Physics 374 Homework 7 Due Nov. 7

1. A cylinder of radius R contains charge according to the following charge density: $r(\vec{x}) = c\, x^2 J(R-r)$ where c is a constant, $r = \sqrt{x^2+y^2}$ and θ is the usual step function. The goal of this problem is to find the electric field for the region r>R.

   a) As a first step calculate the multipole coefficients. You should find only the m=0 and m=2 terms contribute. Hint: write the x^2 term in polar coordinates.
   b) Find the potential Φ. As the m=0 term is nonzero you must pick an arbitrary r_0.
   c) Find an expression for the electric field.

2. For the problem in 1 find the potential for r<R. Hint: use the general solution.

3. Consider 4 line charges oriented along the z axis, 2 with a charge per unit length of +λ located at (x,y)=(d,d) and (x,y)=(-d,-d) and 2 with a charge per unit length of -λ located at (x,y)=(d,-d) and (x,y)=(-d,d).

   a) Show that the exact expression for the potential is given by

   $$\Phi(\vec{r}) = -\,2\,l\,\log\left(\frac{\sqrt{\left(r^2+2d^2+2^{3/2}r\,d\cos(f-\frac{p}{4})\right)\left(r^2+2d^2+2^{3/2}r\,d\cos(f-\frac{5p}{4})\right)}}{\sqrt{\left(r^2+2d^2+2^{3/2}r\,d\cos(f-\frac{3p}{4})\right)\left(r^2+2d^2+2^{3/2}r\,d\cos(f-\frac{7p}{4})\right)}}\right)$$

   Hint: Use the alternative general expression and note that the integral just gets replaced by a discrete sum over the line charges. Recall $|\vec{x}-\vec{y}| = \sqrt{x^2+y^2+2\vec{x}\cdot\vec{y}}$
   b) Show that the multipole coefficients are given by

   $$b_o = 0$$

   $$a_m = \frac{\left(\sqrt{2}\,d\right)^m}{m}\left(\exp(i\tfrac{mp}{4}) + \exp(i\tfrac{m5p}{4}) - \exp(i\tfrac{m3p}{4}) - \exp(i\tfrac{m7p}{4})\right)$$

   Hint: Again note that that the integral just gets replaced by a discrete sum over the line charges.
   c) Write an approximate expression for the potential based on the multipole expansion and including up to the m=4 term.
   d) Test how well the multipole expression works by using Mathematica to plot the exact expression, and multipole truncated at m=2,3 and 4 as a function of r for ϕ=π/4.

4. It was argued in class that multipole coefficient with smallest nonvanishing m is independent of the choice of origin. Show that the m=2 multipole for the problem does not change if move the origin to the position of the charge at the lower left.