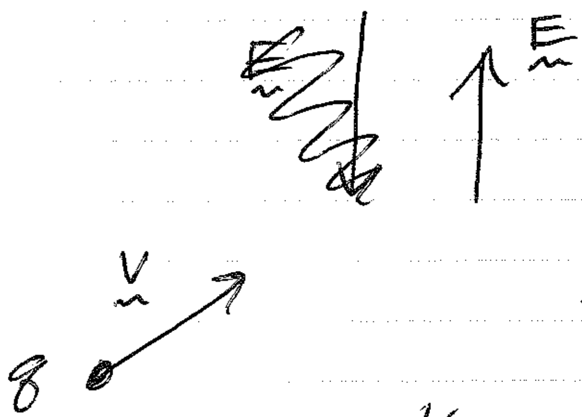
Energy and Electrical Potential

Consider a charge q' moving in a uniform electric field. If q' is ~~greater than~~



positive then the charge is ~~repelled~~ pushed along the direction of $\mathbf{E}$ similar to the motion in a gravitational field.

We can also define a potential function U similar to the gravitational potential.

To show this, we write

$$m\frac{d\mathbf{v}}{dt} = q'\mathbf{E}$$

take dot product with $\mathbf{v}$

$$m\,\mathbf{v}\cdot\frac{d\mathbf{v}}{dt} = q\mathbf{E}\cdot\mathbf{v} = qE\,\cancel{\frac{dy}{dt}}\,v_y$$

$$m\frac{d}{dt}\frac{1}{2}v^2 = qE\frac{dy}{dt}$$

$$\frac{d}{dt}\left(\tfrac{1}{2}mv^2 - q'Ey\right) = 0$$

Let $U = -q'Ey$ = potential energy

$$\frac{d}{dt}(K+U) = 0 \qquad \text{Energy} = K+U = \text{const.}$$

Can also define the potential per unit charge

$$V = \frac{U}{q'} = \text{electrical potential} = -Ey$$

$$\text{Energy} = K + q'V = \text{const.}$$

units of $V \longrightarrow$ volts $= \frac{\text{joules}}{\text{coulomb}}$.

$\Rightarrow$ potential goes down as move in the direction of E.

$E = -\frac{\partial V}{\partial y}$

Advantage of using the potential to calculate the motion of charged particles is that the energy is a scalar

$\Rightarrow$ don't have to worry about vector directions.

To calculate the change in KE as the charge q' moves from y_1 to y_2

$$K_1 - q'Ey_1 = K_2 - q'Ey_2$$

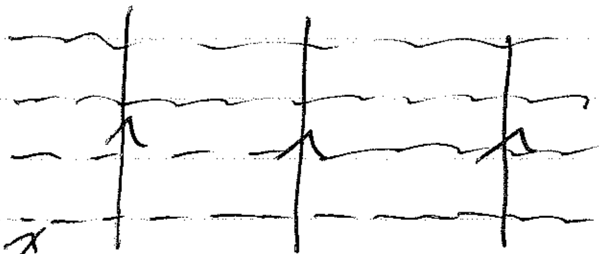
$$K_2 = K_1 + q' E (y_2 - y_1)$$

KE increases for $q' > 0$ if $y_2 > y_1$

positive charge moves to lower potential V

~~positive~~ negative charge moves to higher potential V.

$\underset{\sim}{E}$



$\uparrow$ V decreases in this direction

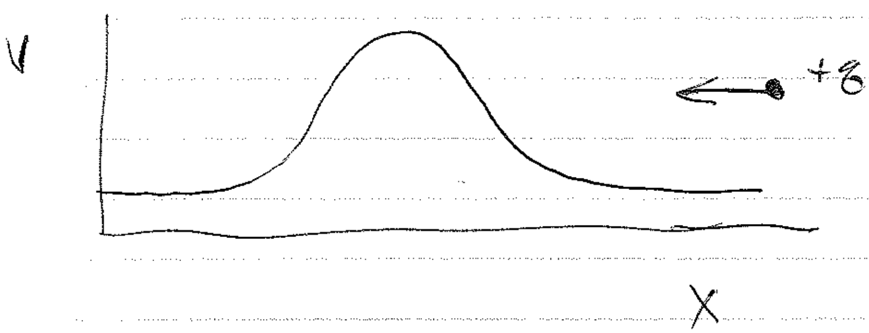
$\downarrow$ ~~V increases in this direction~~

E points in direction of decreasing potential.

