Electric charges, forces and the electric field

The electromagnetic force is one of the fundamental forces of nature. Electric charges are the basic sources of these forces.

We will first discuss the basic characteristics the electric charge and then show how the fields from these charges, the electric and magnetic fields, are formed and can be used to calculate forces.

Electric charge

Simple experiments reveal that there are two basic kinds of charge: positive and negative.

⇒ plastic rods rubbed with fur repell each other

⇒ glass rods rubbed with silk repell each other

⇒ however, the plastic rod and glass rod attract each other

⇒ the "charge" on the plastic and glass rods must be different

~~positive (glass rod) and negative~~

$\Rightarrow$ describe the charge as positive (glass rod) or negative (plastic)

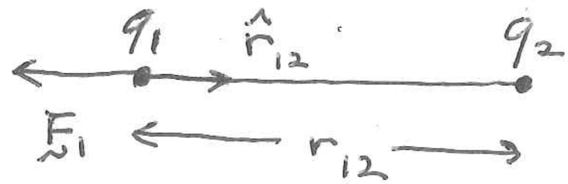
Coulomb's Law

The force between two charges q_1 and q_2 scales like

$$F \sim \frac{q_1 q_2}{r_{12}^2}$$

with r_{12} the separation between q_1 and q_2

$\Rightarrow$ doubling the separation reduces the force by a factor of four



$$\underset{\sim}{F}_1 = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

$\hat{r}_{12}$ = unit vector (magnitude unity) pointing from 1 to 2.

q measured in Coulombs

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N m^2} \quad \text{SI units}$$

= permittivity (electric constant)

$$\frac{1}{4\pi\epsilon_0} \simeq 9 \times 10^9 \frac{N m^2}{C^2}$$

What about $\underset{\sim}{F}_2$?

units:

$$F \sim \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \sim \frac{N m^2}{C^2} \frac{C\,C}{m^2} \sim N$$

$\Rightarrow$ force in Newtons

Characteristics of charge

The smallest unit of charge is

$$e = 1.6 \times 10^{-19}\ C$$

An electron has a charge $-e$ and a proton $+e$.

$\Rightarrow$ charge is conserved

Can move charge around a separate positive and negative charge but cannot create or destroy charge.

Characteristic forces

Take two charge of $1\,C$ and $1\,m$ apart.

$$F \sim 9 \times 10^9 \frac{N m^2}{C^2} \frac{(1C)^2}{(1m)^2} \sim 10^{10}\ N$$

$\Rightarrow$ very large force $\gg$ gravitational force

Avagadro's number of electrons

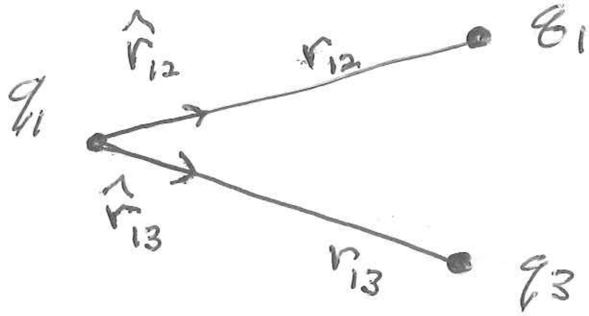
$$Q_A \sim -6 \times 10^{23} \otimes 1.6 \times 10^{-19}\ C$$

$$\sim 10^5\ C$$

What does this tell you about the number of electrons and protons in a normal physical environment?

Force between several charges

In situations with many charges you must do the <u>vector</u> <u>sum</u> of all of the forces acting on a charge



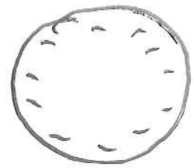
$$\underset{\sim}{F_1} = -\frac{1}{4\pi\epsilon_0}\left[\frac{\hat{r}_{12}\, q_2}{r_{12}^2} + \frac{\hat{r}_{13}\, q_3}{r_{13}^2}\right] q_1$$

Conductors and insulators

Some materials allow charge to move from one location to another $\Rightarrow$ typically electrons move while ions are stationary

