Comment on "Tank-treading and tumbling frequencies of capsules and red blood cells"

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In a recent computational investigation, Yazdani *et al.* [Phys. Rev. E **83**, 046305 (2011)] reported an exponential (logarithmic) dependence of the tank-treading frequency F_{tt} with the viscosity ratio λ at low (moderate) viscosity ratios for erythrocytes in shear flows. We argue that the authors misinterpreted the inverse linear dependence on λ of the tank-treading frequency F_{tt}, as found by Wodson and Dimitrakopoulos [Biophys. J. **99**, 2906 (2010)], owing to the fact that Yazdani *et al.* plotted their frequency-versus-λ results in a log-log plot.

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Recently, Yazdani *et al.* [1] determined computationally the tank-treading frequency of capsules and erythrocytes in shear flows as a function of the viscosity ratio $\lambda = \mu_c/\mu$ (where μ_c is the cytoplasm viscosity and μ the viscosity of the surrounding fluid). The authors reported that for erythrocytes they found two regimes of the viscosity dependence of the tank-treading frequency: an exponential regime at low viscosity ratios (typically $\lambda < 0.3$) in which the tank-treading frequency decreases at a slower rate with increasing viscosity ratio, and a logarithmic range (for $0.5 < \lambda < 3$) in which it decreases at a much faster rate. (See also Fig. 13 in Ref. [1].)

We note that the complicated behavior of the erythrocyte's tank-treading frequency reported in the recent study of Yazdani *et al.* [1] is not in contradiction with that found and analyzed in our earlier computational work for erythrocytes [2] and also shown here in Fig. 1(a). In particular, in our earlier work [2] we verified that the tank-treading frequency F_{tt} increases linearly with the shear rate G, as reported in the experimental work of Fischer and co-workers [3,4]. In addition, our computations revealed that the erythrocyte's tank-treading period $G\,P_{tt}$ increases linearly with the viscosity ratio λ. The increase of the tank-treading period with the viscosity ratio can be understood from the increased hydrodynamic forces in the cytoplasm which slow down the rotation of the inner fluid and thus the erythrocyte membrane [2].

We argue that Yazdani *et al.* [1] misinterpreted the dependence of the tank-treading frequency on the viscosity ratio because they plotted their frequency-versus-λ results in a log-log plot in their Fig. 13. To show this, in Fig. 1(b) we replot our computational results for the tank-treading frequency versus the viscosity ratio in a log-log plot. As seen in this figure, the tank-treading frequency, which is an inverse linear function of λ, as clearly shown in Fig. 1(a) for the entire range of viscosity ratios, does appear to have the same complicated behavior reported by Yazdani *et al.* [1].

We note that our computational results on the linear increase of the tank-treading period with the viscosity ratio are in quantitative agreement with the predictions of the Skotheim-Secomb model [5] when we provide the erythrocyte lengths (from our computations) in this model. This is due to the addition of the (correct) shape memory effects in the Skotheim-Secomb model, as we discussed in our recent publication [6].

Based on the above, we can easily verify our findings via simple scaling analysis. Taking into consideration the balance of the rate of work done by the surrounding fluid on the ellipsoidal cell during tank treading with the internal dissipation and the rate of elastic energy storage owing to shape memory, Eq. (6) in the work of Skotheim and Secomb [5]

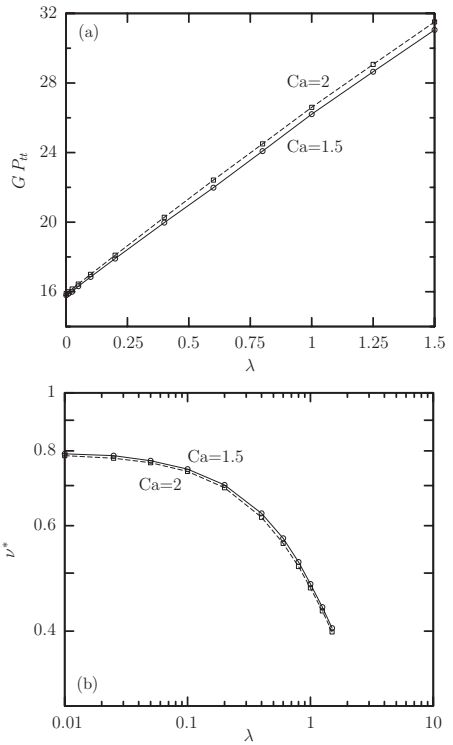

FIG. 1. (a) Tank-treading period $G\,P_{tt}$ as a function of the viscosity ratio λ for an erythrocyte in a shear flow with capillary number Ca = 1.5,2 based on the computational results of Dodson and Dimitrakopoulos reported in Fig. 5(b) of Ref. [2]. (b) As in (a) but for the tank-treading frequency ν^* versus the viscosity ratio λ in a log-log plot. Note that we define the tank-treading frequency $\nu^* = 4\pi/G\,P_{tt} = 4\pi\,F_{tt}/G$ as in Ref. [1], while our capillary number $\mathrm{Ca} = \mu\,G\,a/G_s$ (where a is the radius of a sphere with the same volume as the erythrocyte and G_s the membrane shear modulus) is twice that used in Ref. [1].

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suggests that, for cells with significant shape-memory effects, the tank-treading frequency F_{tt} scales as

$$F_{tt} \sim \frac{E_0}{\lambda V_c \mu f_1 - V_c \mu f_2}, \tag{1}$$

where E_0 is the elastic energy change, V_c the cell volume, and f_1, f_2 dimensionless functions of the cell's lengths [5]. Therefore the erythrocyte's tank-treading period $P_{tt} = F_{tt}^{-1}$ is a linear function of the viscosity ratio λ :

$$G P_{tt} \sim \frac{V_c \mu(-f_2) G}{E_0} + \frac{V_c \mu f_1 G}{E_0} \lambda. \tag{2}$$

In the (swinging) tank-treading regime, $f_1 > 0$ while $f_2 < 0$ [6], and thus the tank-treading period $G P_{tt}$ increases linearly with the viscosity ratio λ. It is of interest to note that this dependence is not restricted to erythrocytes but represents other types of capsules, for the same physical reasons.

Finally we emphasize that available experimental data on the dependence of the erythrocyte's tank-treading frequency with the viscosity ratio should not be depicted as simple (straight) curves as seen in Fig. 13 in the work of Yazdani *et al.* [1]. This is true even for the experimental data of Fischer [4] (probably the highest quality available data) which show a wide variation (probably owing to the inherent differences between erythrocytes), as discussed extensively in the earlier experimental study [4] and our recent computation work [2]. (See Figs. 4 and 5(b) and the associated discussion in Ref. [2].)

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