Physics 270, Assignment 4

31.14

We wish to know what the power dissipated by each resistor in the configuration is. We know that the power in a resistor is given by $P = I^2 R$. We know that both resistors will have the same current because they are in series. Furthermore, their equivalent resistance R_{eq} is given by

$$R_{eq} = R_1 + R_2 = 12\ \Omega + 15\ \Omega = 27\ \Omega$$

The current is then obtained by Ohm's law Kirchoff's loop rule

$$9\ \text{V} - IR_{eq} = 9\ \text{V} - I\,(27\ \Omega) = 0$$

or

$$I = 1/3\ \text{A}$$

Therefore, the powers dissipated in R_1 and R_2 are given by

$$\begin{aligned} P_1 &= I^2 R_1 = (1/3\ \text{A})^2\, 12\ \Omega = 4/3\ \text{J/s} \\ P_2 &= I^2 R_2 = (1/3\ \text{A})^2\, 15\ \Omega = 5/3\ \text{J/s} \end{aligned}$$

31.32

We now want the equivalent resistance between the points a and b in the figure. We can model the system as three resistors in parallel. The top wire configuration has three resistors in series and therefore an equivalent resistance of $R_t = 3\,(100\ \Omega) = 300\ \Omega$. The middle wire has only two resistors in series. Thus, we have an equivalent resistance of $R_m = 2\,(100\ \Omega) = 200\ \Omega$. The bottom wire has resistance $R_b = 100\ \Omega$. We can think of these three wires as three resistors in parallel. Hence, the total equivalent resistance R_{eq} of the system is given by:

$$\frac{1}{R_{eq}} = \frac{1}{R_t} + \frac{1}{R_m} + \frac{1}{R_b}$$

or

$$R_{eq} = \frac{1}{\frac{1}{R_t} + \frac{1}{R_m} + \frac{1}{R_b}} = \frac{1}{\frac{1}{300\ \Omega} + \frac{1}{200\ \Omega} + \frac{1}{100\ \Omega}} = \frac{600}{11}\ \Omega = 54.54\ \Omega$$

33.17

Now we would like the energy stored in a 0.03 m diameter, 0.12 m long solenoid with 200 turns and a current of 0.80 A. We can think of the solenoid as an inductor with an energy given by

$$U_L = \frac{1}{2} L I^2$$

where L is the inductance of a solenoid given by

$$L_{\text{solenoid}} = \frac{\mu_0 N^2 A}{l}$$

This gives us

$$U_L = \frac{1}{2}\frac{\mu_0 N^2 A I^2}{l} = \frac{1}{2}\frac{\mu_0 N^2 \pi R^2 I^2}{l} = \frac{1}{2}\frac{\left(1.26\times 10^{-6}\,\frac{\text{Tm}}{\text{A}}\right) 200^2 \left(\pi\,(0.015\ \text{m})^2\right)(0.80\ \text{A})^2}{(0.12\ \text{m})} = 1.90\times 10^{-4}\ \text{J}$$

33.66

We are now given that the current rhough inductance L is given by $I = I_0 \sin(\omega t)$. We would like an expression for the potential difference ΔV_L across the inductor. This is given by

$$\Delta V_L = -L\frac{dI}{dt} = -LI_0\omega\cos(\omega t)$$

Next we are given that the maximum voltage across the inductor is 0.20 V when $L = 50 \times 10^{-6}$ H and $f = 500$ kHz. We now want I_0. We may merely plug these values into the formula above. Note that the maximum voltage is achieved when $\cos(\omega t) = -1$.

$$\Delta V_{L,\max} = LI_0\omega$$

Thus,

$$I_0 = \frac{\Delta V_{L,\max}}{L\omega} = \frac{\Delta V_{L,\max}}{L(2\pi f)} = \frac{0.20\text{ V}}{(50\times10^{-6}\text{ H})(2\pi(500\times10^3\text{ s}^{-1}))} = 0.0012\text{ A}$$

33.68

Next, we are given an LC circuit with a 10×10^{-3} H inductor and an 8.0×10^{-6} F capacitor. The current attains its maximum value of 0.5 A at $t = 0$. We want to know how long it will be before the capacitor is fully charged. We know that when the current is at its maximum, the charge across the capacitor is zero. Therefore, the charge will be at a maximum after 1/4 of the period of oscillation has passed. That period T is given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{1}{LC}}} = 2\pi\sqrt{LC} = 2\pi\sqrt{(10\times10^{-3}\text{ H})(8.0\times10^{-6}\text{ F})} = 0.0017\text{ s}$$

Hence, the capacitor will be charged after $T/4$ where

$$\frac{T}{4} = 4.44\times10^{-4}\text{ s}$$

We would also like the voltage across that capacitor at that time. By conservation of energy, we know that the energy of the inductor at $t = 0$ will be equal to the energy of the capacitor when it is fully charged (and therefore no current goes through the inductor.) That energy relation is

$$\frac{1}{2}LI_{\max}^2 = \frac{1}{2}CV^2$$

or

$$V = \frac{LI_{\max}^2}{C} = \frac{\left(10\times10^{-3}\text{ H}\right)(0.5\text{ A})^2}{(8.0\times10^{-6}\text{ F})} = 312.5\text{ V}$$