$\Rightarrow$ <u>conductors</u>: silver, copper, water with disolved salts

Materials in which charges do not move are called <u>insulators</u>: ceramics, pure water, air charge on a spherical conductor:



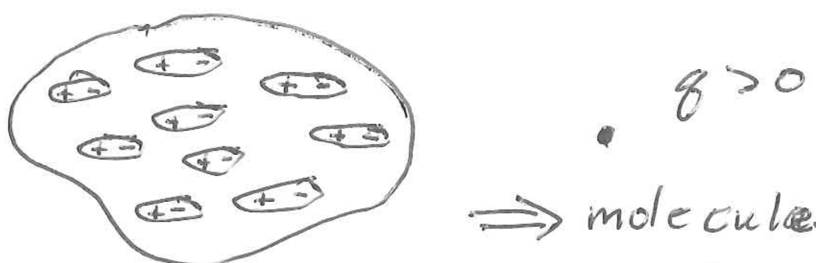
⇒ charges distribute to be as far apart as possible

charge near uncharged conductor:

⇒ electrons are attracted to the charge

⇒ is there a net force on the uncharged conductor?

polarization of an insulator:



⇒ molecules in the insulator orient with negative charge towards q.

⇒ is there a net force on the insulator?

Electric field

Consider a charge q at some location.

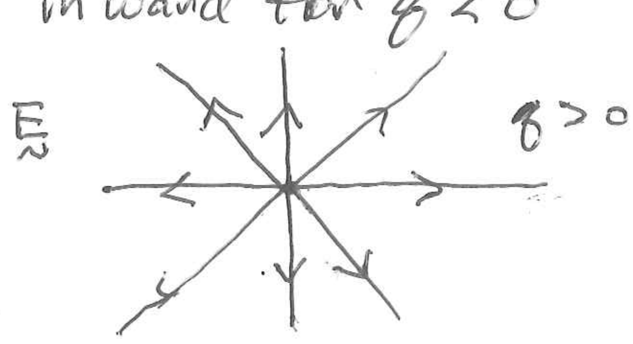
We can calculate the force on any other charge q' due to q by using Coulomb's law.

Alternatively, we can think of the charge q as modifying the space around it by forming an <u>electric field</u> similar to the gravitational field of the Earth. This field depends on q but not q'.

$$\underset{\sim}{E} = \frac{\underset{\sim}{F}}{q'} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

units: $E \sim \frac{F}{q'} \sim \frac{N}{C}$

For a single charge q we can draw the electric field as radial lines pointing outward from q if $q>0$ and radially inward for $q<0$



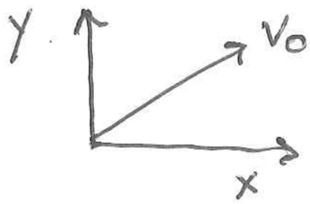
$\Rightarrow$ electric fields only start and end on charges

Can calculate the force ~~[illegible]~~ on any change q'

$$\Rightarrow \quad \underset{\sim}{F} = q' \underset{\sim}{E}$$

The electric field exists even if there are no other changes than q present.

Motion in a uniform electric field: electron



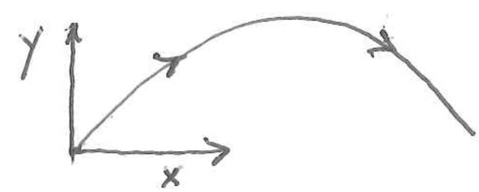
$\uparrow \underset{\sim}{E}$

$$\underset{\sim}{E} = E_y \hat{j}$$

$$q = -e$$

initial velocity:

$$\underset{\sim}{v}_0 = v_{x0} \hat{i} + v_{y0} \hat{j}$$



Force: $\underset{\sim}{F} = -e \underset{\sim}{E} = -e E_y \hat{j}$

Equation of motion: $m \frac{d}{dt} \underset{\sim}{v} = \underset{\sim}{F}$

$$m \frac{dv_x}{dt} = F_x = 0 \qquad v_x = v_{x0} = \text{const.}$$

$$\frac{dx}{dt} = v_x \Rightarrow x = x_0 + v_{x0} t$$

$$m \frac{dv_y}{dt} = F_y = -e E_y$$

$$v_y = v_{y0} - \frac{e E_y}{m} \int_0^t dt = - \frac{e E_y}{m} t$$

$$y = y_0 + \int_0^t dt' \, v_y(t') = y_0 + v_{y0} t - \frac{e E_y}{2m} t^2$$

Superposition of electric field

Consider the force on charge q from q_1 and q_2

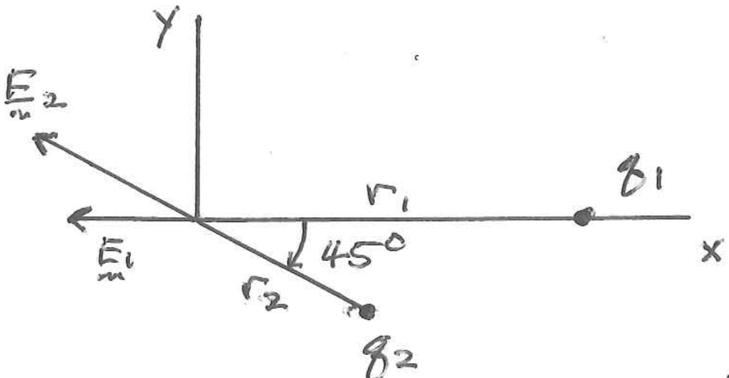
$$\vec{F} = \vec{F}_1 + \vec{F}_2 = q\vec{E}_1 + q\vec{E}_2$$

where q_1 produces $\vec{E}_1$ and q_2 produces $\vec{E}_2$.

$$\vec{E} = \frac{\vec{F}}{q} = \vec{E}_1 + \vec{E}_2$$

$\Rightarrow$ the total $\vec{E}$ is the vector sum of the electric field from all the charges

<u>example</u>: calculate $\vec{E}$ from two charges



$r_1 = 2m$

$r_2 = 1m$

$q_1 = 1.0\,nC$

$q_2 = 1.0\,nC$

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1^2} = 9\times10^9\,\frac{Nm^2}{C^2}\,\frac{10^{-9}C}{4m^2}$$

$$= \frac{9}{4}\,N/C$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2^2} = 9\,N/C$$

$$E_x = E_{1x} + E_{2x} = -\frac{9}{4}\frac{N}{C} - 9\frac{N}{C}\cos 45^\circ$$

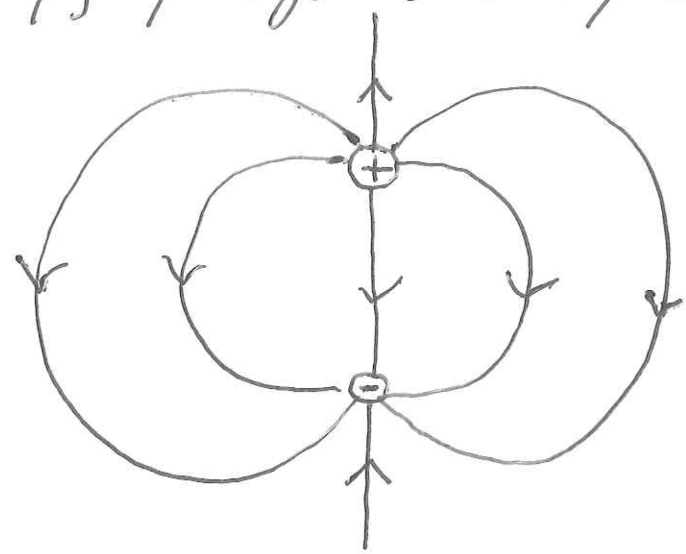
$$= -\left(\frac{9}{4} + \frac{9}{\sqrt{2}}\right)\frac{N}{C} \qquad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$E_y = +E_2 \sin 45^\circ = +\frac{9}{\sqrt{2}}\frac{N}{C}$$

Electric field from a dipole

Consider two charges $q, -q$ separated by a distance d.

