PHYS 374 Homework---Due October 24

1. Two particles of mass *m* collide and stick together forming a new particle.
   a) Supposing their initial velocities where equal and opposite with a magnitude of *v*, find the mass of the final particle.
   b) Suppose instead that we studied the problem in the frame where one of the particles is at rest. Show that the magnitude velocity of the moving particle is $\frac{2v}{1+v^2}$
   c) Find the energy and momentum in this new frame.
   d) Use the relation between energy and momentum to calculate the momentum in the new frame and verify it is the same as the result in a).

2. Consider the collision between two particles of mass *m*. These two particle are annihilated in the collision and two massless particles are produced. This can occur for example when an electron and a positron annihilate into two photons.
   a) Suppose that the initial velocities of the two massive particle are equal and opposite and have magnitude *v* and that the two massless particle come out in directions perpendicular to the initial velocities of the massive particles. Find the energy and momentum of the two massless particles.

   b) Consider the same problem as in a) but viewed in a frame where one of the massive particles is at rest and the other is moving with a velocity whose magnitude is $\frac{2v}{1+v^2}$. Find the energy and momentum of the two massless particles in this case. What angle do they make relative to the initial velocity?

   c) Now again suppose that the initial velocities of the two massive particle are equal and opposite and have magnitude *v* but that the two massless particle come out in directions parallel to the initial velocities of the massive particles. Find the energy and momentum of the two massless particles.
   d) Consider the same problem as in c) but viewed in a frame where one of the massive particles is at rest and the other is moving with a velocity whose magnitude is $\frac{2v}{1+v^2}$. Find the energy and momentum of the two massless particles in this case.

3. Consider the following covariant equation of motion $\vec{a}^4 = \frac{d\vec{u}^4}{d\tau} = \vec{f}^4 - \vec{u}^4\left(\vec{u}^4 \cdot \vec{f}^4\right)$ where $\vec{f}^4$ is a 4-vector.

   a) Show that for any 4-vector this satisfies the constraint equation $\vec{a}^4 \cdot \vec{u}^4 = 0$.

   b) Suppose that in the "lab frame" $\vec{f}^4 = \begin{pmatrix} 0 \\ a \\ 0 \\ 0 \end{pmatrix}$ find the differential equations for the components of $\vec{u}^4$ as a function of τ.