# Solving the Close Enough Traveling Salesman Problem

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# Outline

- Problem Definition
- Literature Review
- Example
- Description of Heuristic
- Results
- Future Directions

# Problem Definition

- Classical traveling salesman problem (TSP) visits every node in tour

- Close Enough TSP (CETSP) visits within distance *r* every node in tour
  - A disc of radius *r* implicitly surrounds every node
  - Tour must touch every disc

- Applications
  - Reconnaissance aircraft route planning
  - Ship tracking
  - Aerial forest fire detection
  - Robot monitoring of wireless sensor networks

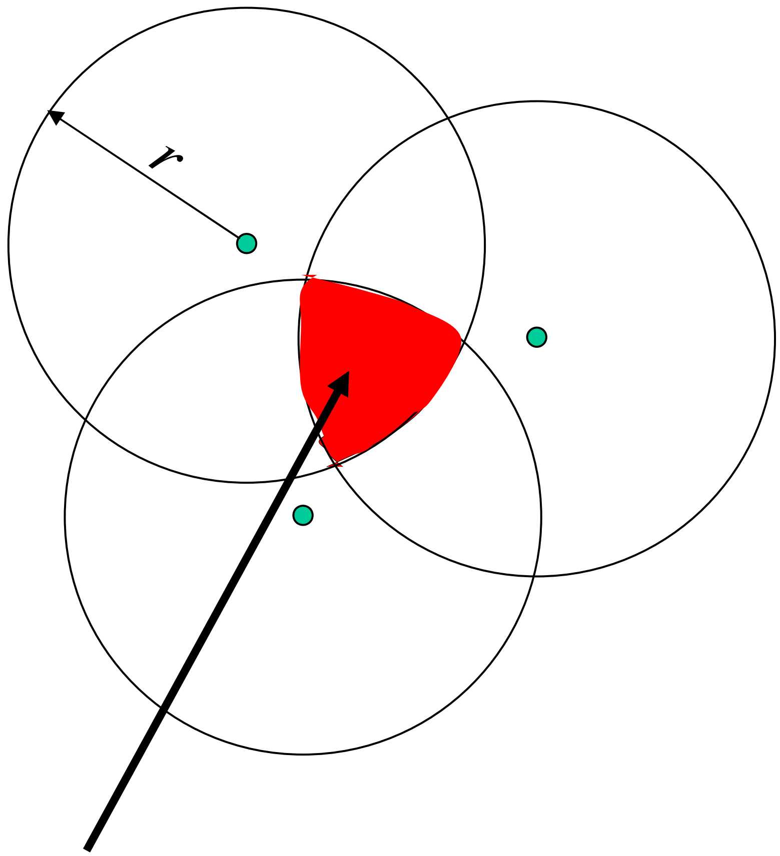
# Literature Review

- Close Enough Traveling Salesman Problem
  - Dong, Yang, Chen (2007)
  - Shuttleworth, Golden, Smith, and Wasil (2007)
  - Yuan, Orlowska, Sadiq (2007)
- Covering Tour Problem
  - Gendreau, Laporte, and Semet (1997)
  - Arkin and Hassin (1994)
- Generalized Traveling Salesman Problem
  - Fischetti, Gonzalez, and Toth (1997)
  - Silberholz and Golden (2007)
- TSP with Neighborhoods
  - Computational geometry literature seeking polynomial time approximation schemes
  - Mitchell (2007)
  - Dror, Efrat, Lubiw, and Mitchell (2003)

# Definitions

- Every node is surrounded by a disc of radius *r*
- *Steiner Zone (SZ) of degree k*