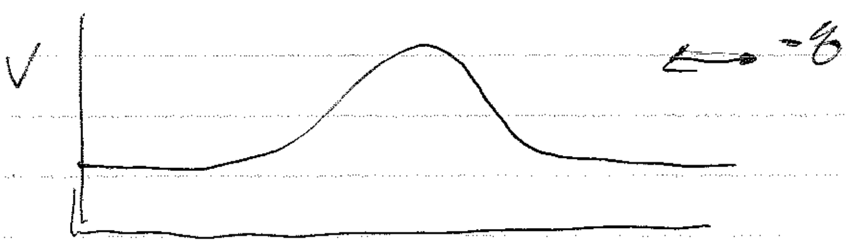
contours of constant potential V

Note that $\underset{\sim}{E}$ is $\perp$ to contours of V } this is a general result

$$\Longrightarrow \quad E = -\frac{dV}{dy}$$



what happens?



what happens?

example

An electron is accelerated through a potential difference of 1 kV. What is its velocity?

Energy = const

$$K_i + qV_i = K_f + qV_f$$

$$K_f = q(V_i - V_f)$$

$$= +e\,\Delta V$$

$$\frac{1}{2}mv^2 = e\,\Delta V$$

$$v^2 = 2\,\frac{1.6\times10^{-19}\,\cancel{C}}{9.11\times10^{-31}\,\cancel{kg}}\;\frac{\cancel{\text{Joule}}}{\cancel{C}}\;10^{3} \qquad \frac{\cancel{kg}\,m^2}{s^2}$$

$$= 3.51\times10^{14}\ \frac{m^2}{s^2}$$

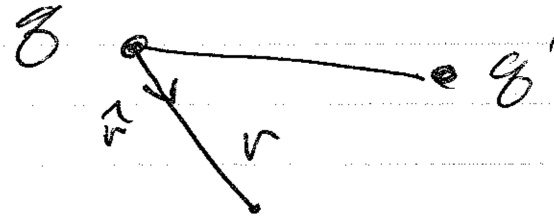
$$v = 1.9\times10^{7}\ \frac{m}{s}$$

~~$K_2 = K_1 + q'E(y_2 - y_1)$~~

~~KE increases for q' so if $y_2 > y_1$~~

Potential Energy of Point Charges

Consider a charge q. Take a charge q' and move it towards q. If $q' > 0$ then have to do work



to move q' towards q.

$\Longrightarrow$ q' moves into a region of higher potential.

To calculate the potential, we can again study the motion of q' in the electric field due to a fixed charge q.

The electric field due to q is

$$\underset{\sim}{E} = \frac{q}{4\pi\epsilon_0 r^2}\,\hat{r}$$

~~motion of q'~~

Motion of q'

$$m \frac{d\mathbf{v}}{dt} = q' \mathbf{E}$$

$$m\, \mathbf{v} \cdot \frac{d}{dt} \mathbf{v} = q' \mathbf{E} \cdot \mathbf{v}$$

$$\frac{d}{dt} \frac{1}{2} m v^2 = q' \frac{q}{4\pi\epsilon_0 r^2}\, \hat{r} \cdot \mathbf{v}$$

