Electrostatics

Glass rod rubbed with silk — positive.

Plastic rod rubbed with fur. — negative

Glass rod → pith +
plastic rod → pith −

glass repells glass
plastic repells plastic
glass attracts plastic

⟹ two types of charge +, −

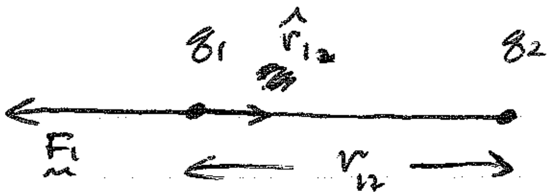
~~plus repells plus~~
⟹ like charges repell
⟹ opposite charges attract.

Coulombs Law

Double the distance between two sets of charges

⟹ forces decreases by a factor of four.

$$F \propto \frac{q_1 q_2}{r^2}$$



$$\underset{\sim}{F}_1 = - \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

$\hat{r}_{12}$ = unit vector pointing from 1 to 2.

q measured in Coulombs

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N m^2}$ SI units $\Leftrightarrow$ permittivity

$\frac{1}{4\pi\epsilon_0} \simeq 9 \times 10^9 \frac{N m^2}{C^2}$

If take a charge and start dividing it

then find the smallest unit of charge e is

$$e = 1.6 \times 10^{-19} C$$

An electron has a charge $-e$ and a proton $+e$.

$\Rightarrow$ charge is conserved.

<u>example</u>

two charges of 1C and 1m apart.

$$F = 9 \times 10^9 \frac{N m^2}{C^2} \frac{(1C)^2}{1 m^2} \sim 9 \times 10^9 N$$

$\Rightarrow$ very large force.

~~What is force~~

Avogadro's # of electrons

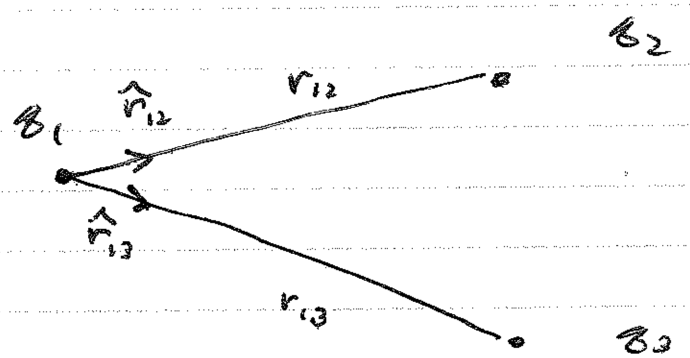
$$Q_A = 6.023 \times 10^{23}\ 1.6 \times 10^{-19}\ C$$

$$\sim 10^5\ C.$$

What does this tell you about the number of electrons and positive charges in a normal physical environment?

$\Longrightarrow$ Coulomb force is much greater than gravitational force.

Force between several charges



$$\underset{\sim}{F}_1 = \frac{1}{4\pi\epsilon_0}\left[\frac{\hat{r}_{12} q_2}{r_{12}^2} + \frac{\hat{r}_{13} q_3}{r_{13}^2}\right] q_1$$

Must take the vector sum of the all forces acting on charge q_1.

Conductors and Insulators

Some materials allow charge to move from one place to another (electrons ~~more while~~ usually move while ions are usually stationary) $\Rightarrow$ called conductors

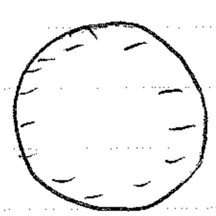
metals are generally good conductors

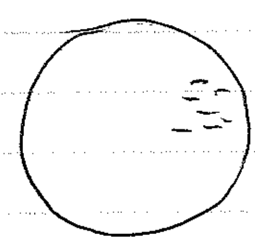
~~to~~ silver, copper ...
$\Rightarrow$ water with disolved salts.

Materials in which charges do not move are called insulators $\Rightarrow$ ceramics, purewater, air

conductor

insulator





Electric Field

Consider a charge q at some location.

We can calculate the force on any other charge q' due to this charge q by using Coulombs Law.