Field pattern:



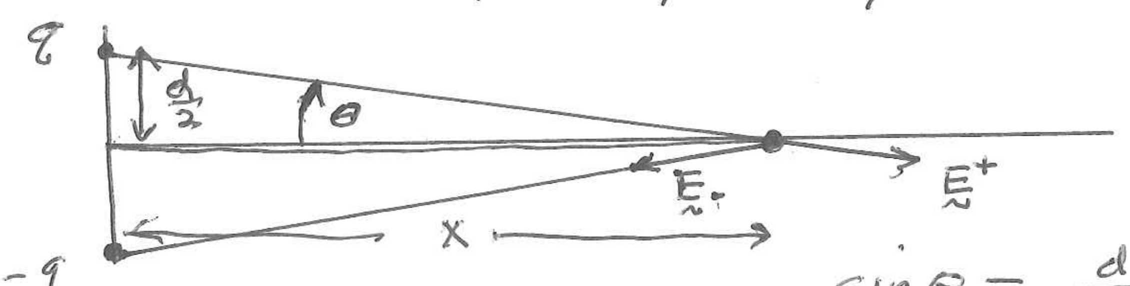
$\Rightarrow$ use symmetry arguments to draw the field lines

$\Rightarrow$ E from a single charge falls off like $1/r^2$. What about a dipole?

$\Rightarrow$ scaling argument

$$E \sim \underbrace{\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}}_{\text{from single charge but } q \text{ and } -q \text{ cancel if } d/r \to 0}\underbrace{\left(\frac{d}{r}\right)}_{\text{correction factor for } d \neq 0 \text{ must be dimensionless}} \sim \frac{qd}{4\pi\epsilon_0 r^3}$$

Calculate $\underset{\sim}{E}$ along the symmetry line:



$$\sin\theta = \frac{d}{2r}$$

$$r^2 = x^2 + d^2/4$$

what is the direction of $\underset{\sim}{E} = \underset{\sim}{E}^{+} + \underset{\sim}{E}^{-}$?

$\Rightarrow$ exploit symmetry

$\Rightarrow$ along x $\underset{\sim}{E}^{+}$ and $\underset{\sim}{E}^{-}$ cancel

$\Rightarrow$ $\underset{\sim}{E}$ along negative y.

$$E^{+} = E^{-} = \frac{q}{4\pi\epsilon_0 r^2} \equiv E$$

$$E_y^{+} = -E\sin\theta$$

$$E_y^{-} = -E\sin\theta$$

$$E_y = -2E\sin\theta = -2\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\frac{d}{2r}$$

$$= -\frac{qd}{4\pi\epsilon_0}\frac{1}{\left(x^2+d^2/4\right)^{3/2}}$$

$p = qd$ = dipole moment

$$E_y = -\frac{p}{4\pi\epsilon_0}\frac{1}{\left(x^2+d^2/4\right)^{3/2}}$$

large $x \gg d/2$

$$E_y \simeq -\frac{p}{4\pi\epsilon_0}\frac{1}{x^3}$$

Electric field from continuous charge distributions

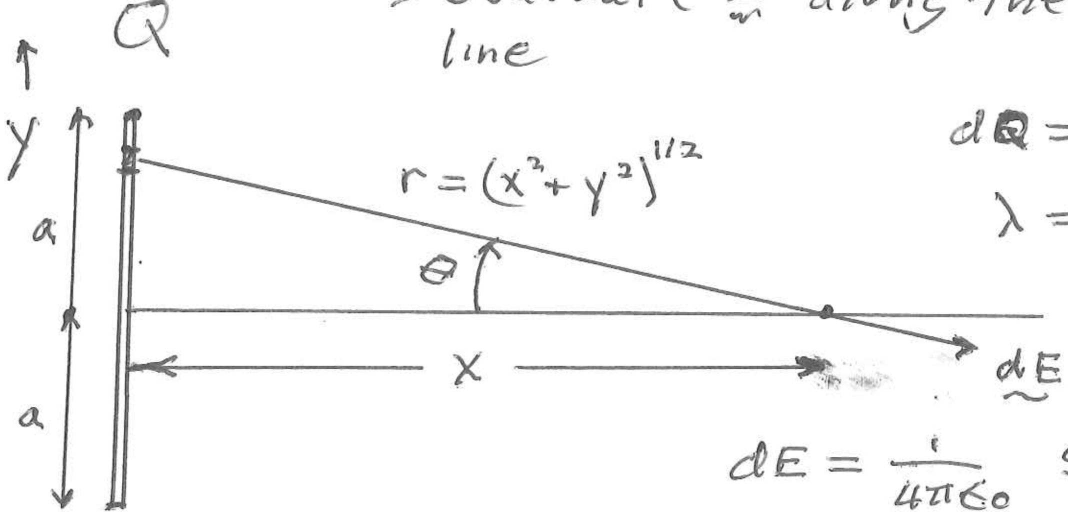
Generally have to integrate the electric field contributions from the charge distribution ⇒ vector sum