$\mathbf{r} = r\hat{r}$

$$= \frac{q'q}{4\pi\epsilon_0 r^2}\, \hat{r} \cdot \frac{d\mathbf{r}}{dt}$$

$$= \frac{q'q}{4\pi\epsilon_0 r^3}\, \mathbf{r} \cdot \frac{d\mathbf{r}}{dt}$$

$$= \frac{q'q}{4\pi\epsilon_0 r^3}\, \frac{1}{2} \frac{d}{dt} r^2$$

$$= \frac{q'q}{4\pi\epsilon_0} \frac{1}{r^3}\, r \frac{dr}{dt}$$

$$= -\frac{q'q}{4\pi\epsilon_0} \frac{d}{dt} \frac{1}{r}$$

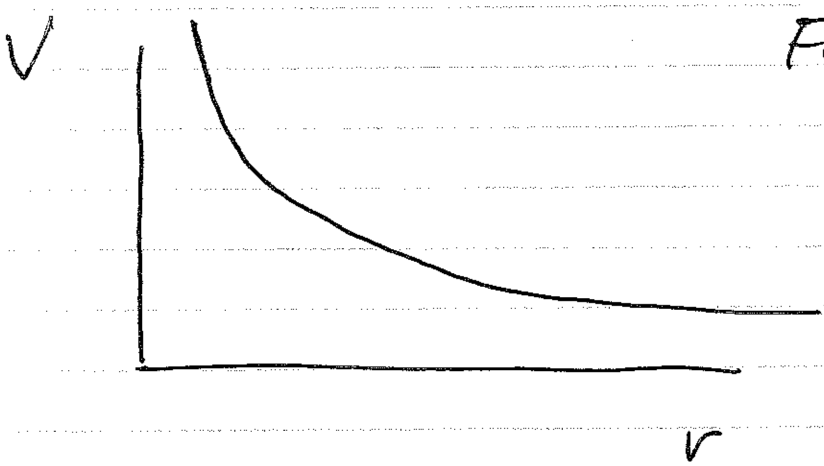
$$\frac{d}{dt}\left(\frac{1}{2} m v^2 + \frac{q'q}{4\pi\epsilon_0} \frac{1}{r}\right) = 0$$

$$U = \frac{q'q}{4\pi\epsilon_0} \frac{1}{r}$$ = potential energy stored as q' moves to a distance r of q.

As with uniform field can define the potential energy per unit charge q' as

$$V = \frac{U}{q'} = \frac{q}{4\pi\epsilon_0}\frac{1}{r}$$

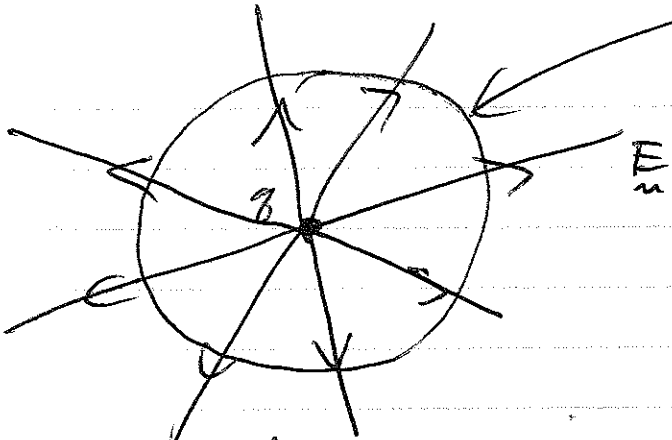
V is an electric potential
units: volts = joule/coulomb
which surrounds the charge q.



Falls off like $\frac{1}{r}$ at large r.

Positive charges $q' > 0$ are repelled from the potential,

Negative charges $q' < 0$ are attracted by this potential.



contour of constant V

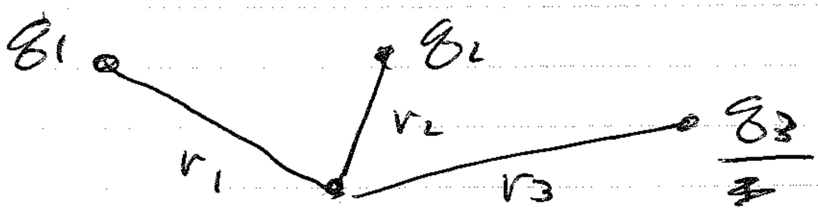
Again $\underset{\sim}{E}$ $\perp$ to constant potential contours

potential increases toward the charge q.

$$\boxed{E = -\frac{\partial V}{\partial r}}$$

What if we have several charges.

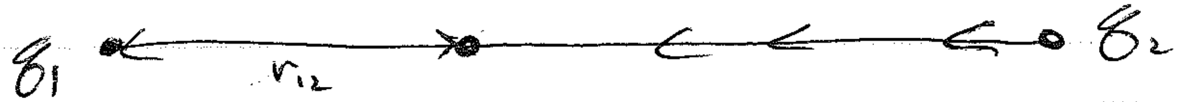
What is the potential at r if several charges are present?



