Electromagnetic Induction

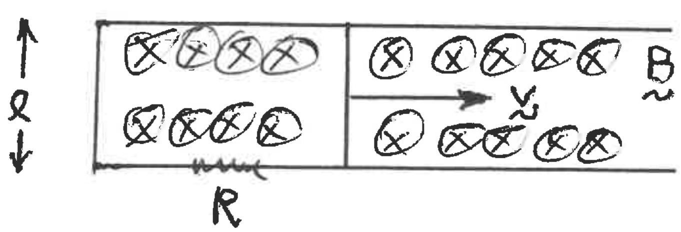
Experiments reveal that moving a bar magnetic toward or away from a coil induces a current in the coil.

Move a bar magnet toward the coil $\Rightarrow$ measure a current in the coil

Reverse the motion of the magnet $\Rightarrow$ direction of current reverses

Stationary magnet $\Rightarrow$ no current.

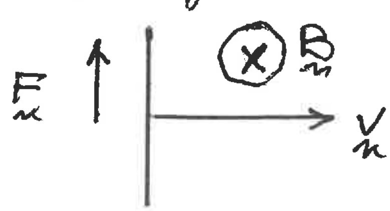
Motional emf : an example of electromagnetic induction



Consider a sliding conductor of length ℓ moving in a magnetic field as shown

$\Rightarrow$ sliding wire has a velocity V.

There will be a force on the free charges in the wire (pretend that the free charges are positive)



$$\underset{\sim}{F} = q\, \underset{\sim}{V} \times \underset{\sim}{B}$$

This force does work on the charge $W = F\ell = qVB\ell$

This work acts like an emf to drive the current

$$W = q\mathcal{E} = q\,vBl$$

$$\Rightarrow \mathcal{E} = \text{emf} = vBl = Bvl$$

example:

$v = 1\,m/s$
$l = 0.1\,m$
$B = 1\,T$

$\mathcal{E} = 0.1$ volt

Take the wire resistance to be $1\,\Omega \Rightarrow I = \mathcal{E}/R$

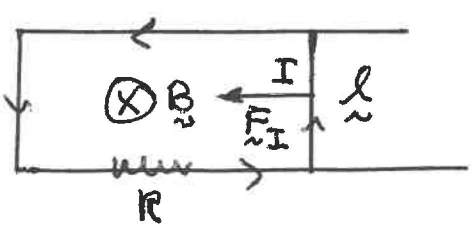
$I = 0.1\,A$

what is the power dissipated?

$$I = \mathcal{E}/R$$

$$P = I^2 R = \frac{v^2B^2l^2}{R^2}R = \frac{v^2B^2l^2}{R}$$

What is the energy source?
The current carrying wire must have a force acting on it.



$$\underset{\sim}{F}_I = I\,\underset{\sim}{l} \times \underset{\sim}{B}$$

The force on the wire $\underset{\sim}{F}_I$ is to the left. To keep the wire moving, must apply an opposite force

The work done to keep the wire moving

$$P_F = F_I v = \left(\frac{vB\ell}{R}\right)\ell B v = \frac{B^2 v^2 \ell^2}{R}$$

$\Rightarrow$ same as power dissipated in the resistor

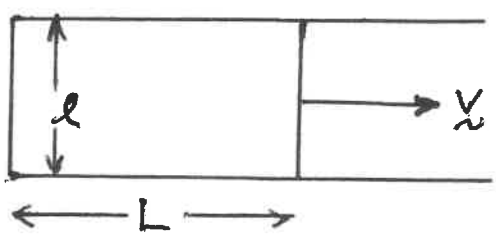
$\Rightarrow$ the power dissipated in the resistor is supplied by the force required to push the wire across the magnetic field

Induced emf

Can express the induced emf $\mathcal{E}$ as

$$\mathcal{E} = (v\ell)B$$

What is $v\ell$? $\Rightarrow$ the rate of change of the area of the loop



$A = \ell L$ = area of loop

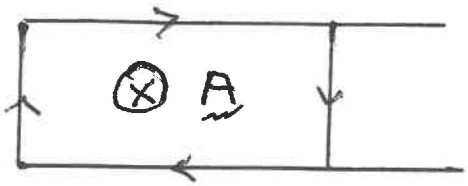
$$\frac{dA}{dt} = \ell \frac{dL}{dt} = \ell v$$

$$\Rightarrow \quad \mathcal{E} = B\frac{dA}{dt} = \frac{d}{dt}(BA)$$

$\Phi \equiv BA$ = flux of magnetic field through the loop

more generally $\displaystyle \Phi \equiv \int_A \underset{\sim}{B} \cdot d\underset{\sim}{A}$