~~Alternatively we can think of q as modify~~

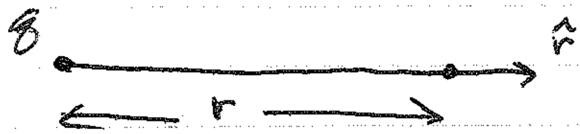
Alternatively, we can think of the charge q as modifying the space around it by forming ~~the~~ an Electric field similar to the gravitational field of the Earth.

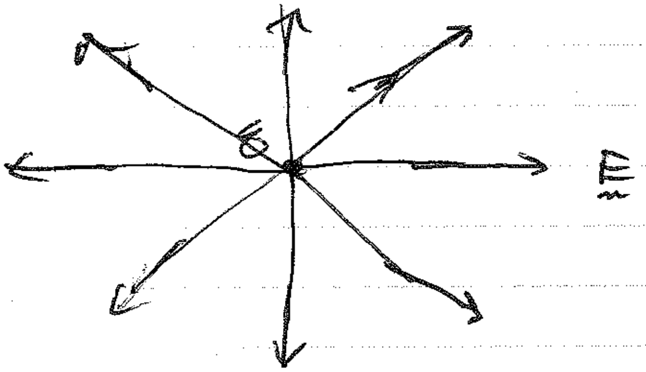
~~This force field is~~ This electric field depends on q and not q'

$$\underset{\sim}{E} = \frac{\underset{\sim}{F}}{q'} = \frac{1}{4\pi\epsilon_0}\frac{q\,\hat{r}}{r^2}$$

$\underset{\sim}{F}$ = force on q' due to q.

$$\boxed{E \sim \frac{N}{C}}$$



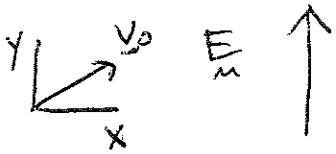


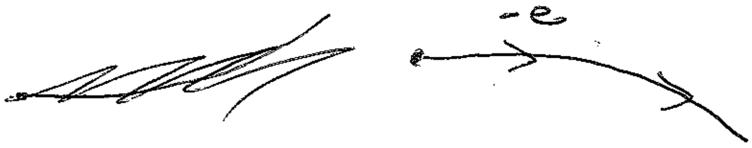
The magnitude and direction of $\underset{\sim}{E}$ varies with position.

The electric field points <u>radially outwards</u> <u>from a positive charge.</u>
<u>Inwards for a negative charge.</u>
The electric field lines only start and end on charges.
To calculate the force $\underset{\sim}{F}$ on any charge q'

$$\underset{\sim}{F} = q'\,\underset{\sim}{E}.$$

$\Longrightarrow$ the electric field exists even if there are no other charges than q present.





Motion in a uniform electric field

Consider a uniform electric field $\underset{\sim}{E} = E_y \hat{j}$:

An electron $(q = -e)$ has an initial velocity

Find the trajectory of the electron. $\underset{\sim}{V}_0 = V_{x0}\hat{i} + V_{y0}\hat{j}$