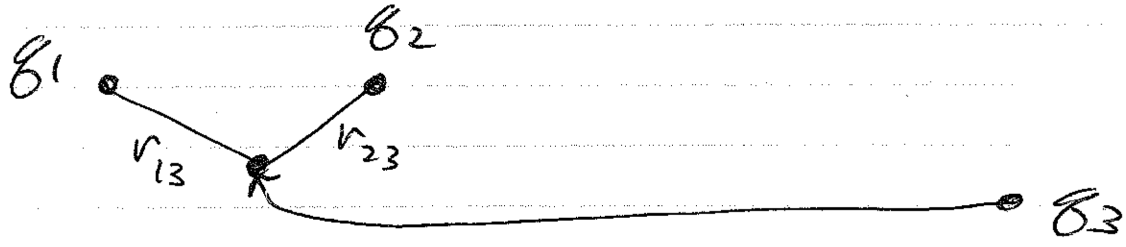
$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

$\Rightarrow$ note that the potential is a scalar so can add the potentials without doing vector addition.

What is the energy required to assembly a group of charges?



To move q_2 from infinite to r_{12} requires energy $\quad U_{12} = \dfrac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$



To bring q_3 requires

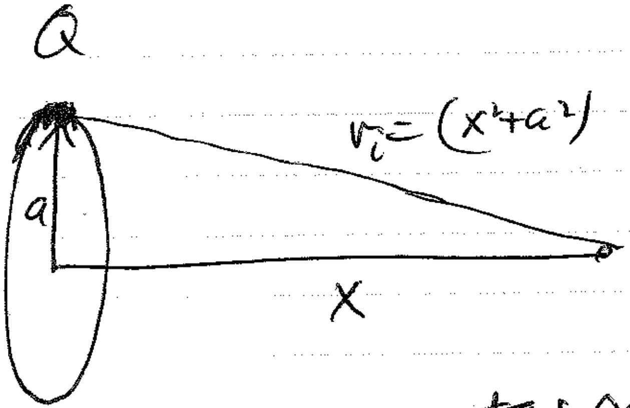
$$U_{13} = \frac{q_1 q_3}{4\pi\epsilon_0 r_{13}}$$

$$U_{23} = \frac{q_2 q_3}{4\pi\epsilon_0 r_{23}}$$

Total potential energy,

$$U = U_{12} + U_{23} + U_{13} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{13}} + \frac{q_1}{q_{13}}\right.$$

Potential Due to A Charged Ring



r_i = same for all charges.

$$V = \sum_i \frac{\Delta q_i}{4\pi\epsilon_0 r_i} = \sum_i \frac{\Delta q_i}{4\pi\epsilon_0 (x^2+a^2)^{\frac{1}{2}}}$$

$$V = \frac{1}{4\pi\epsilon_0 (x^2+a^2)^{\frac{1}{2}}} Q$$

Techniques for calculating the potential

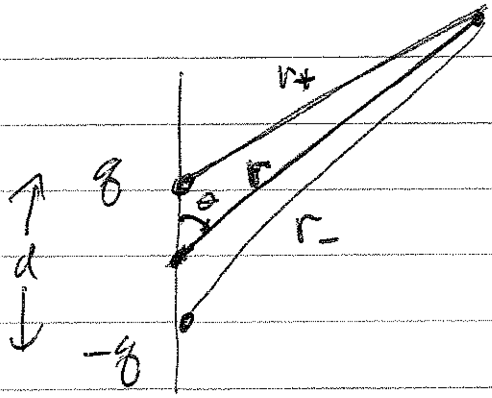
$\Rightarrow$ sum over the potential due to individual charges

$\Rightarrow$ can calculate the change in potential by integrating $\underset{\sim}{E}$

$$\frac{d}{dt} K = q' \underset{\sim}{E} \cdot \underset{\sim}{v} = q' \underset{\sim}{E} \cdot \frac{d\underset{\sim}{x}}{dt}$$

$$\int_1^2 dK = K_2 - K_1 = q' \int_{\underset{\sim}{x}_1}^{\underset{\sim}{x}_2} \underset{\sim}{E} \cdot d\underset{\sim}{x}$$

