PHYS 374 Homework 4---Due October 10

1. Consider a particle of mass *m* in an anharmonic potential given by $U(x) = D\left(2 - \cos(x/L) - \cos(3x/L)\right)$ with *D* a constant with units of energy.
   a) Suppose the mass is released from rest at an initial position of $x_i << L$. Find an approximate expression for the motion including the first nontrivial correction due to anharmonicity.
   b) Suppose the system is from rest at an initial position of x_i where $L - x_i << L$ find an approximate expression for the period of oscillation.

Hint: This problem is quite straightforward given what we have derived in class---you do *not* need to reinvent the wheel.

2. Suppose we have two frames related by a Lorentz boost in the x direction:

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

Show explicitly that $t'^2 - x'^2 - y'^2 - z'^2 = t^2 - x^2 - y^2 - z^2$ as claimed in lecture.

3. In this problem I want you to derive an expression for how velocities in the same direction add in relativity . To do this consider three frames labeled 0, 1, and 2. Frame 1 is obtained from 0 by a boost of v_1 in the x direction and 2 is obtained from 1 by a boost of v_2 in the x direction.

$$\begin{pmatrix} t_2 \\ x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} \gamma_2 & -\beta_2\gamma_2 & 0 & 0 \\ -\beta_2\gamma_2 & \gamma_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t_1 \\ x_1 \\ y_1 \\ z_1 \end{pmatrix} \qquad \begin{pmatrix} t_1 \\ x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} \gamma_1 & -\beta_1\gamma_1 & 0 & 0 \\ -\beta_1\gamma_1 & \gamma_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t_0 \\ x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

where $\beta_1 = v_1$, $\gamma_1 = \dfrac{1}{\sqrt{1-\beta_1^2}}$, $\beta_2 = v_2$, $\gamma_{21} = \dfrac{1}{\sqrt{1-\beta_2^2}}$.

By doing a simple matrix multiplication one can express frame 2 in terms of frame zero. Show that the total velocity is given by $v_T = \dfrac{v_1 + v_2}{1 + v_1 v_2}$.

4. Verify that the expression derived in 3 has the correct limits when

   a) Both v_1 and v_2 are much less than unity (*i.e.* much less than *c* if we did not use dimensionless units).
   b) Either v_1 or v_2 is equal to 1 (*i.e. c* in dimensionful units).