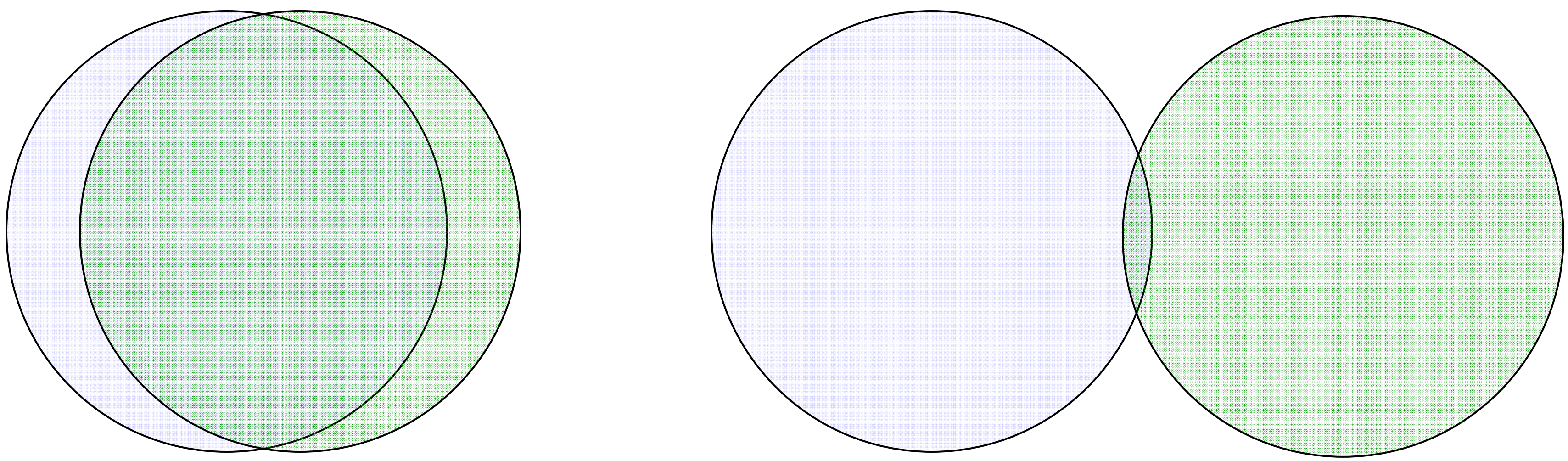
  -a region in which any point is simultaneously close enough to all k member nodes




SZ of degree 3

# Definitions

- Amount of overlap does not matter to us



- Both intersections are SZs of degree 2

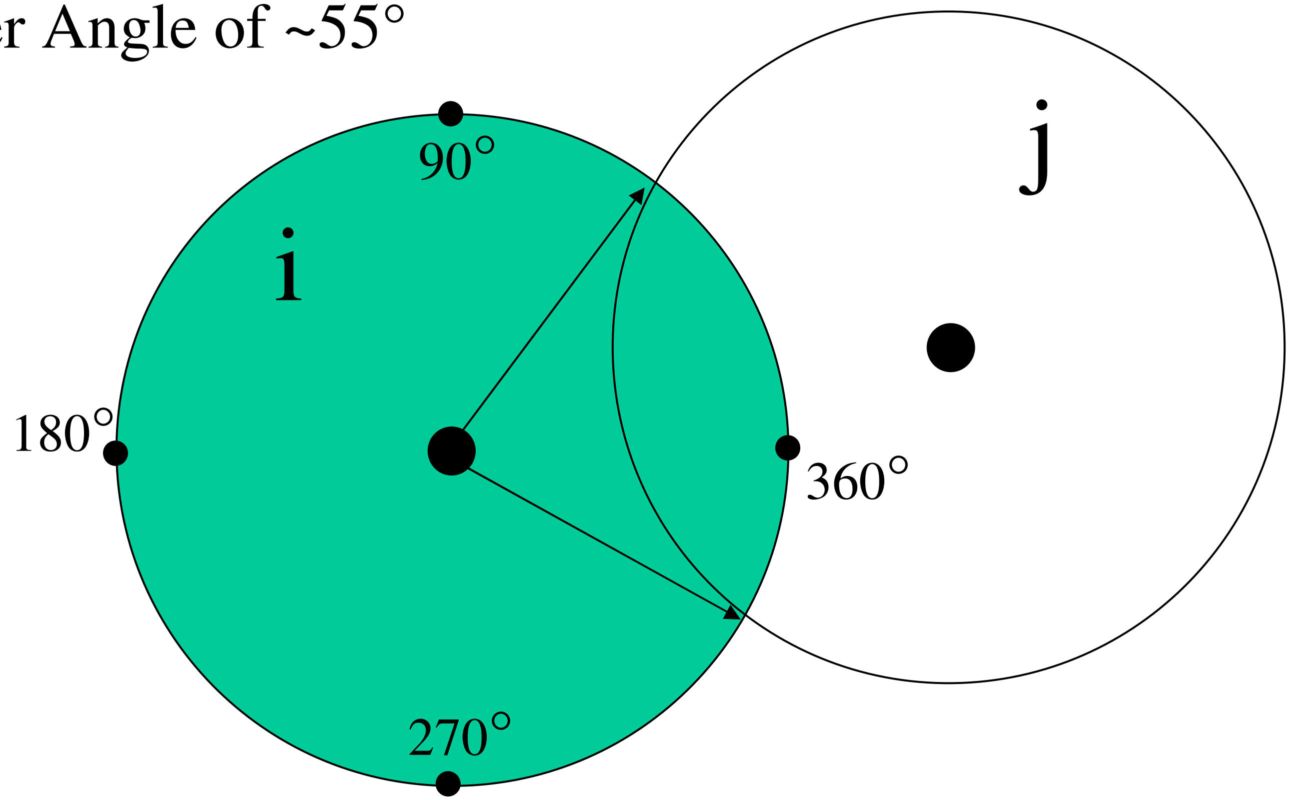
# Geometry

- The origin of a disc is its center point

- Given discs i and j, each of radius *r* and origins Or(i) and Or(j) respectively, it is straightforward to compute
  - The two points of intersection, if they exist
  - The angle of these intersections, with respect to Or(i)