Force: $\underset{\sim}{F} = -e\,\underset{\sim}{E} = -e E_y \hat{j}$

$$m \frac{d\underset{\sim}{V}}{dt} = \underset{\sim}{F}$$

$$m \frac{dV_x}{dt} = F_x = 0 \qquad \boxed{V_x = \text{const} = V_{x0}}$$

$$\frac{dx}{dt} = V_x \qquad \boxed{X = X_0 + V_{x0} t}$$

$$m \frac{dV_y}{dt} = F_y = -eE_y$$

$$V_y - V_{y0} = -\frac{eE_y}{m} \int_0^t dt = -\frac{eE_y}{m} t$$

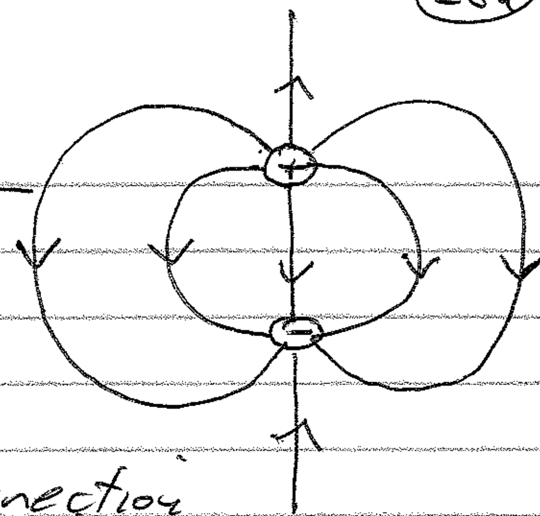
$$\boxed{V_y = -\frac{eE_y}{m} t + V_{y0}}$$

$$\frac{dy}{dt} = V_y$$

$$y - y_0 = \int_0^t dt\, V_y(t) = -\frac{eE_y}{m} \underbrace{\int_0^t dt\, t}_{\frac{t^2}{2}} + V_{y0} \underbrace{\int_0^t dt}_{t}$$

$$\boxed{y = y_0 - \frac{eE_y}{m2} t^2 + V_{y0} t}$$

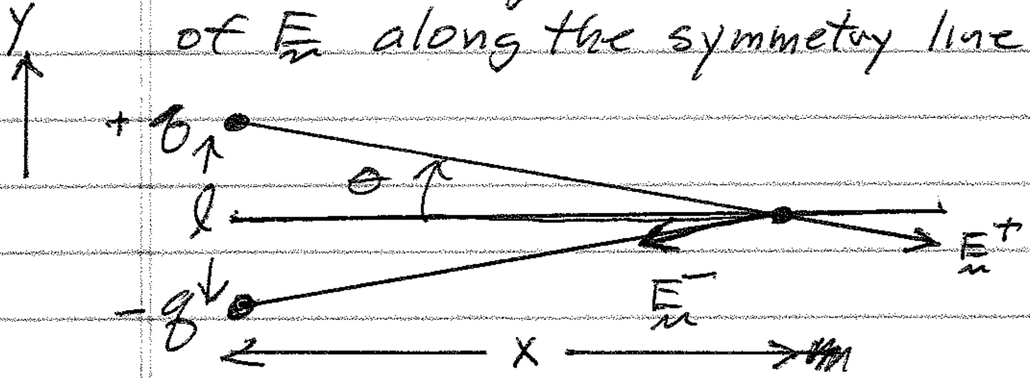
Field from electric dipole



Consider two charges $q, -q$ separated by a distance l.

Calculate magnitude and direction of $\underline{E}$ along the symmetry line

field pattern



$$\sin\theta = \frac{l}{2r}$$

By symmetry what is the direction of $\underline{E}$?

$\Rightarrow$ always exploit symmetry

$\Rightarrow$ $\underline{E}$ in y direction $\Rightarrow$ see diagram

$$E_y^+ = -E\sin\theta$$

$$E_y^- = -E\sin\theta$$

$$E_y = -2E\sin\theta = -2\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\frac{l}{2r}$$

$$= -\frac{ql}{4\pi\epsilon_0}\frac{1}{\left[x^2+\left(\frac{l}{2}\right)^2\right]^{3/2}}$$

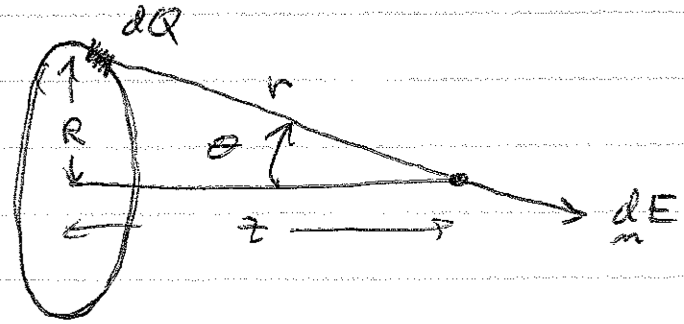
$p = ql =$ dipole moment

$$E_y = -\frac{p}{4\pi\epsilon_0}\frac{1}{\left(x^2+\left(\frac{l}{2}\right)^2\right)^{3/2}}$$

large $x \gg \frac{l}{2}$ $\qquad E_y \simeq \frac{p}{4\pi\epsilon_0}\frac{1}{x^3}$

Electric field from a charged ring

Consider the electric field due to a ring of charge Q (radius R) along its axis.



$$r = \left(z^2 + R^2\right)^{\frac{1}{2}}$$

By symmetry the net electric field will be in the z direction.

