Physics 270, Assignment 7

21.24

A strong reflection occurs if the path length difference if there is constructive interference resulting in the condition 21.32

$$\lambda = \frac{2nd}{m}$$

where m is an integer, d is the thickness of the sample, and n is the index of refraction for the film. Thus, we have the smallest suitable thickness is

$$d = \frac{\lambda}{2n} = \frac{600 \text{ nm}}{2\,(1.39)} = 215.82 \text{ nm}$$

22.4

Here we have light from a sodium lamp with $\lambda = 589$ nm in a double slit experiment. The fringe spacing on a screen $L = 150$ cm behind the slits is $\Delta y = 4.0$ mm. We want the space d between the slits. We will simply use

$$\Delta y = \frac{\lambda L}{d}$$

giving us

$$d = \frac{\lambda L}{\Delta y} = \frac{\left(589 \times 10^{-9} \text{ m}\right)\left(150 \times 10^{-2} \text{ m}\right)}{\left(4 \times 10^{-3} \text{ m}\right)} = 2.20 \times 10^{-4} \text{ m} = 0.22 \text{ mm}$$

22.10

Here we have light with $\lambda = 633$ nm on a diffraction grating. The distance between the $m = 1$ fringes is $\Delta y = 32$ cm on a screen 2.0 m behind the grating. We want the spacing between slits on the grating. We may use the fact that 32 cm$= y_m - y_{-m} = 2y_1$ and equation 22.16

$$y_1 = L \tan \theta_1$$

where θ_1 is given by equation 22.15:

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{d}\right)$$

Thus, we have

$$y_1 = \frac{\Delta y}{2} = L \tan\left[\sin^{-1}\left(\frac{\lambda}{d}\right)\right] = L\frac{\lambda/d}{\sqrt{1 - (\lambda/d)^2}}$$

Solving for λ/d, we have

$$\lambda/d = \frac{\Delta y}{\sqrt{(\Delta y)^2 + 4L^2}}$$

Which gives us a spacing of

$$d = \frac{\lambda}{\Delta y}\sqrt{(\Delta y)^2 + 4L^2} = \frac{633 \times 10^{-9} \text{ m}}{32 \times 10^{-2} \text{ m}}\sqrt{(32 \times 10^{-2})^2 + 4\,(2 \text{ m})^2} = 7.93 \times 10^{-6} \text{ m}$$

22.18

Here we are using a cell phone, with a signal of 800 MHz behind to buildings with a space of 15 m between them. This can be modeled as a single slit experiment. That angle θ has the form

$$\theta = \frac{180}{\pi}\frac{w}{L}$$

where w is the width of central maximum and L is the distance from the buildings at which we measure w. The factor of $\frac{180}{\pi}$ is there to convert from radians to degrees. We may now use the equation

$$w = \frac{2\lambda L}{a}$$

where a is the space between the buildings to give us

$$\theta = \frac{180}{\pi}\frac{w}{L} = \frac{180}{\pi}\frac{2\lambda}{a}$$

Now we use the fact that for electromagnetic radiation, we have $c = \lambda f$ to give us

$$\theta = \frac{180}{\pi}\frac{2c}{af} = \frac{180}{\pi}\frac{2\left(2.99 \times 10^8 \text{ m/s}\right)}{(15 \text{ m})\left(800 \times 10^6 \text{ s}^{-1}\right)} = 2.85 \text{ degrees}$$

22.44

Now we have a diffraction grating that we want to disperse light in the spectrum 400 nm to 700 nm over 30 degrees in the first order. How many lines n per millimeter will our grating need? We have $n = \frac{1}{d}$ where d is the line spacing for our grating. We also know from equation 22.15 that

$$d = \frac{m\lambda}{\sin\theta}$$

For 400 nm and 700 nm light, this corresponds to a distance

$$\begin{aligned} d_{400} &= \frac{\lambda}{\sin 30°} = \frac{400 \times 10^{-9} \text{ m}}{\sin 30°} = 8.0 \times 10^{-7} \text{ m} \\ d_{700} &= \frac{\lambda}{\sin 30°} = \frac{700 \times 10^{-9} \text{ m}}{\sin 30°} = 1.4 \times 10^{-6} \text{ m} \end{aligned}$$

We know that smaller line-spacings disperse light at greater angles. Thus, while a grating with d_{400} will disperse 400 nm light at 30 degrees, it will disperse everything with bigger wavelengths at even higher angles. So this is the spacing our grating will have. It leads to a line density of

$$n = \frac{1}{d_{400}} = 1.25 \times 10^6 \text{ lines/m} = 1250 \text{ lines/mm}$$

The first order diffraction angle of 589 nm light will be given by equation 22.15

$$\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{589 \times 10^{-9} \text{ m}}{8.0 \times 10^{-7} \text{ m}}\right) = 47.41°$$

22.56

Now there is a radar broadcasting 12GHz from a 2.0 m diameter circular radar antenna. This can be modeled as a circular aperture of diameter $D = 2.0$ m. At a distance of 30km, the diameter w of the radar beam is

$$w = \frac{2.44\lambda L}{D} = \frac{2.44cL}{Df} = \frac{2.44\left(2.99 \times 10^8 \text{ m/s}\right)\left(30 \times 10^3 \text{ m}\right)}{(2.0 \text{ m})\left(12 \times 10^9 \text{ s}^{-1}\right)} = 911.95 \text{ m}$$

Now if the antenna is operating at a power of 100 kW, then at 30 km, the intensity will be

$$I = \frac{P}{A} = \frac{P}{\pi\left(\frac{w}{2}\right)^2} = \frac{4P}{\pi w^2} = \frac{4\left(100 \times 10^3 \text{ W}\right)}{\pi\left(911.95 \text{ m}\right)^2} = 0.15 \text{ W/m}^2$$