⇒ first use symmetry, to determine the direction of the field

⇒ Find the field from a small portion of the charge dQ

⇒ Integrate

example : Electric field from a line charge
⇒ evaluate $\underset{\sim}{E}$ along the symmetry line



$$dQ = \left(\frac{Q}{2a}\right) dy$$

$$\lambda = \frac{Q}{2a} = \frac{\text{charge}}{\text{length}}$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2}$$

Direction of $\underset{\sim}{E}$ along x ⇒ discard dE_y

$$dE_x = dE \cos\theta \quad \Rightarrow \cos\theta = \frac{x}{(x^2+y^2)^{\frac{1}{2}}}$$

$$E_x = \int dE_x = \lambda\, x \int_{-a}^{a} dy \frac{1}{(x^2+y^2)^{3/2}} \frac{1}{4\pi\epsilon_0}$$

$$\int dy \frac{1}{(x^2+y^2)^{3/2}} = \frac{y}{x^2(x^2+y^2)^{1/2}}$$

$$E_x = \left(\frac{Q}{2a}\right)\frac{2x}{4\pi\epsilon_0}\frac{a}{x^2(x^2+a^2)^{1/2}} = \frac{Q}{4\pi\epsilon_0}\frac{1}{x}\frac{1}{(x^2+a^2)^{1/2}}$$

⇒ In what limit do you know the answer?

⇒ For $x \gg a$, looks like a simple point charge

$$E_x = \frac{Q}{4\pi\epsilon_0}\frac{1}{x^2}$$

⇒ What about $x \ll a$?

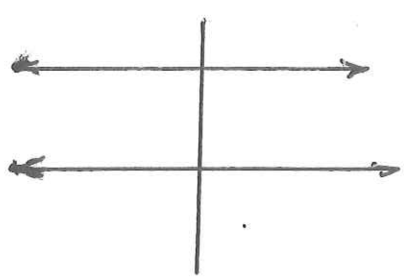
$$E_x = \frac{Q}{4\pi\epsilon_0}\frac{1}{ax} = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{x}$$

⇒ field from an infinite line charge falls off as $\frac{1}{x}$

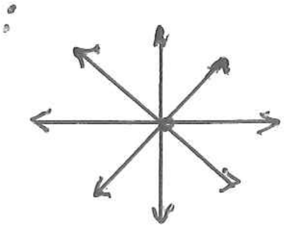
Depends only on charge / length λ and not total Q.

<u>Infinite line charge</u> $\underset{\sim}{E}$

side:

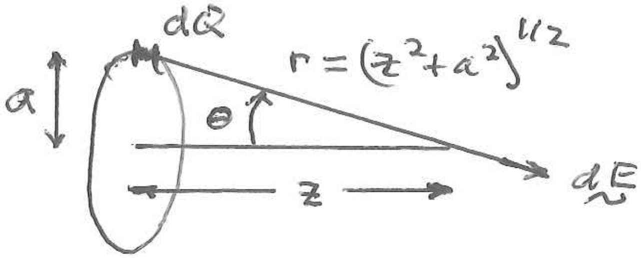


endon:



<u>example</u>: Electric field from charged ring

Charged ring of radius "a" along symmetry axis.