$\Rightarrow$ first consider a small element of charge dQ

$\Rightarrow$ produces an electric field of magnitude

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2}$$

$\Rightarrow$ component in z direction

$$dE_z = dE\cos\theta = dE\,\frac{z}{r}$$

$$dE_z = \frac{1}{4\pi\epsilon_0} \frac{dQ\, z}{r^3}$$

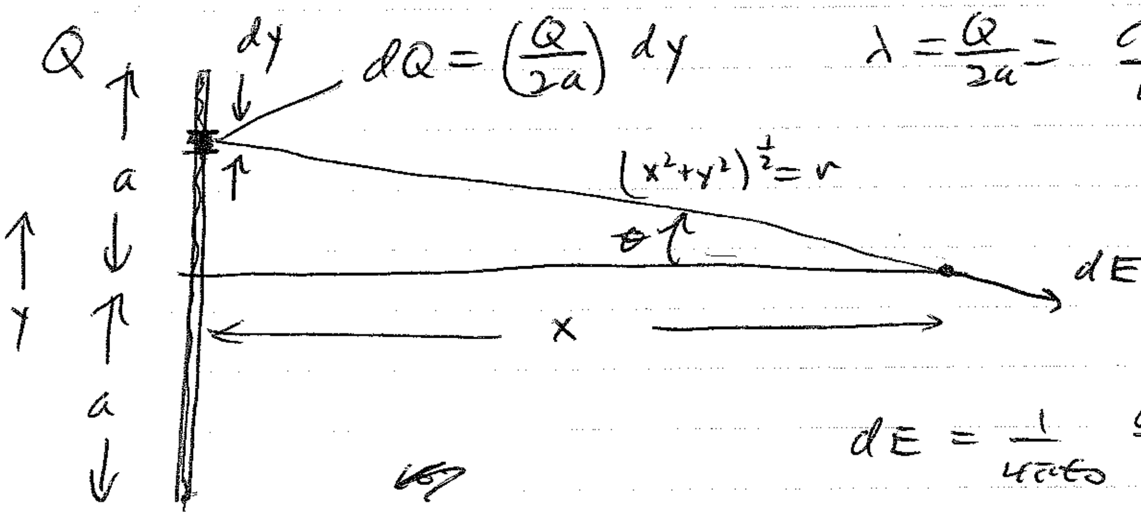
$$E_z = \int dE_z = \frac{1}{4\pi\epsilon_0} \int \frac{dQ\, z}{r^3}$$

$$E_z = \frac{zQ}{4\pi\epsilon_0 r^3} = \frac{zQ}{4\pi\epsilon_0 \left(R^2 + z^2\right)^{3/2}}$$

large z $\quad \dfrac{Q}{4\pi\epsilon_0 z^2} \Rightarrow$ acts like pt. charge.

small z?

Electric field from a Line Charge



$$\lambda = \frac{Q}{2a} = \frac{\text{charge}}{\text{unit length}}$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}$$

In what direction is $\underset{\sim}{E}$? $\Longrightarrow$ ~~B~~ x direction

$$dE_x = dE\cos\theta \qquad \cos\theta = \frac{x}{(x^2+y^2)^{\frac{1}{2}}}$$

$$= \frac{\frac{1}{4\pi\epsilon_0}\left(\frac{Q}{2a}\right)dy\left(\frac{x}{(x^2+y^2)^{\frac{1}{2}}}\right)}{(x^2+y^2)}$$

$$E_x = \frac{Q}{2a}\frac{1}{4\pi\epsilon_0}\,x\int_{-a}^{a} dy\,\frac{1}{(x^2+y^2)^{3/2}}$$

$$\int\frac{dy}{(x^2+y^2)^{3/2}} = \frac{y}{x^2(x^2+y^2)^{\frac{1}{2}}}$$

$$E_x = \frac{Q}{\cancel{2a}}\frac{1}{4\pi\epsilon_0}\cancel{x}\,\frac{a}{x^{\cancel{2}}(x^2+a^2)^{\frac{1}{2}}} = \frac{Q}{4\pi\epsilon_0}\,\frac{1}{x}\,\frac{1}{(x^2+a^2)^{\frac{1}{2}}}$$

