Physics 374 Homework 5 Due October 17

1. A rod in its rest frame is of length L and lies in the x-y plane making an of ϕ relative to the x axis. Find the angle that the rod makes to the x-axis in a frame moving along the x-axis with a velocity v relative to the rest frame.

2. In class we work through the twin paradox problem in the frame of twin 1, the twin who stayed home. Here I want you to work through the same problem in the frame co-moving with twin 2 on her way out. In the rest frame of twin 1 the trajectories of the two twins are $\vec{x}_1 = 0$, $\vec{x}_2 = v\,t\theta\left(\frac{T}{2}-t\right)\theta(t) + \theta\left(t-\frac{T}{2}\right)\theta(T-t)\left(\frac{v\,T}{2}-v\left(t-\left(\frac{T}{2}\right)\right)\right)$ where θ is the usual step function, which is zero for negative arguments and 1 for positive arguments. Note that the form given for the trajectories is not explicitly covariant in terms of Lorentz transformations.

   a) Use the expression $\tau = \int dt\sqrt{1-\vec{v}^2}$ to find the proper time of each of the two twins from the event where they leave each other to the event where they meet again. Do this calculation in the rest frame of twin 1. (This just redoes what we did in class using these parameters). You should find that Twin 1is older by a factor of γ.
   b) Find the trajectories in the frame comoving with twin 2 on his outward voyage.
   c) Working in this new frame use the expression $\tau = \int dt\sqrt{1-\vec{v}^2}$ to find the proper time of the two twins. You should find your result identical to those of part a).

3. In class we have derived the longitudinal Doppler shift, *i.e.* the shift in frequency for motion in the direction of propagation of a light wave. We did this by exploiting the fact that one can combine the frequency and wave-vector into a four vector:

$$\vec{k}^4 = \begin{pmatrix} w \\ k_x \\ k_y \\ k_z \end{pmatrix}.$$

   Here I want you to derive the transverse Doppler shift. That is the shift in frequency due to motion perpendicular to the propagation of the wave. That is, assuming you have a light wave propagating in the y direction with a frequency of ω to find the frequency in a frame moving relative to the initial frame in the x direction with a velocity v.

4. In this problem we revisit the twin paradox from the point of view of the Doppler shift. Suppose the clock used by twin 1 one was synchronized to a propagating radio wave is that each "tick" of the clock corresponded to one cycle of the radio wave. These "ticks" can be measured by twin 2---by counting cycles of the radio wave she can determine how much time has elapsed for her twin. Of course these radio waves shifted down in frequency (red-shifted) on the way out and shifted up in frequency (blue-shifted) due to the Doppler effect. Using the situation discussed in problem 2. and the Doppler formula compute how many "ticks of the clock" are measured by twin 2 during her voyage. This number is the amount by which her twin has aged. Does this agree with the result in 2?