# Geometry

- For example, Steiner Zone (ij) would be characterized by:
  - Origin of i
  - Lower Angle of ~330°
  - Upper Angle of ~55°

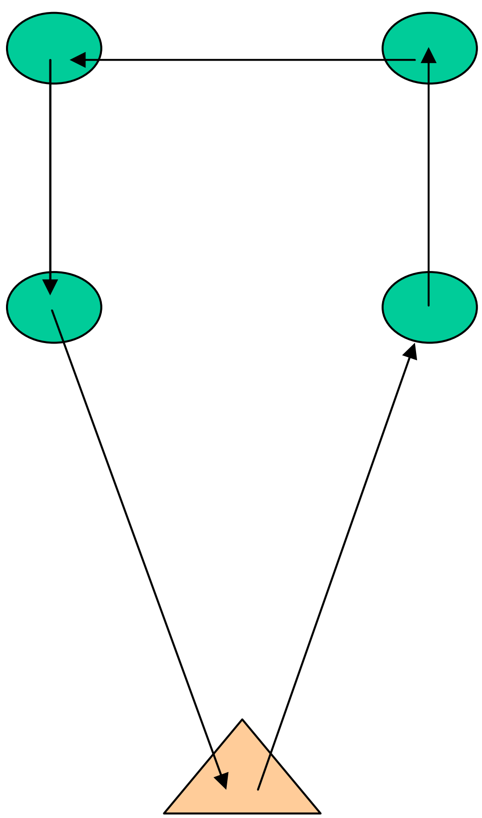


# Definition

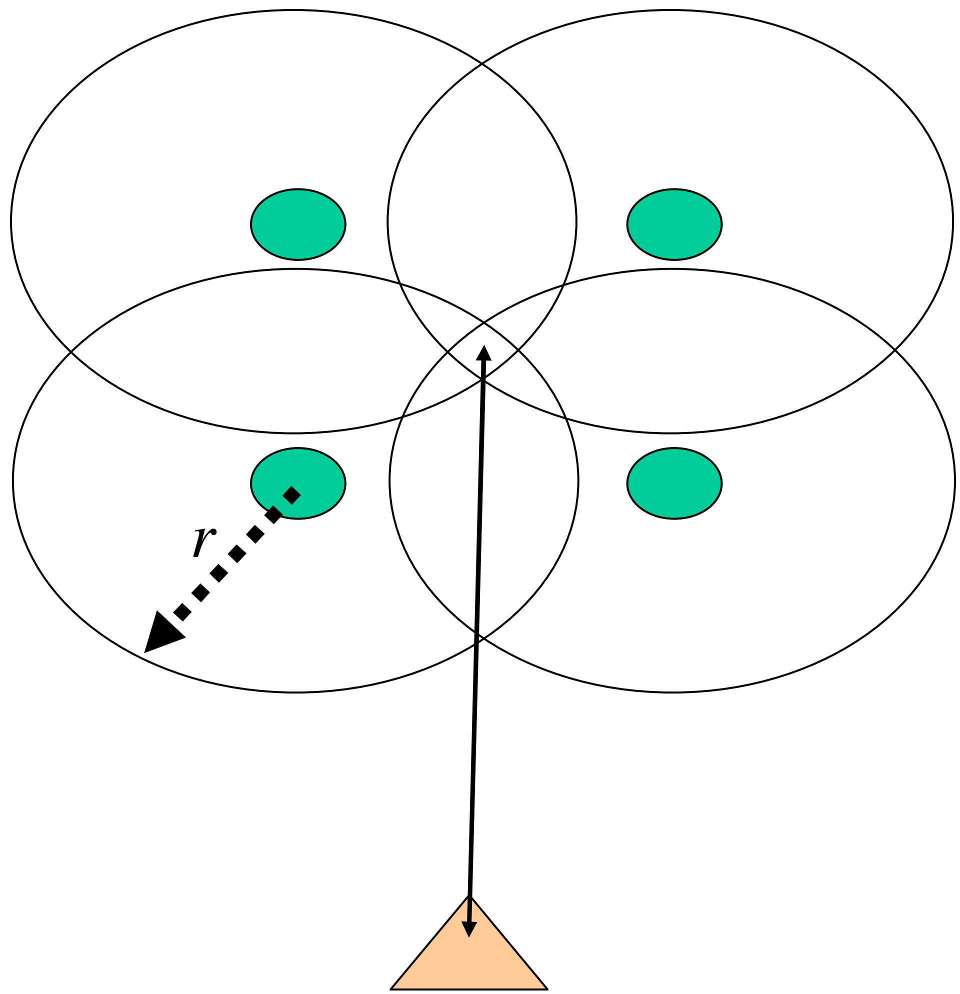
- As the radius grows, problem difficulty grows but we also expect shorter tours
  - More discs overlap → more complexity
  - More overlap → smaller number of points needed

