PHYS 374 Homework 3---Due October 2

1. Consider a particle of mass *m* in a harmonic potential around the origin characterized by a spring constant *k*. In this problem I want you to use the integral expression based on energy conservation to solve for the motion. The integral you get can be analytically---look it up on a table or use Mathematica and $t(x)$ can easily be inverted to get $x(t)$.

   a) Find the motion if the particle is initially at rest at position *A*.
   b) Find the motion if the particle is initially at $x=0$ and moving with velocity v (with $v>0$).

2. Consider a particle of mass *m* moving in the following anharmonic potential: $U(x) = \frac{1}{2}k\,x^2 + \frac{1}{24}\alpha\, x^4$ with $\alpha>0$. The particle is released from rest at $x=A$.
   a) Use energy conservation to determine the speed of the particle when it reaches $x=0$.
   b) Use the approximate expression based on Fourier analysis to determine the velocity at $x=0$. You should work at lowest nontrivial order (including the 3ω term in the expansion)
   c) Specify the condition on *A* for which you expect your answer in b) to be valid.

3. Show that the two expressions derived in problem 2a. and 2b are consistent, *i.e.* they differ only beyond the order calculated in the approximation

4. Consider the system described in problem 2.
   a) Derive an expression for the period of this system as a function of the amplitude using the energy conservation method described in class. The expression can be left in the form of an integral. Hint: how is the period related to the time it takes to go from $-A$ to A where *A* is the amplitude
   b) Derive an approximate expression for the period based on the Fourier technique used in class. The lowest order expression is $2\pi/\omega_0$. Here I want you to find the correction to this valid up to quadratic order in the amplitude---*i.e.* up to the first nontrivial correction.
   c) For $A=\frac{1}{10}\sqrt{k/\alpha}$ find the period using the exact expression in part a), the lowest order approximate expression in part b) and the approximate expression in b) including the first nontrivial correction. In using the exact expression you must do it numerically. However, doing the numerical integral requires that one scale out various factors and do the integral using appropriate dimensionless variables.