<u>$x \gg a$</u>

$$E_x = \frac{Q}{4\pi\epsilon_0} \frac{1}{x^2}$$

<u>$a \gg x$</u>

$$E_x = \frac{1}{2\pi\epsilon_0} \lambda \frac{1}{x} \quad = \text{field from infinite line charge}$$

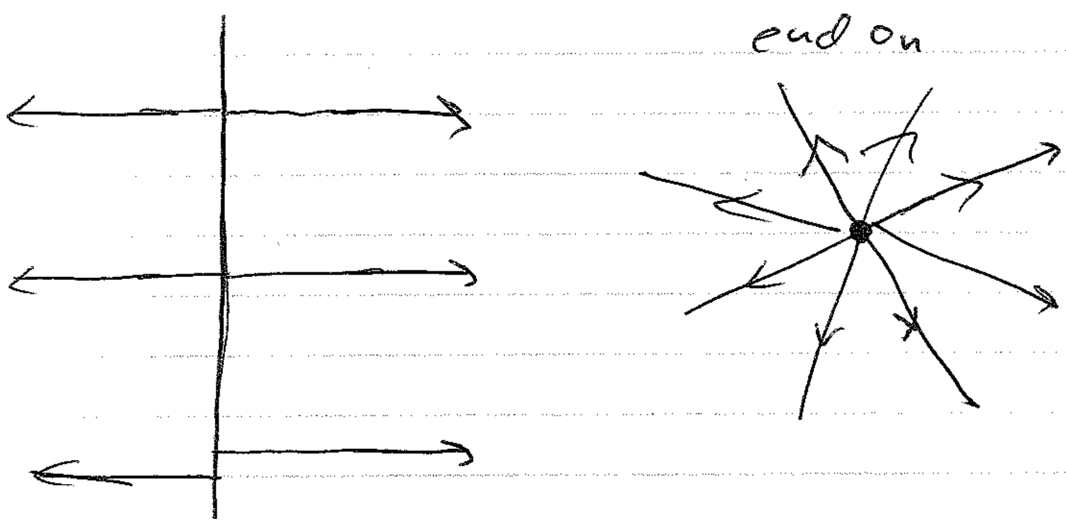
falls off as $\frac{1}{x}$.

$$\lambda = \frac{Q}{2a} = \text{charge per unit length}$$

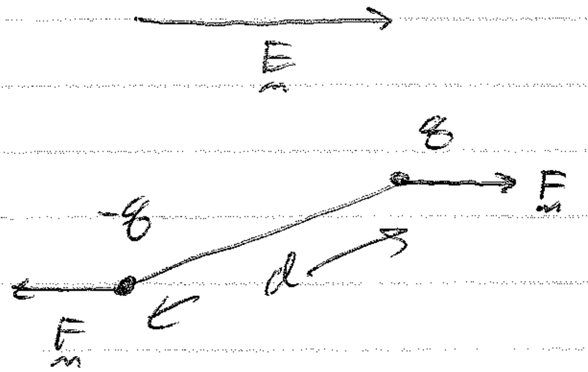
~~Electric Field from Plane~~

~~Gauss' Law~~

What does the electric field from a line charge look like?



Dipole in a uniform Electric Field



net force is zero.

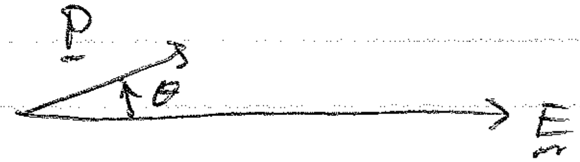
$$\mathbf{F}_{TOT} = q\mathbf{E} - q\mathbf{E} = 0$$

$\Rightarrow$ dipole wants to rotate

$\Rightarrow$ torque.