Define the direction of $\underset{\sim}{A}$ by choosing a direction for the wire loop and using the RH rule



Fingers follow loop and area $\underset{\sim}{A}$ into the page.

$$\Phi = \int_A \underset{\sim}{B} \cdot d\underset{\sim}{A}$$

valid even if $\underset{\sim}{B}$ not parallel to $\underset{\sim}{A}$ and if $\underset{\sim}{B}$ not uniform

The relation

$$\boxed{\mathcal{E} = -\frac{d}{dt}\Phi}$$

Faraday's Law

In this case $\mathcal{E} < 0$ because the emf $\mathcal{E}$ is opposite in the sense of the path around the loop

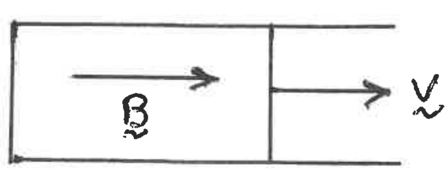
$\Rightarrow$ path around loop is clock wire

$\Rightarrow$ driven current is counter clockwise since $\mathcal{E} < 0$.

If the direction of motion of the wire is reversed, $dA/dt < 0$ and $\mathcal{E} > 0$

$\Rightarrow$ current flows clockwise

What if $\underset{\sim}{B}$ is parallel to the loop?



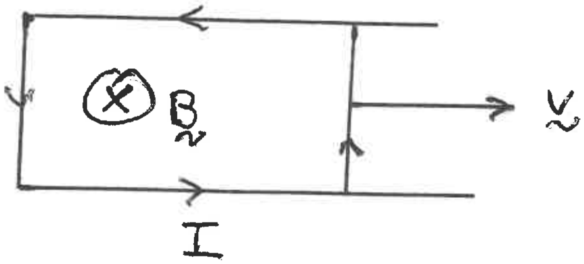
$$\underset{\sim}{B} \cdot d\underset{\sim}{A} = 0$$

$$\Phi = \int_A \underset{\sim}{B} \cdot d\underset{\sim}{A} = 0$$

$$\Rightarrow \mathcal{E} = 0 \Rightarrow I = 0$$

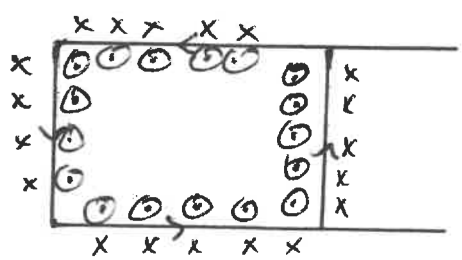
Lenz's Law

The easiest way ~~the~~ to determine the direction of the induced emf is with Lenz's Law.



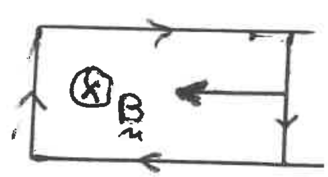
As the wire moves to the right, the magnetic flux linking the loop increases.

The direction of the magnetic field produced by I points outward on the inside of the loop. This magnetic field tends to stop the increase in magnetic flux through the loop.



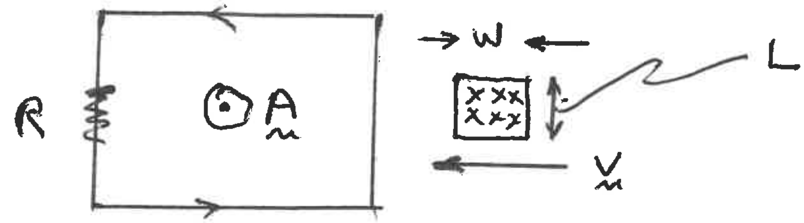
$\Longrightarrow$ The current flows so as to oppose the change in magnetic flux

Reverse bar motion



In this case the magnetic flux linking the loop decreases in time.

$\Rightarrow$ the current flows to increase the magnetic flux example

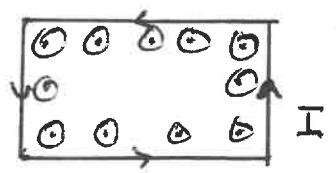


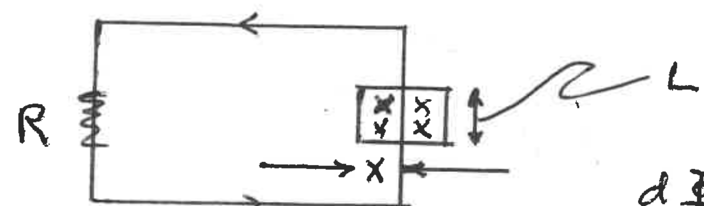
$\underset{\sim}{B}$ of magnet into the plane

Choice of loop direction arbitrary

What direction will current flow as the magnet crosses into the loop?

