Physics 374 Due December 11

1. The potential on the surface of a sphere of radius R is fixed at $\Phi(R,\theta)=V_0\cos^2(\theta)$ with the potential defined to be zero at infinity. There are no charge for r>R or r<R

   a) Use the known Legendre polynomials to express the potential on the surface as the sum of two terms---one with l=0 and the other with l=2, *i.e.* express the potential as $\Phi(R,\theta)=c_0\ P_0\left(\cos(\theta)\right)+\ c_2\ P_2\left(\cos(\theta)\right)$ where $c_0\ ,c_2$ are coefficients.

   b) Find the potential and electric field for r>R.

   c) Find the potential for r<R.

2. In class we used δ functions to prove that the Fourier transform of a convolution is the product of the Fourier transforms. In this problem I want you to prove the converse namely that the Fourier transform of a product is the convolution of the Fourier transforms. That is:

$$\int dx\, e^{ikx} f(x)g(x)=\int dk'\ \tilde{f}(k-k')\tilde{g}(k')$$

3. Consider a "field" ψ which satisfies the following wave equation: $\left[\frac{\partial^2}{\partial t^2}-v_0{}^2\frac{\partial^2}{\partial x^2}+\beta^2\frac{\partial^4}{\partial x^4}\right]\psi(x,t)=0$. Show that solutions to the wave equation can be written in the form :

$$\psi(x,t)=\int dk\left[f_r(k)\,e^{i\left(kx-\omega(k)t\right)}+\ f_l(k)\,e^{i\left(kx+\omega(k)t\right)}\right]$$

with $\omega(k)=k\sqrt{v_0{}^2+\beta^2k^2}$

4. For the system described in problem 3. suppose that we know the value of the field and its time derivative at t=0: $\psi(x,t=0)=Ae^{-(x/L)^2}$ and $\left.\frac{\partial\psi}{\partial t}\right|_{t=0}=0$. Show that the field as a function of time is given by $\psi(x,t)=\int dk\ \ e^{ikx}\left(\frac{L}{2}\right)e^{-\left(\frac{kL}{2}\right)^2}\ \cos\left(k\sqrt{v_o{}^2+\beta^2k^2}\ t\right)$