- How then do we measure the potential for improvement?
  - Define the overlap for a problem as the ratio of *r* to the length of the smallest square surrounding all n discs

# Example



Classical TSP

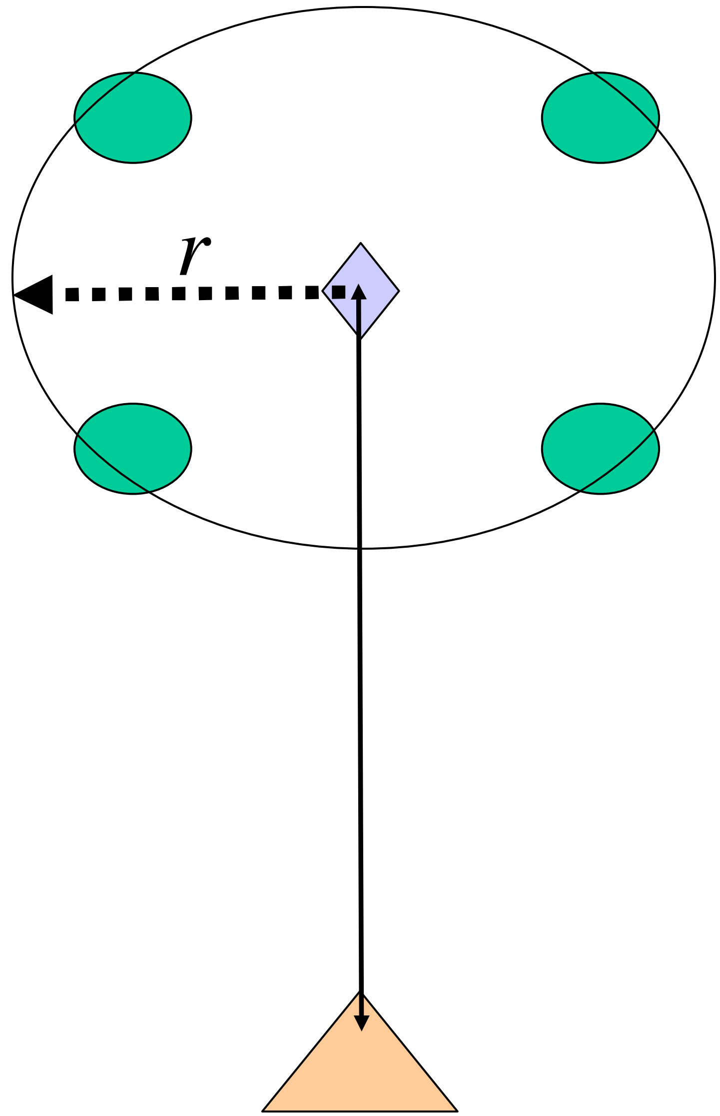



CETSP

# Example

*How is such a short tour feasible?*

-Every node is within *r* units of the visited location

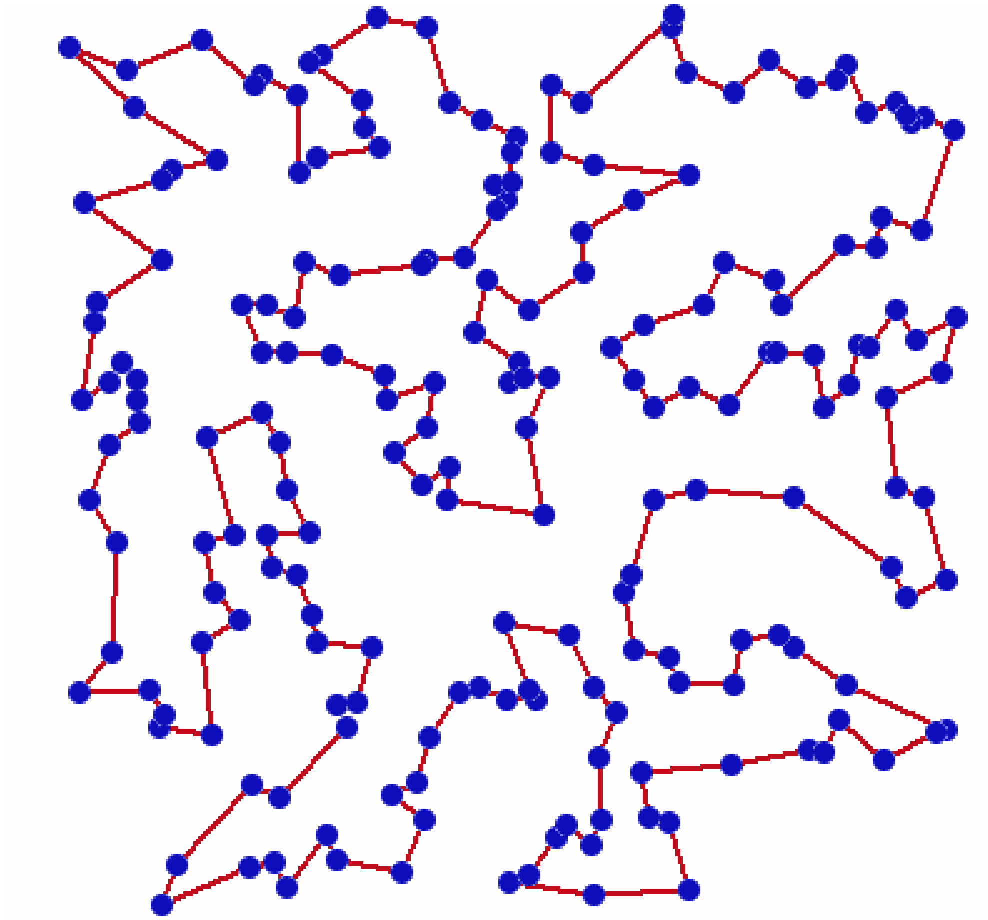


# The Steiner Zone Heuristic

- Three key steps
  - Graph reduction
  - Solve the underlying TSP
