

PHYS 401 Homework---Due September 28

Griffiths---Problems 1.12, 1.14

If you do problem 1,14 correctly you will find that uncertainty is consistent with the Heisenberg uncertain relation but the inequality will become an equality. The type of wavefunction in 1.14 is called a minimum uncertainty wave packet.

In addition do:

1. Show that if \hat{A} and \hat{B} are both hermitian operators, then $i[\hat{A}, \hat{B}]$ is also hermitian where the square bracket indicates the commutator: $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$. Recall that a hermitian operator is one which has a real expectation value in any wave function (that value in any wavefunction that vanishes at $\pm\infty$). You may find it useful to use the fact that for any hermitian operator \hat{A} and any wave functions which vanishes at $\pm\infty$:

$$\int dx \psi_1^*(x) (\hat{A} \psi_2(x)) = \int dx (\hat{A} \psi_1(x))^* \psi_2(x).$$

2. Show that if \hat{A} is a Hermitian operator associated with a physical observable, the time derivative of its expectation value is given by

$$\frac{d\langle \hat{A} \rangle}{dt} = \frac{-i}{\hbar} \langle [\hat{A}, \hat{H}] \rangle$$

where \hat{H} is the Hamiltonian operator. Hint: Use the Schrödinger equation and the relation given in problem 1. Note that this result implies if an operator has a zero commutator with the Hamiltonian (*i.e.* it commutes) then its expectation value is a constant of the motion.