Lenz's Law: current flows to stop the increase in flux into page $\Rightarrow \underset{\sim}{B}$ ~~current~~ from wire out of page (inside loop)





From Faraday's Law

$$\Phi = -XLB$$

$$\Rightarrow \Phi < 0 \text{ since } \underset{\sim}{B} \cdot \underset{\sim}{A} < 0$$

$$\frac{d\Phi}{dt} = -\frac{d}{dt}(XLB) = -LB\frac{dX}{dt}$$

$$= -VLB$$

$$\mathcal{E} = VLB = IR > 0 \Rightarrow \text{along loop direction}$$

$$\Rightarrow I = \frac{VLB}{R}$$

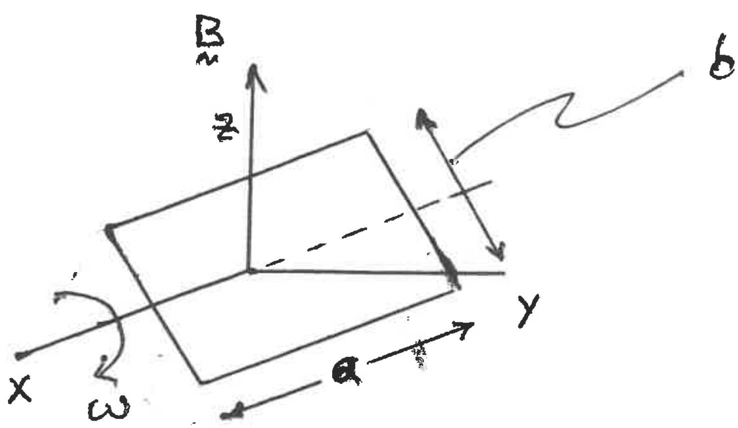
Force on wire $\quad \underset{\sim}{F} = I \underset{\sim}{L} \times \underset{\sim}{B}$

$\Rightarrow$ points to the left

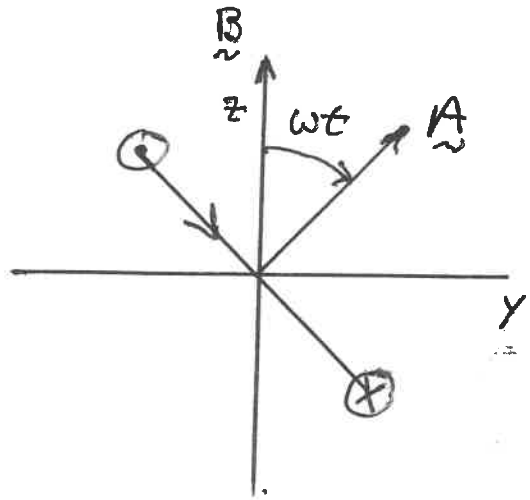
Once magnet is inside the loop no current

$$\Rightarrow \frac{d}{dt}\Phi = 0$$

<u>example</u> wire loop rotating in a magnetic field



$A = ab$



Direction around loop gives direction of $\underset{\sim}{A}$ using RH rule

$$\Phi = \int_A \underset{\sim}{B} \cdot d\hat{A} = BA\cos\omega t$$

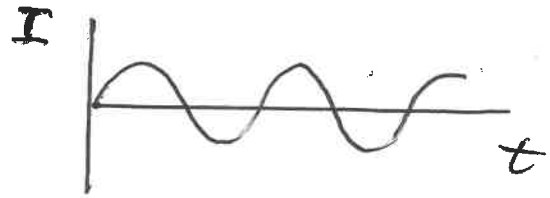
$$\mathcal{E} = -\frac{d\Phi}{dt} = BA\omega\sin\omega t$$

$$I = \frac{\mathcal{E}}{R} = \frac{BA\omega}{R}\sin\omega t$$

I is maximum when $\omega t = \pi/2$

$\Rightarrow$ plane of loop is $\parallel$ to $\underset{\sim}{B}$.