  - Optimize the TSP tour with respect to the Steiner Zones

# Before Step 1

**200-node problem**

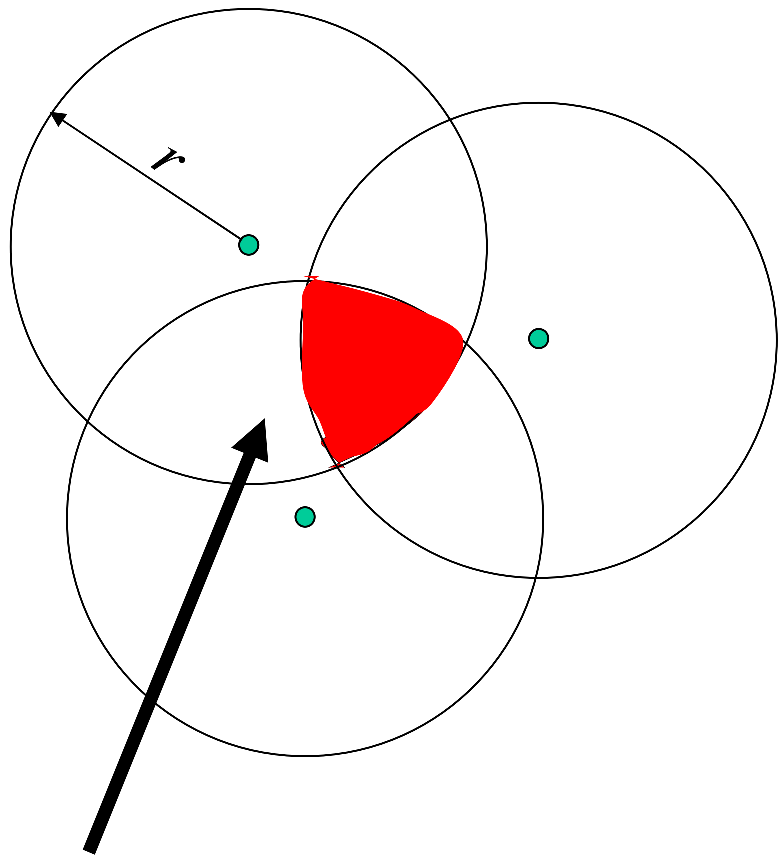


Optimal TSP solution: 1074.4

# 1. Graph Reduction

- Naïve reduction method
  - Step 1. Compute all SZs with respect to a chosen node in the graph
    - stored in descending order by degree
  - Step 2. Remove from the graph the member nodes of the highest degree SZ found
  - Step 3. If any nodes remain uncovered, go to Step 1
    - otherwise, we are done