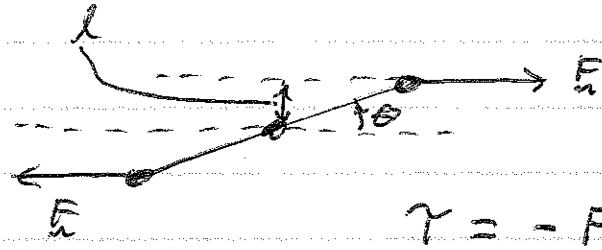
Define a vector dipole moment $\mathbf{p}$ with $p = qd$

$\Rightarrow$ $\mathbf{p}$ points from negative to positive



$\mathbf{p}$ wants to align with $\mathbf{E}$.

$\Rightarrow$ calculate the torque



$$l = \frac{d}{2}\sin\theta$$

$$\tau = -Fl - Fl = -\cancel{2}\, qE \frac{d}{\cancel{2}} \sin\theta$$

$$\boxed{\tau = -Ep\sin\theta}$$

vector language $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$

$\Rightarrow$ note $\tau < 0$ since acts to decrease θ.

potential and Energy

$$\frac{d}{dt} I\omega = \tau \qquad \omega = \frac{d\theta}{dt}$$

$$= -Ep\sin\theta$$

$$\omega \frac{d}{dt} I\omega = -Ep\frac{d\theta}{dt}\sin\theta$$

$$\frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = Ep\frac{d}{dt}\cos\theta$$

$$\frac{d}{dt}\left(\frac{1}{2}I\omega^2 - Ep\cos\theta\right) = 0$$

$$U = -Ep\cos\theta = -\underset{\sim}{p}\cdot\underset{\sim}{E}$$

$\Longrightarrow$ minimum energy when $\underset{\sim}{p}$ is aligned with $\underset{\sim}{E}$.

Electric Field in Conductors

Charge in a conductor moves in response to an electric field. The charge moves so as to shield the electric field out of the conductor. The charge only stops moving when $\underset{\sim}{E} = 0$. Thus, a stationary state is only reached when $\underset{\sim}{E} = 0$ inside a conductor. If it were not zero, charge would continue to move.

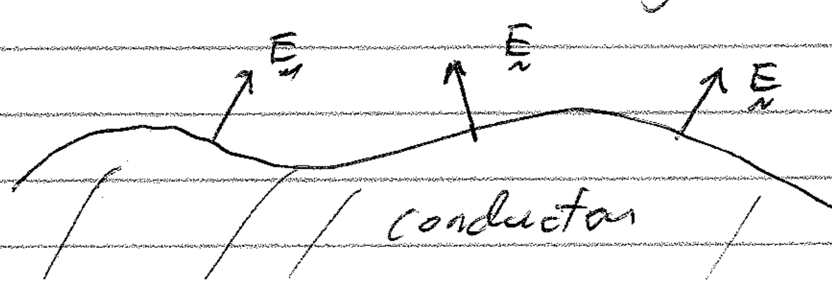
$\Rightarrow$ $\underset{\sim}{E} = 0$ in a conductor for time independent system

$\Rightarrow$ no net charge inside a conductor or $\underset{\sim}{E}$ would not be zero

$\Rightarrow$ net charge must lie on the surface of a conductor.