33.72

We want to know the maximum voltage to which we can charge the 1200μF capacitor is. We know that however we do it, the voltage will be transfered by means of a circuit that can be modelled as an LC circuit. As such, we can be sure that the maximum voltage on the1200μF capacitor will be attained when the voltage across the inductor is zero. Hence, we want all the energy of the system to be on the1200μF capacitor, under the constraint that energy of the total system is conserved. Initially, the energy is

$$E = \frac{1}{2}C_1V_1^2$$

where $C_1 = 300\mu$F and $V_1 = 100$ V. At a later time, we must have

$$E = \frac{1}{2}C_1V_1^2 = \frac{1}{2}C_2V_2^2$$

where $C_1 = 1200\mu$F and V_2 is unknown. This gives us

$$V_2 = \sqrt{\frac{C_1}{C_2}V_1^2} = \sqrt{\frac{300\mu F}{1200\mu F}V_1^2} = \frac{1}{4}V_1 = \frac{1}{4}100 \text{ V} = 25 \text{ V}$$

33.74

We are given that the switch in the figure has been open for a long time, but it is closed at $t = 0$. We want the current through the 20 Ω resistor. Initially, when the switch is just closed $\frac{dI}{dt}$ through the inductor is very big. As such, all the current flows through the path of least resistance: through the 20 Ω resistor as if the inductor were cut out of the circuit. Hence the current is

$$I = \frac{V}{R_{eq}} = \frac{30}{10\ \Omega + 20\ \Omega} = 1 \text{ A}$$

When the switch is closed for a long time, the voltage difference across the inductor is zero because $\frac{dI}{dt} = 0$. Thus, all current flows through the inductor and nothing at all flows through the 20 Ω resistor.

Lastly, if the switch is immediately reopened, then we have an LR circuit. Hence, there will be an exponential decay with an initial current (the current we are trying to find) of

$$I_0 = \frac{30 \text{ V}}{20\ \Omega} = 3/2 \text{ A}$$

33.81

We will now find the inductance per meter of coaxial cable. We remember that inductance is defined to be the magnetic flux through a surface per unit current applied. They tell us to use a rectangle with length l spanning the gap between inner and outer conductors. The magnetic field induced by the inner conductor can be derived from Ampere's law.

$$\oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I$$

or $B = \frac{\mu_0 I}{2\pi r}$. The magnetic flux through the rectangle will then be the integral.

$$\Phi = \oint \vec{B} \cdot d\vec{A} = \int_{r_1}^{r_2} dr \int_0^l dx \frac{\mu_0 I}{2\pi r} = \int_{r_1}^{r_2} dr \frac{\mu_0 I l}{2\pi r} = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

Thus, the inductance is

$$L = \frac{\Phi}{I} = \frac{\mu_0 l}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

and the inductance per unit length is

$$\frac{L}{l} = \frac{\mu_0}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

For the parameters given in the problem, this is

$$\frac{L}{l} = \frac{\mu_0}{2\pi} \ln\left(\frac{r_2}{r_1}\right) = \frac{1.26 \times 10^{-6}\ \frac{\text{Tm}}{\text{A}}}{2\pi} \ln\left(\frac{3 \text{ mm}}{0.5 \text{ mm}}\right) = 3.59 \times 10^{-7}\ \frac{\text{Tm}}{\text{A}}$$

31.38

We want the time-constant for the configuration in the figure. We may model this as a system with an equivalent resistance R_{eq} and an equivalent capacitance C_{eq} where

$$\begin{aligned} R_{eq} &= \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} = \frac{1}{\frac{1}{1\times10^3\ \Omega}+\frac{1}{1\times10^3\ \Omega}} = 500\ \Omega \\ C_{eq} &= C_1 + C_2 = 2\times10^{-6}\ \mathrm{F} + 2\times10^{-6}\ \mathrm{F} = 4\times10^{-6}\ \mathrm{F} \end{aligned}$$

Thus, the time constant is

$$\tau = R_{eq}C_{eq} = (500\ \Omega)\left(4\times10^{-6}\ \mathrm{F}\right) = 0.002\ \mathrm{s}$$

31.80

We are given an RC circuit driven by a battery. We want the total energy supplied to by the battery as the capacitor is being charged. The charge on the capacitor is given by equation 31.40

$$Q = Q_{\max}\left(1 - e^{-t/\tau}\right)$$

where $\tau = RC$ and $Q_{\max} = CV$. Differentiating this expression gives us the current through the system.

$$I = \frac{dQ}{dt} = \frac{CV}{\tau}e^{-t/\tau} = \frac{V}{R}e^{-t/\tau}$$

To get the total energy supplied by the battery, we shall need to integrate the power $P_{\mathrm{batt}} = IV$ from $t = 0$ to $t = \infty$. This gives us

$$E_{\mathrm{batt}} = \int_0^\infty dtIV = \int_0^\infty dt\frac{V^2}{R}e^{-t/\tau} = \frac{V^2}{R}\tau = \frac{V^2}{R}RC = CV^2$$

Next, we want the total energy dissipated by the resistor. We may do the same thing: integrate power $P_{\mathrm{res}} = I^2R$.

$$E_{\mathrm{res}} = \int_0^\infty dtI^2R = \int_0^\infty dt\left(\frac{V}{R}e^{-t/\tau}\right)^2 R = \frac{V^2}{R}\int_0^\infty dte^{-2t/\tau} = \frac{V^2}{R}\frac{\tau}{2} = \frac{1}{2}CV^2$$

The capacitor's energy, once fully charged, is well-known:

$$E_{\mathrm{cap}} = \frac{1}{2}CV^2$$

Lastly, we note that our results show that energy is indeed conserved. The energy supplied by the batter gets divided evenly between the resistor and the capacitor.

$$E_{\mathrm{batt}} = CV^2 = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = E_{\mathrm{res}} + E_{\mathrm{cap}}$$