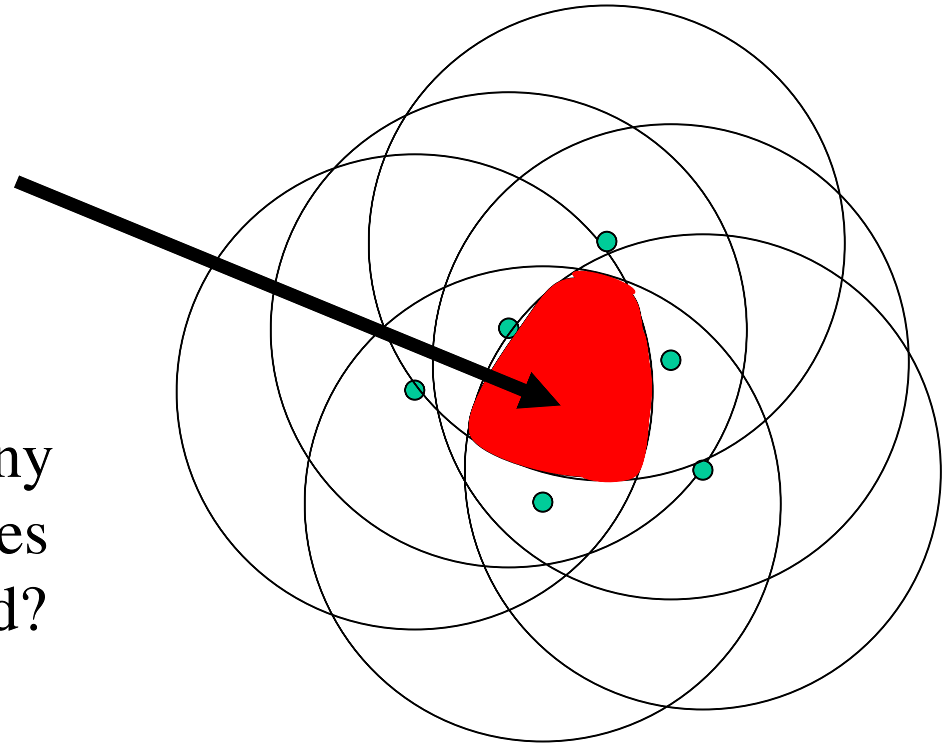
# Three Sub-Steiner Zones




Sub-Steiner
Zones for three
nodes

# Computing all Steiner Zones

Steiner Zone of
degree 6 – call it B



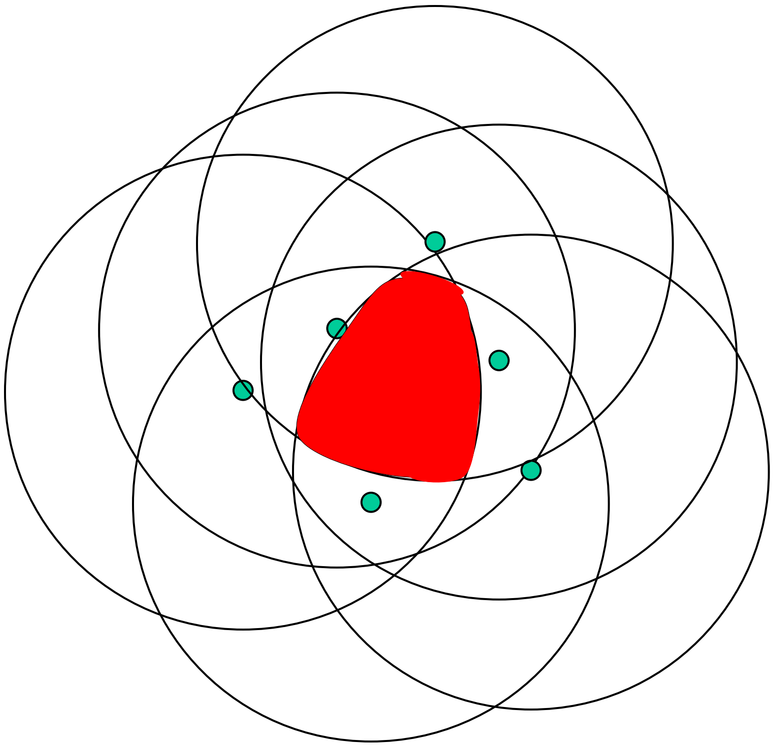
When B is created, how many potentially new Steiner Zones would the naïve method find?