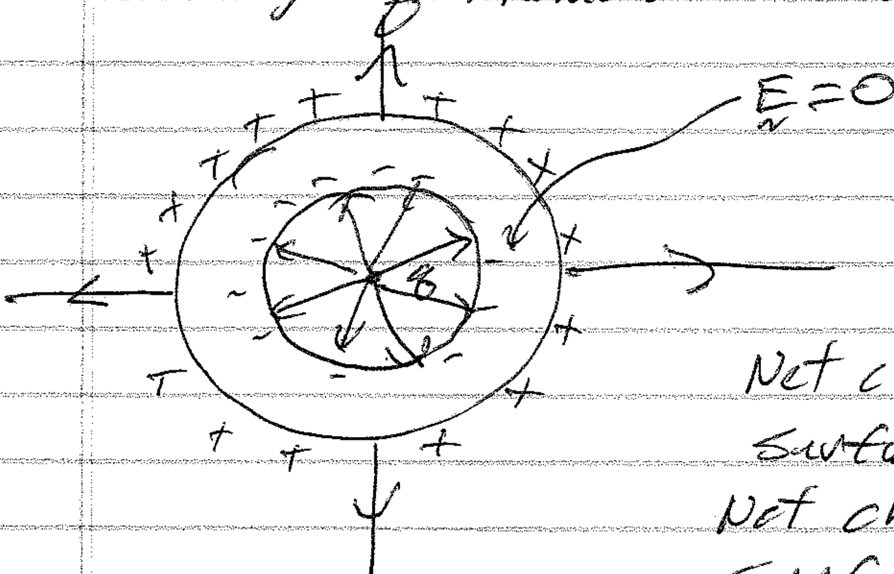
At the surface of a conductor $\underset{\sim}{E}$ is $\perp$ to the surface

$\Rightarrow$ any $E_{\parallel}$ at the surface would again cause charge to move along the surface



example

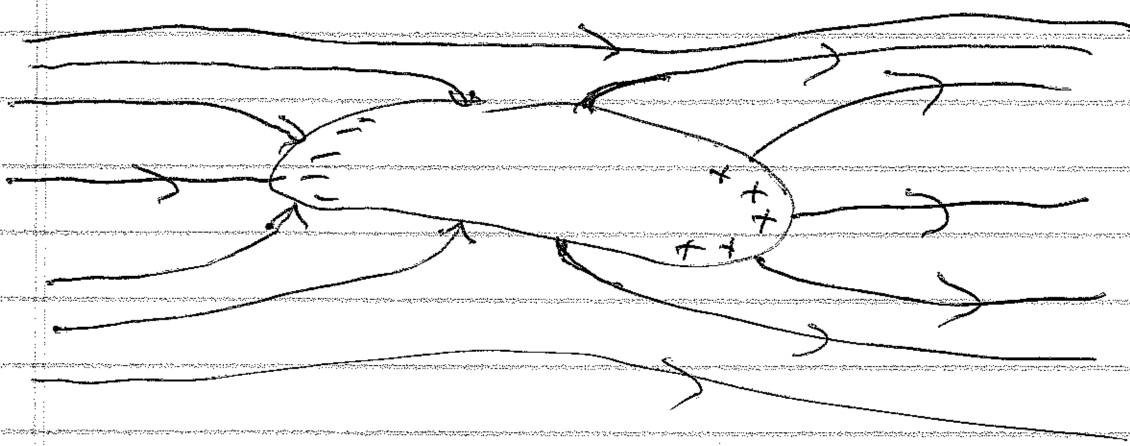
Charge q inside a conducting shell.



Net charge on inner surface is $-q$.
Net charge on outer surface is $+q$.

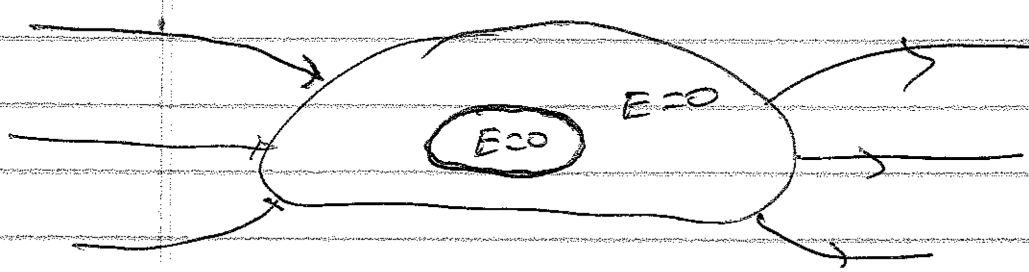
example

Conductor in an external field



example

Volume inside a conductor



$\Rightarrow E=0$
will show this later.