AMSC/MATH 420, Spring 2013
Modeling Epidemics: Team Homework 5

due Friday April 8

In class we'll discuss the two-group SI model with two types of interventions, each of which may affect the two groups differently:

$$\begin{aligned} dS_1/dt &= -p_{11}S_1\mathcal{I}_1 - p_{12}S_1\mathcal{I}_2 - a_1S_1 \\ d\mathcal{I}_1/dt &= p_{11}S_1\mathcal{I}_1 + p_{12}S_1\mathcal{I}_2 - (a_1+b_1)\mathcal{I}_1 \\ dS_2/dt &= -p_{21}S_2\mathcal{I}_1 - p_{22}S_2\mathcal{I}_2 - a_2S_2 \\ d\mathcal{I}_2/dt &= p_{21}S_2\mathcal{I}_1 + p_{22}S_2\mathcal{I}_2 - (a_2+b_2)\mathcal{I}_2. \end{aligned}$$

Let's call the interventions "type a", which affects susceptible and infectuous people equally, and "type b", which only affects infectuous people.

To get a sense of how these parameters affect the size of an outbreak, let's first consider them in the basic SI model:

$$\begin{aligned} dS/dt &= -pS\mathcal{I} - aS \\ d\mathcal{I}/dt &= pS\mathcal{I} - (a+b)\mathcal{I}. \end{aligned}$$

(If $a = 0$, this is equivalent to the SIR model.) One way to account for the total number of people infected (according to the model) is to add a "removed" population $R(t)$ with $R(0) = 0$ and

$$dR/dt = (a+b)\mathcal{I}.$$

Notice that R only counts people removed from the infectuous population, not from the susceptible population. Then $I(T) + R(T)$ is the total number of people infected up to time T. To assess the size of the entire outbreak, you should use a value of T large enough so that $S'(T)$ is close to zero. Remember that if $a = b = 0$, the total number of people infected will eventually be $N = S(0) + I(0)$.

1. Choose values of N and p appropriate to one your data sets (the exact values are not so important), and set $I(0) = 0.01N$. Find the approximate value (2 significant digits is fine) of a_{50} such that the size of the outbreak is 50% of N when $a = a_{50}$ and $b = 0$. Similarly, find a_{25} that reduces the size of the outbreak to 25% of N when $b = 0$. Also, find b_{50} and b_{25} that reduce the size of the outbreak to 50% and 25% of N when $a = 0$. Next suppose that you divide your resources between the two interventions in the following sense: let $a = 0.5a_{50}$ and $b = 0.5b_{50}$. What is the size of the outbreak for this split strategy? Finally, same question for splitting between a_{25} and b_{25}.

2. Next, consider the 2-group model using the values of the parameters and initial conditions you found from fitting each of your data sets in the previous assignment (set $p_{12} = 0$ if your fit yielded a negative value). The idea here is to compare type a intervention targeted primarily at group 1 versus type a intervention directed equally toward both groups. For the targeted strategy, set $b_1 = b_2 = 0$ and $a_1 = 10a_2$ and find how big you have to make (a_1, a_2) to reduce the size of the outbreak to 50% of N and to 25% of N. Then, set $b_1 = b_2 = 0$ and $a_1 = a_2$ and see how big you have to make the a's to get the same reductions. Finally, determine the effects of the split strategies that average the two sets of "50%" parameters you found and the two sets of "25%" parameters you found. Do these tasks for each of your two cities.

3. Same question but for type b intervention.

4. Summarize and interpret your results.