Potential From a Dipole



$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$

$$=$$

$$\frac{\underline{d}}{2} + \underline{r}_+ = \underline{r}$$

$$\underline{r}_+ = \underline{r} - \frac{\underline{d}}{2}$$

$$r_+^2 = r^2 - 2\,\underline{r}\cdot\underline{d}\,\frac{1}{2} + \frac{d^2}{4}$$

$$\simeq r^2 - \cancel{2}\,r\,d\cos\theta$$

$$r_+ \simeq r\left(1 - \frac{d}{r}\cos\theta\right)^{\frac{1}{2}}$$

$$\simeq r\left(1 - \frac{1}{2}\frac{d}{r}\cos\theta\right)$$

$$r_+ = r - \frac{1}{2}d\cos\theta$$

$$r_- = r + \frac{1}{2}d\cos\theta$$

$$\frac{p\cos\theta}{4\pi\epsilon_0 r^2}$$

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r - \frac{1}{2}d\cos\theta} - \frac{1}{r + \frac{1}{2}d\cos\theta}\right) \,\Big\|$$

$$= \frac{1}{r}\frac{q}{4\pi\epsilon_0}\left(\cancel{1} + \frac{1}{2}d\cos\theta - \left(\cancel{1} - \frac{1}{2}\frac{d}{r}\cos\theta\right)\right)$$

$$\underline{K_2 - K_1 - q'\int_{x_1}^{x_2} E\cdot dx} = 0$$

$$\Delta U = U_2 - U_1 = q'(V_2 - V_1)$$

$$V_2 - V_1 = \Delta V = -\int_{x_1}^{x_2} E\cdot dx$$

w = work done by E on charge. If $w > 0$ then U must be negative

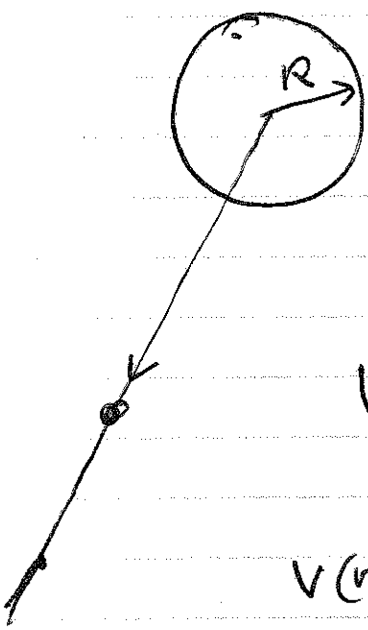
Also note that $w = \int_{x_1}^{x_2} qE\cdot dx$ so

$$-q(V_2 - V_1) = w$$

$$w = -(U_2 - U_1)$$

<u>example</u>

potential around a conducting sphere with charge Q



$r > R$

$$E = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}\hat{r}$$

Let $x_1 \longrightarrow \infty$ and $V_1 = 0$ at ∞.

$$V = -\int_{\infty}^{r} E\cdot dr$$

dr opposite in direction to E

$$V(r) = -\int_{\infty}^{r}\left(\frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}\right)dr$$

$$= -\frac{Q}{4\pi\epsilon_0}\int_{\infty}^{r}\frac{dr}{r^2} = -\frac{Q}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)\Big|_{\infty}^{r}$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{1}{r}$$

