Modeling Portfolios that Contain Risky Assets

Optimization III: Conclusion

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Optimization III: Conclusion

Selecting the Risk Aversion Coefficient.

We now know how for find an optimal portfolio for a given choice of the risk aversion coefficient χ, but we have not said how to select a value of χ. The idea is the following.

1. We compute the optimal portfolio $\mathbf{f}_{\mathsf{opt}}(\chi)$ for several values of χ.

2. For each $\mathbf{f}_{\mathsf{opt}}(\chi)$ identify and compute relevant measures of uncertainty $\{\widehat{\omega}_k(\chi)\}$.

3. Based on historical data, decide what find a χ that performs best for a given choice of $\{\widehat{\omega}_k\}$, call it $\chi_{\mathsf{best}}(\widehat{\omega})$.

4. Select a χ for which $\chi \approx \chi_{\mathsf{best}}(\widehat{\omega})$.

Summary. The MPT models that we have developed illustrate three basic principles of portfolio management. (As we have seen, there are others.)

1. *Diversification reduces the volatility of a portfolio.*

2. *Increased volatility lowers the expected growth rate of a portfolio.*

3. *Diversification raises the expected growth rate of a portfolio.*

Remark. The last of these follows from the first two.

Reacll that two kinds of people hold risky assets: *traders and investors*.

Traders often take positions that require constant attention. They might buy and sell assets on short time scales in an attempt to profit from market fluctuations. They might also take highly leveraged positions that expose them to enormous gains or loses depending how the market moves. They must be ready to handle margin calls. Trading is often a full time job.

Investors operate on longer time scales. They buy or sell an asset based on their assessment of its fundamental value over time. Investing does not have to be a full time job. Indeed, most people who hold risky assets are investors who are saving for retirement. Lured by the promise of high returns, sometimes investors will buy shares in funds designed for traders. At that point they have become gamblers, whether they realize it or not.

It is important to realize that MPT is designed to help balance investment portfolios, not trading portfolios.

Advice. If you invest or plan to invest, I highly recommend that you read

- *The Investment Answer* by Daniel C. Goldie and Gordon S. Murray, Business Plus, New York, 2011.

This short, straight talking book provides a framework for investing with a minimum of mathematics. It tells you most things you need to know to invest wisely except how to allocate your assets. But that is what these lectures have been about. The two complement each other.

An important topic discussed in the book is how to engage professional advice. Professionals will be generally more knowledgeable than you about investment products that are available. Their advice will become more valuable as your portfolio grows. Your interactions with them will be more productive if you engage them as an informed customer.

The MPT models we have studied can be used to manage your investment portfolio provided certain caveats are kept in mind.

1. Never invest money in risky assets that you are not prepared to lose.

2. Develop an understanding of the economic, technological, political, and natural events that might effect your investments. Invest for the long term based on this understanding.

3. Study each risky asset before you invest in it. Decide if its business plan makes sense in the context of your larger understanding.

4. Remember that investing is not a science and models are not reality. Use models for guidence with a full awareness of their limitations.

One major limitation of the models we have studied is that they assume the validity of an underlying IID model. The truth is that all agents who buy and sell risky assets are influenced by the past. An IID model will be valid when the motives of enough agents are sufficiently diverse and uncorrelated. You can test the validity of this assumption with the historical data. But even when the historical data supports this assumption, you must be on guard for correlations that might arise due to changing circumstances.

Another major limitation is that they assume the probability densities in the underlying IID model are sufficiently narrow that second moments exist. When this assumption is not valid this theory breaks down completely.

Yet another major limitation is that dependencies between different assets are only captured by the covariances in historical data. Such models can lose validity when a major event occurs that has no analog in the period spanned by the historical data that you used to calibrate your model.

Recall that certain aspects of MPT that are unrealistic. These generally arise from simplifications that were adopted to make the analysis easier. These include:

- the fact portfolios can contain fractional shares of any asset;
- the fact portfolios are rebalanced every trading day;
- the fact transaction costs and taxes are neglected;
- the fact dividends are neglected.

In practice, any portfolio with a distribution that is nearby the one for the optimal Markowitz portfolio will perform nearly as well. Consequently, most investors rebalance no more than a few times per year, and not every asset is involved each time. Transaction costs and tax issues are thereby limited. Similarly, borrowing costs can be kept to a minimum by not borrowing often. The theory can be modified to account for dividends.

Comments. *Many common criticisms of MPT are simply wrong.* These include the following claims:

- it assumes asset returns are normally distributed;
- it assumes markets are efficient;
- it assumes all investors are rational and risk-adverse;
- it assumes all investors have access to the same information.

Some of these arose because some avocates of MPT did not understand its full generality, and stated more restrictive assumptions in their work that were later attacked by critics. The first claims above are examples of this. We saw that MPT does not assume asset returns are normally distributed, and does not assume an efficient market hypothesis. Other such claims arose because some critics of MPT did not understand it. The last two claims above are examples of this. In fact, without investor diversity it is unlikely that the IID assumptions that underpin our models would be valid.

Many modern portfolio models are more complicated than the ones we have studied. Many of these use mathematical tools that one sees in some graduate courses on stochastic processes. *Stochastic Portfolio Theory* developed by Robert Fernholtz and others is a notable example.

Finally, the simple MPT models that we have studied do not consider any derivative tools that can be used to hedge a portfolio. These tools reduce your risk by paying someone to take it on when certain contingencies are met. In other words, they are insurance polices for risky assets. They thereby transfer the risk held by individual investors to the system as a whole — the so-called *securitization of risk*. Traditional derivatives are *put* and *call* options, but since the 1980s there has been an explosion in derivative products such as exotic options, swaps, futures, and forwards. As we saw in 2008 and 2011, without proper regulation these tools can create ties that critically weaken the entire financial system.