$\Rightarrow$ current oscillates in time



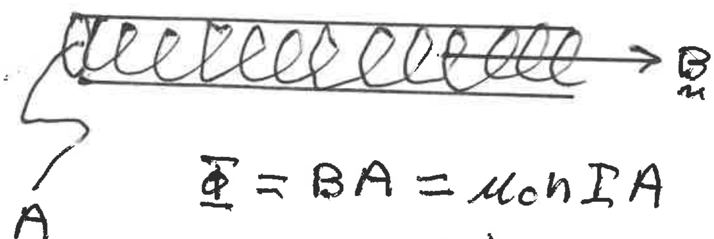
This is an AC generator

$\Rightarrow$ oscillatory emf

Induced Electric Fields

All examples of induced emf's have involved loops in which the flux from a constant magnetic is changing. Suppose that there is a time-dependent magnetic field that links the loop. This also produces an induced emf and takes the form of an induced electric field.

example Consider a solenoid in which the current is increasing in time.



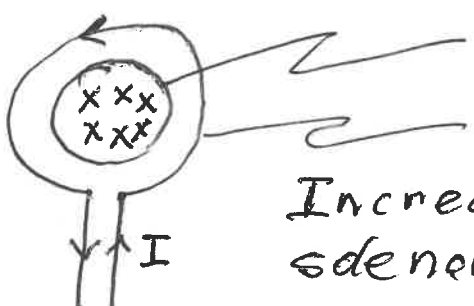
$\Rightarrow B = \mu_0 n I$

n = # of wraps per unit length

$\Phi = BA = \mu_0 n I A$

= magnetic flux through the solenoid.

Wrap a wire loop around the solenoid



Increase the current in the solenoid so that the flux Φ passing through the loop increases.

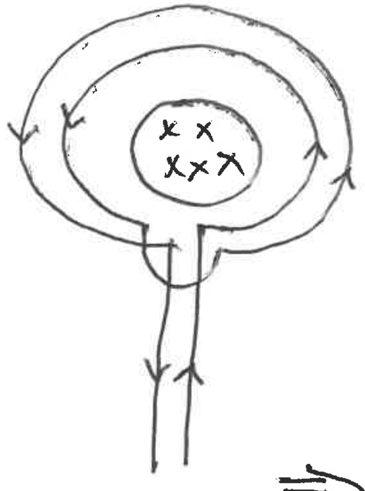
What is the direction of the induced emf and current?

$\Longrightarrow$ From Lenz's law current is counter clockwise to oppose the change in flux.

$$\mathcal{E} = -\frac{d}{dt}\left(\underbrace{-\mu_0 n I A}\right) = +\mu_0 n A \frac{d}{dt} I$$

since $\underset{\sim}{A}$ is out of page and $\underset{\sim}{B}$ is into the page, $\underset{\sim}{B} \cdot \underset{\sim}{A} < 0$

Suppose have two loops of wire.



Induced emf

$$\mathcal{E} = 2nA\mu_0 \frac{dI}{dt}.$$

For N loops of wire

$$\mathcal{E} = N\left(nA\mu_0 \frac{dI}{dt}\right)$$

⇒ this is a transformer

⇒ for a given potential drop across the solenoid, we can adjust the number of wraps N in the coil to obtain ~~and~~ any emf in the coil that is needed.

In this problem there is no $\underset{\sim}{v} \times \underset{\sim}{B}$ force so what is producing the emf?

⇒ there is an induced electric field

$$\mathcal{E} = \oint \underset{\sim}{E} \cdot d\underset{\sim}{l} = -\frac{d}{dt}\Phi$$

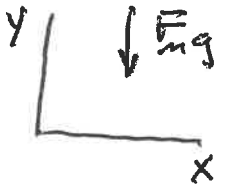
⇒ where the integral is along a closed path.

⇒ note that this is not an electrostatic field since a closed integral of the

electrostatic field would be zero.
For an electrostatic field $\mathcal{E}_V$

$$\mathcal{E}_V = \oint \underset{\sim}{E} \cdot \underset{\sim}{dl} = - \oint \nabla V \cdot dl = - \oint dV = 0$$

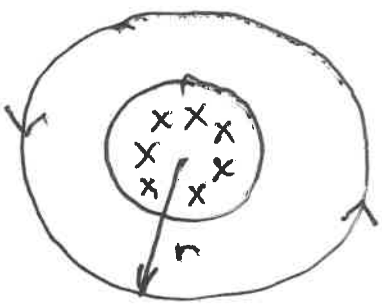
$\Rightarrow$ similar to gravitational potential where the net work done by the gravitational force along a path that closes on itself is zero.



$$W_g = \oint \underset{\sim}{F_g} \cdot \underset{\sim}{dl} = - \oint mg\,dy = -mg\,(Y_{final} - Y_{init}) = 0$$

Induced Electric Field

For the solenoid with a wire loop can evaluate the induced $\underset{\sim}{E}$.