| | |
|---|---|
| Degree 2: | 6 |
| Degree 3: | 10 |
| Degree 4: | 10 |
| Degree 5: | 4 |
| Degree 6: | 1 |
| Total: | 31 |

# Compute only the Necessary SZs

- Modify Step 1 of the naïve method
  - For each new SZ created, only add sub-Steiner Zones of degree 2 or 3

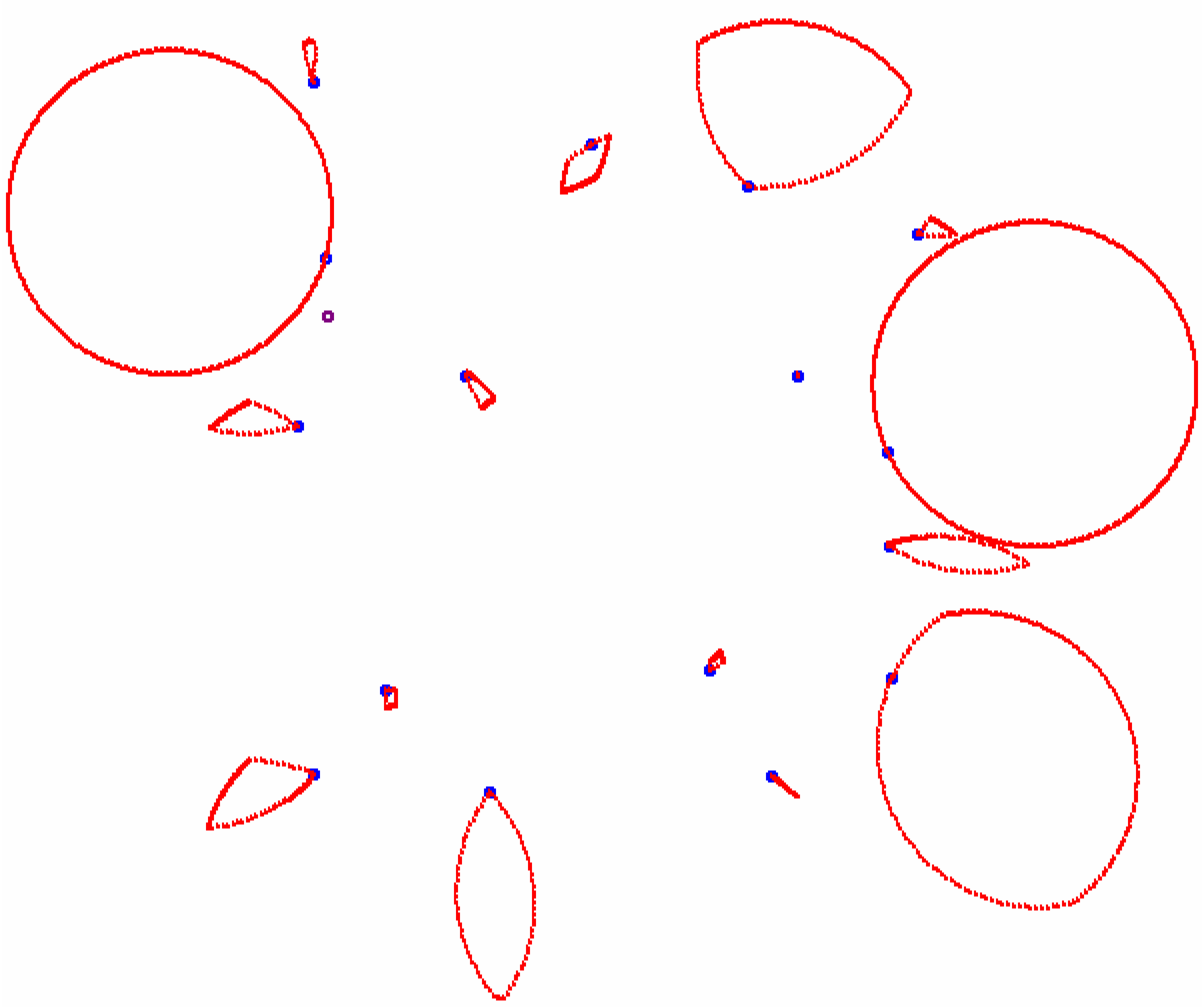
| | |
|---|---|
| Degree 2: | 6 |
| Degree 3: | 10 |
| Degree 6: | 1 |
| Total: | 17 |