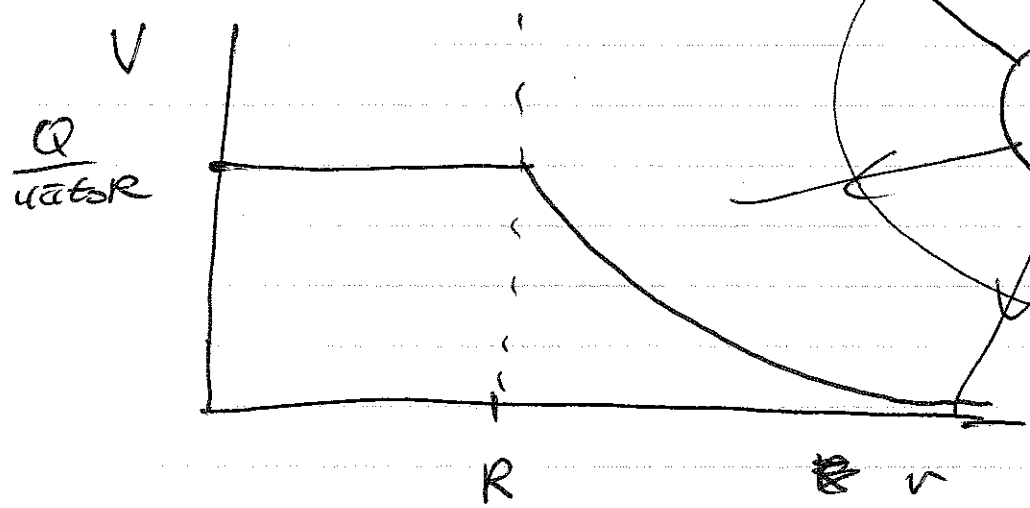
Same as point charge

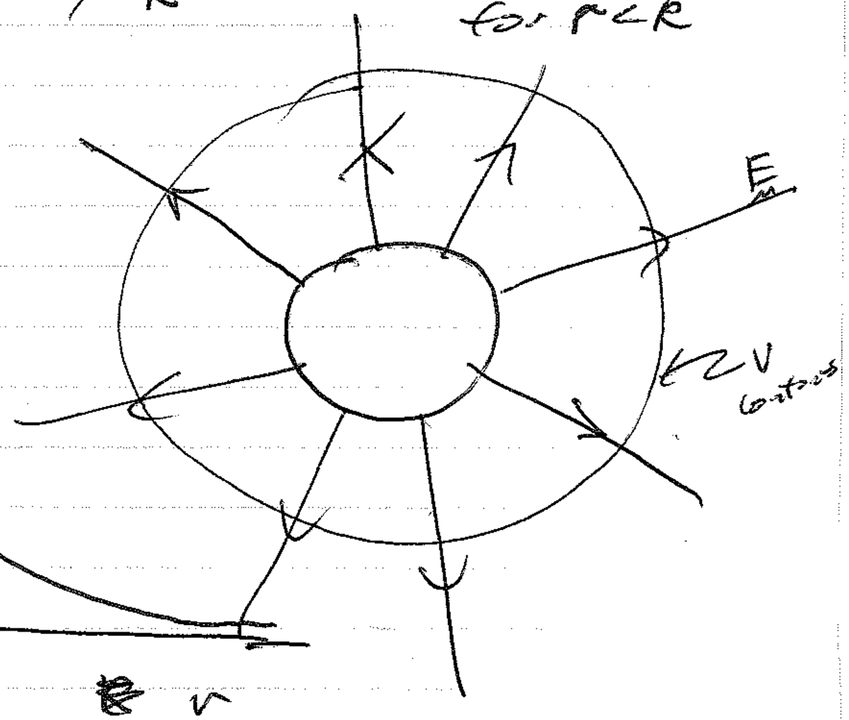
For $r < R$

$$V = -\int_{\infty}^{R} E\,dr + \cancel{\int_{R}^{r} E\,dr} \xrightarrow{} 0$$

since $E = 0$ for $r < R$

$$= \frac{Q}{4\pi\epsilon_0 R}$$





Potential Due to a Line Charge

$\lambda = \frac{\text{charge}}{\text{unit length}}$

$$\Delta V = -\int_{\underline{x}_1}^{\underline{x}_2} \underline{E} \cdot d\underline{x} = -\frac{\lambda}{2\pi\epsilon_0}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$\underline{E} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}\hat{r}$$

$$= -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_2}{r_1}\right)$$

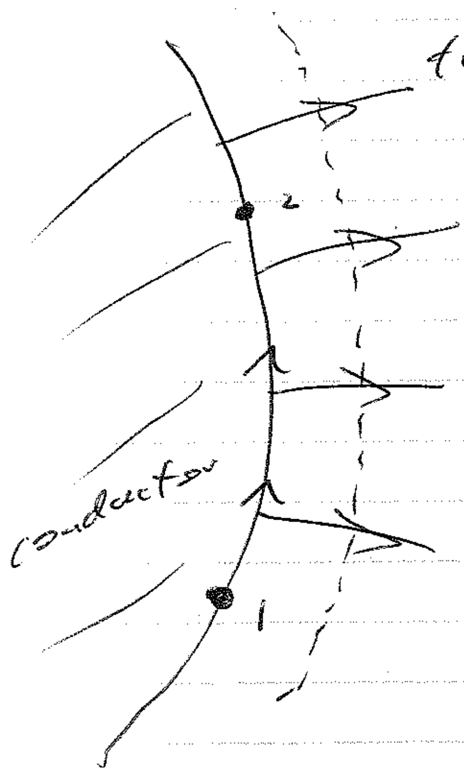
$\Rightarrow$ can't let $r_1 \to \infty$ since $V \to \infty$

$\ln \infty = \infty$

$\Rightarrow$ can only talk about potential difference between points

Potentials Near Conductors

We know that the electric field in a conductor is zero and the electric field at the surface of a conductor is $\perp$ to the conductor. Can calculate