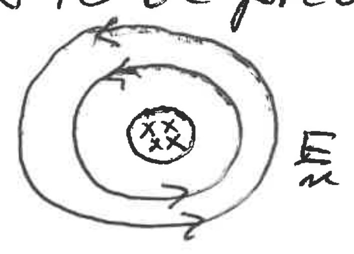
r = radius of loop

$$\oint \underset{\sim}{E} \cdot \underset{\sim}{dl} = E\, 2\pi r = - \frac{d}{dt} \Phi$$

$$E = - \frac{1}{2\pi r} \frac{d}{dt} \Phi$$

If B_z increases $\underset{\sim}{E}$ is in the counter clockwise direction since $\dot{\Phi} < 0$.

The electric field does not require the wire loop to be present. It exists everywhere in space

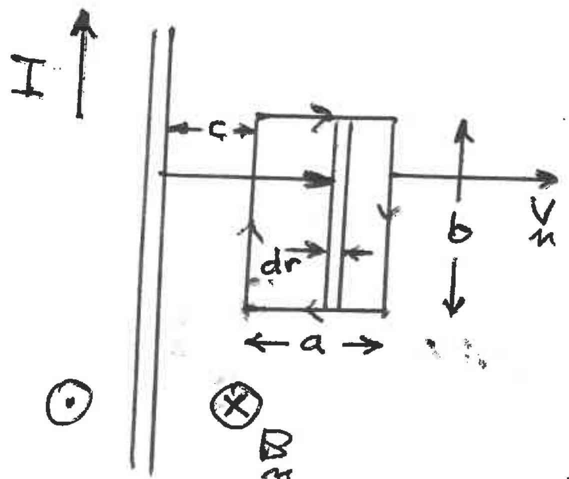


$\Rightarrow$ even within the solenoid

Calculating magnetic flux

To calculate the induced emf's of loops or induced $\underline{E}$, need to calculate the magnetic flux $\Rightarrow$ important even in cases where $\underline{B}$ is not uniform

example Moving wire loop near a current carrying wire



$B = \frac{\mu_0 I}{2\pi r}$ = B from infinite wire

$\Rightarrow$ B depends on r.

$\Rightarrow$ Find magnetic flux $d\Phi$ in a narrow strip of length b and width dr.

$$d\Phi = B\,b\,dr = b\,\frac{\mu_0 I}{2\pi}\,\frac{dr}{r}$$

$$\text{Total } \Phi = \int_c^{c+a} d\Phi = \frac{\mu_0 I}{2\pi}\, b \ln\left[\frac{(a+c)}{c}\right]$$

What is the direction of the current if the loop moves away from the wire?

$\Rightarrow$ flux through the loop decreases so current will flow clockwise to maintain flux

Take the loop velocity to be radially outward at velocity V $\Rightarrow$ $\dot{c} = \frac{d}{dt}c = V$

induced emf

$$\mathcal{E} = -\frac{d}{dt}\Phi = -\frac{\mu_0 I}{2\pi} b \frac{d}{dt}\left[\ln(a+c) - \ln c\right]$$

$$= -\frac{\mu_0 I}{2\pi} b \left[\frac{\dot c}{a+c} - \frac{\dot c}{c}\right]$$

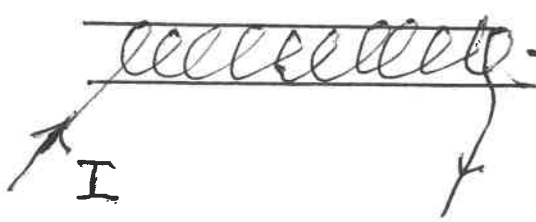
$$= -\frac{\mu_0 I}{2\pi} b v \left[\frac{c-(a+c)}{c(a+c)}\right]$$

$$= \frac{\mu_0 I}{2\pi} v \frac{ab}{c(a+c)}$$

Magnetic materials and magnetization

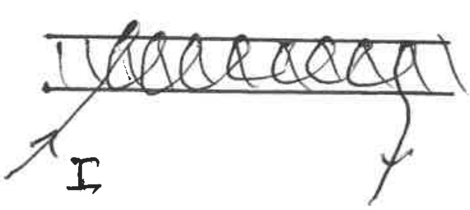
Want to return to the discussion of magnetic materials in a more quantitative discussion. What happens when a material is placed in a magnetic field

$\Longrightarrow$ consider a solenoid



$\longrightarrow B_0 = \mu_0 n I$

$\Longrightarrow$ add a core to the solenoid



$\longrightarrow B = K_m B_0$

K_m = permeability