed{\eta\underset{\sim}{J} = \underset{\sim}{E}}$$

The current density is proportional to the electric field.

1.69×10^{-8} copper

Temperature dependence of resistivity

$$\eta = \frac{m_e \nu_e}{n e^2}$$

ν_e = collisional rate

In classical picture electrons move through the lattice at their thermal velocity

$$\frac{3}{2} k T_e = \frac{1}{2} m_e v_{te}^2$$

$$k = 1.38 \times 10^{-23} \frac{J}{^\circ K} \quad T_e \sim 300\,^\circ K$$

$$m_e = 9.11 \times 10^{-31} Kg$$

$$v_{te} = \left(\frac{3kT_e}{m_e}\right)^{\frac{1}{2}} = \left(\frac{3 \; 1.4 \times 10^{-23} \frac{\cancel{J}}{\cancel{^\circ K}} \; 300 \cancel{^\circ K}}{9.1 \times 10^{-31} \cancel{Kg}}\right)^{\frac{1}{2}} \quad J = \cancel{Kg}\frac{m^2}{s^2}$$

$$= \left(\frac{1.4 \times 10^{-23} \; 10^{2}}{10^{-31}}\right)^{\frac{1}{2}}$$

$$\simeq 1.2 \times 10^{5} \frac{m}{s}$$

$\Rightarrow$ larger than drift speed.

Effective volume swept out by electrons in time t

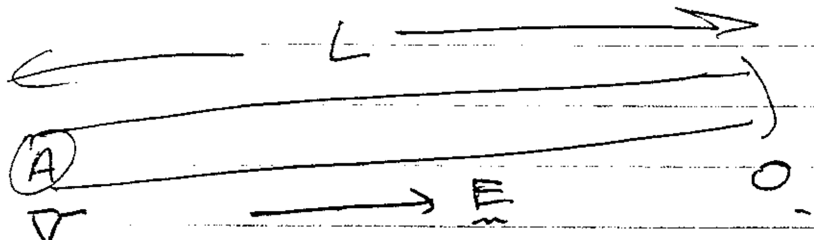
$$V = \sigma v_{te} t$$

$$\# \text{ of collisions} = n \sigma v_{te} t$$

$$\text{rate of collisions} = \underline{\underline{n \sigma v_{te}}} = \nu_e$$

$\Rightarrow$ ~~[illegible]~~ increases with temperature.

Potential Drop along a Wire



current flows along the direction of potential drop,

Apply a potential V along a wire of length L.

$$E = \frac{V}{L}$$

$$\Longrightarrow J = \frac{1}{\rho} E = \frac{1}{\rho} \frac{V}{L}$$

$$I = JA = \frac{A}{\rho} \frac{V}{L}$$

$$R = \frac{\rho L}{A} = \text{resistance}$$

$$\boxed{IR = V}$$ Ohm's Law

$\Longrightarrow$ gives the relation between current and voltage in a wire.

$\Longrightarrow$ large L $\rightarrow$ larger resistance

smaller A $\rightarrow$ " "

units $1\,\text{Ohm} = \dfrac{1\,\text{volt}}{\text{amp}}$ of R $\Longrightarrow 1\,\Omega$

ρ units Ohm-meters

Example

resistivity of copper $\rho = 1.72 \times 10^{-8}\ \Omega m$

steel 20×10^{-8}

How long is a copper wire with $A = (1mm)^2$ which has $R = 1\Omega$.

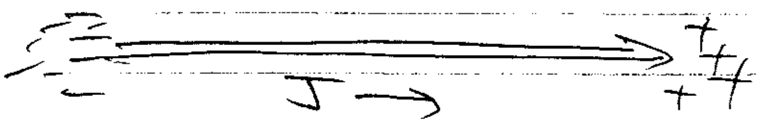
$$R = 1\Omega = \frac{1.72\times \cancel{10^{-8}}^{\,10^{-2}}\ \Omega m\ L}{\cancel{10^{-6}}\ m^2}$$

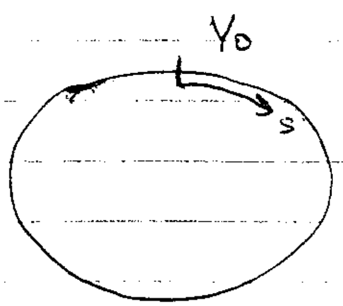
$$L_c = \frac{1}{.0172} m \sim 58 m$$

$$L_s = \frac{1}{.2} m = 5m$$

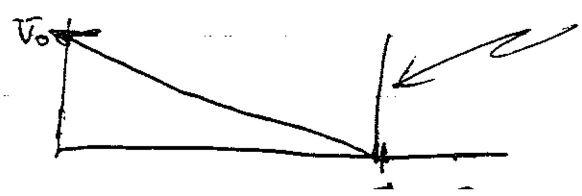
Electromotive Force and Circuits

An electric current always flows down a potential hill. To complete a circuit the current must close on itself or charge will accumulate





V can decrease around a closed loop



need some mechanism to raise the voltage back up.