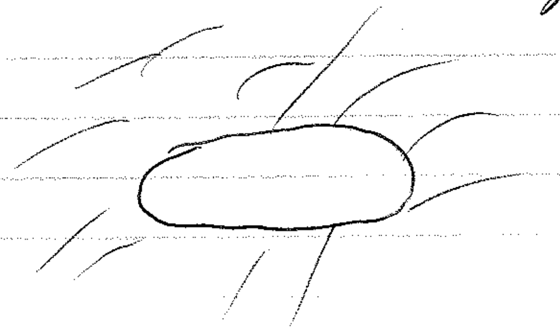
the change in the potential between two points on ~~the~~ a ~~surface of the~~ conductor

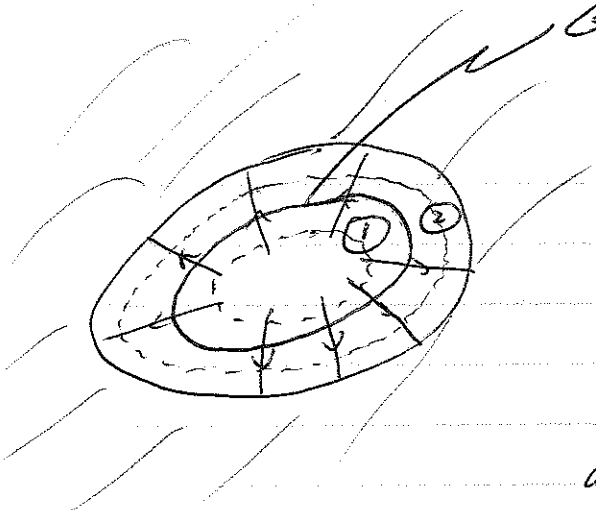
$$\Delta V = -\int_1^2 d\underline{x}\cdot\underline{E} = 0$$

Since $d\underline{x}\cdot\underline{E} = 0$ at surface and $\underline{E} = 0$ inside.

The potential is constant throughout a conductor

What if have an open cavity in a conductor?
Assume no charge inside.





Is potential constant in cavity?
Near surface at ② have ~~Assume~~ const. potential because near a conductor.

Suppose have another const potential contour ① inside ② at diff potential.

Then must have electric field between ① and ② and must be all pointing inwards or outwards. Draw gaussian surface Ⓖ

$$\oint \underset{\sim}{E} \cdot d\underset{\sim}{A} = \frac{\text{charge enclosed}}{\epsilon_0} = 0$$

$$\Rightarrow \underset{\sim}{E} = 0$$

$\Rightarrow$ potential is constant.

You can shield electric fields out of a region of space by enclosing the region with a conducting material $\rightarrow$ copper.

Summary of ~~Pot~~ Properties of Electric Potential

① The electric potential is the energy per unit charge

$$U = q' V$$

② A positive potential surrounds all positive charge.
A negative potential surrounds all negative charges.

③ The potential from a point charge q is

$$V = \frac{q}{4\pi\epsilon_0 r}$$

④ The change in potential ~~that been~~ between $\underset{\sim}{x}_1$ and $\underset{\sim}{x}_2$ is given by

$$\Delta V = V_2 - V_1 = -\int_{\underset{\sim}{x}_1}^{\underset{\sim}{x}_2} d\underset{\sim}{x} \cdot \underset{\sim}{E}$$

⑤ Constant potential contours are $\perp$ to the local electric field.

6 The electric field can be calculated from the ~~magnetic~~ potential

$$\left.\begin{aligned} E_x &= -\frac{\partial V}{\partial x} \\ E_y &= -\frac{\partial V}{\partial y} \\ E_z &= -\frac{\partial V}{\partial z} \end{aligned}\right\} \quad \underset{\sim}{E} = -\nabla V$$

$$\nabla = \hat{i}\,\frac{\partial}{\partial x} + \hat{j}\,\frac{\partial}{\partial y} + \hat{k}\,\frac{\partial}{\partial z}$$

Electron Volt

An electron volt is the energy associated with moving an electron through a potential drop of one volt.

$$1\,ev = 1.60 \times 10^{-19}\,C \cdot \frac{1\ \text{joule}}{\text{Coul.}}$$

$$= \underline{1.6 \times 10^{-19}\ \text{Joules}}$$