dQ

By symmetry $\underset{\sim}{E}$ along z direction

From charge dQ, $dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}$

$\Rightarrow$ component along z

$$dE_z = dE\cos\theta = dE\,\frac{z}{r}$$

$$dE_z = \frac{1}{4\pi\epsilon_0}\,dQ\,\frac{z}{r^3}$$

$$E_z = \int dE_z = \frac{1}{4\pi\epsilon_0}\int\frac{dQ\,z}{r^3}$$

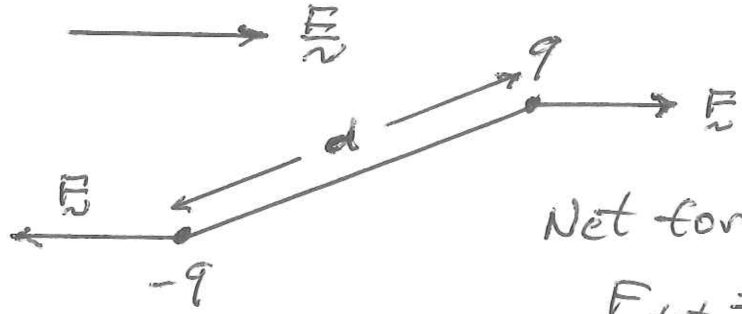
$\Rightarrow$ z and r same for all dQ

$$E_z = \frac{Q}{4\pi\epsilon_0}\,\frac{z}{(a^2+z^2)^{3/2}}$$

$\Rightarrow$ large z $\Rightarrow$ $E_z = \dfrac{Q}{4\pi\epsilon_0 z^2}$ $\Rightarrow$ point charge

$\Rightarrow$ small z $\Rightarrow$ $E_z \propto z$

Dipole in a uniform electric field



Net force is zero

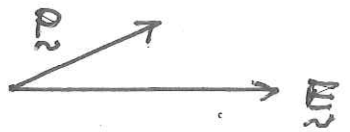
$$\mathbf{F}_{tot} = q\mathbf{E} - q\mathbf{E} = 0$$

$\Rightarrow$ dipole wants to rotate

$\Rightarrow$ torque is not zero

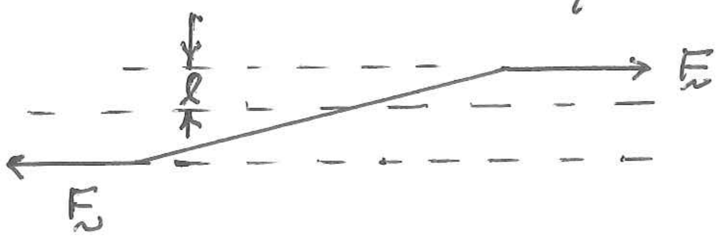
Define a vector dipole moment $\mathbf{p}$ with $p = qd$

$\Rightarrow$ $\mathbf{p}$ points from negative to positive



$\Rightarrow$ $\mathbf{p}$ wants to align with $\mathbf{E}$.

Calculate the torque



l = moment arm

$= \frac{d}{2} \sin\theta$

$$\tau = -Fl - Fl$$

$$= -2\, qE\, \frac{d}{2} \sin\theta$$

$$\tau = -Ep \sin\theta$$

vector form $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$

$\Rightarrow$ note $\tau < 0$ since acts to decrease θ.

Potential energy of a dipole

Equation describing the rotational motion of a dipole

$$I \frac{d\omega}{dt} = \tau = -Ep\sin\theta \qquad \text{with } \omega = \frac{d\theta}{dt}$$

$$I\omega \frac{d\omega}{dt} = -Ep\frac{d\theta}{dt}\sin\theta = Ep\frac{d}{dt}\cos\theta$$

$$\frac{d}{dt}\left(\frac{1}{2}I\omega^2 - Ep\cos\theta\right) = 0$$

$$U = \text{potential energy} = -Ep\cos\theta = -\underset{\sim}{p}\cdot\underset{\sim}{E}$$

$\Longrightarrow$ minimum energy when $\underset{\sim}{p}$ is aligned with $\underset{\sim}{E}$.