The savings grows rapidly as the degree gets larger

# End of Step 1

**200-node problem reduced to 16 SZs**



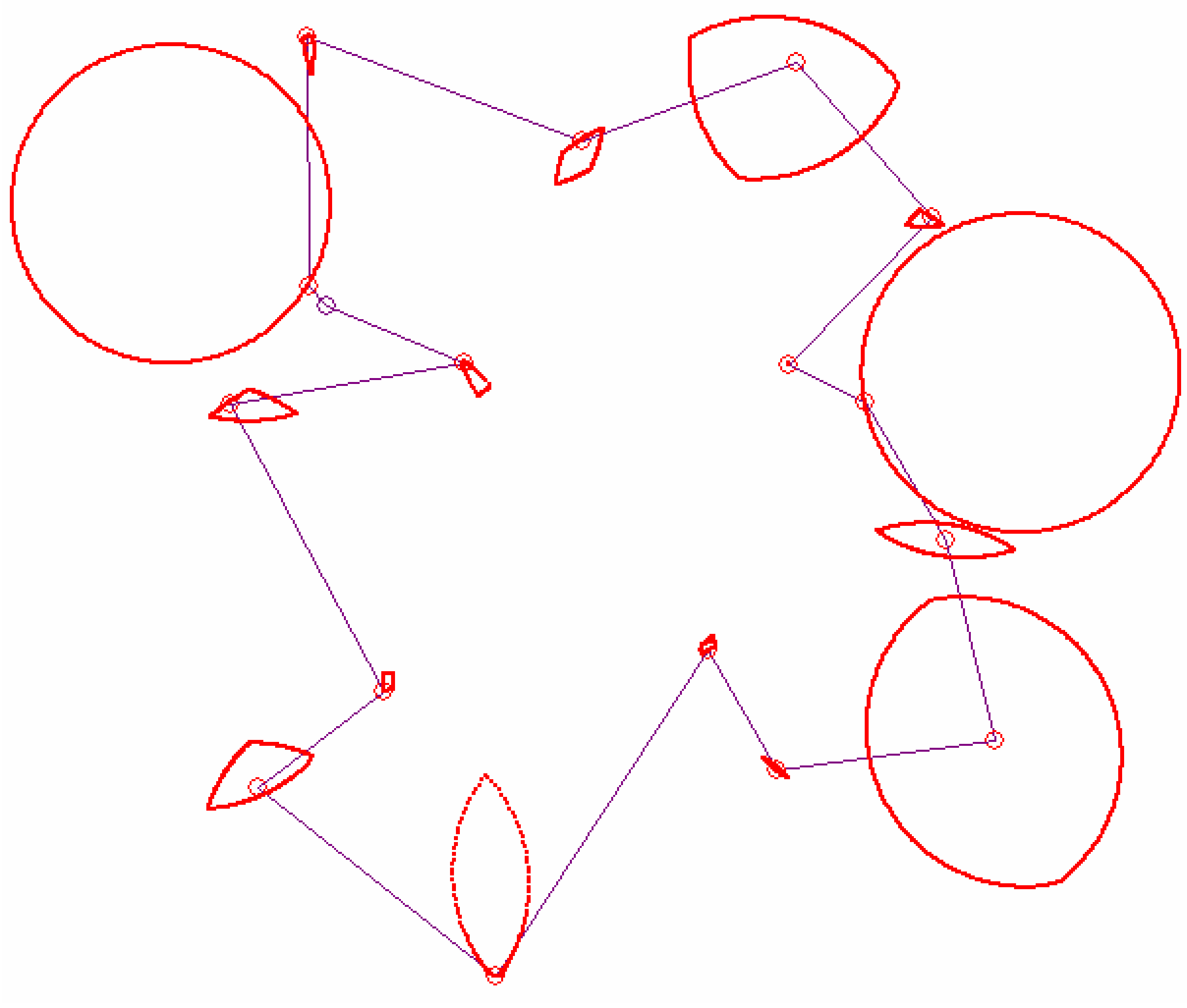
# 2. Solve the Underlying TSP

- Result of Step 1 is a set of disjoint convex regions

- How do we create a TSP on regions instead of nodes?
  - Choose a representative point for each region
    - Closest to depot, closest to centroid, randomly
  - Solve the TSP using those representative points
    - Use Concorde, Lin-Kernighan, …

# End of Step 2

**200-node problem with tour on representative points**



Distance: 404.8

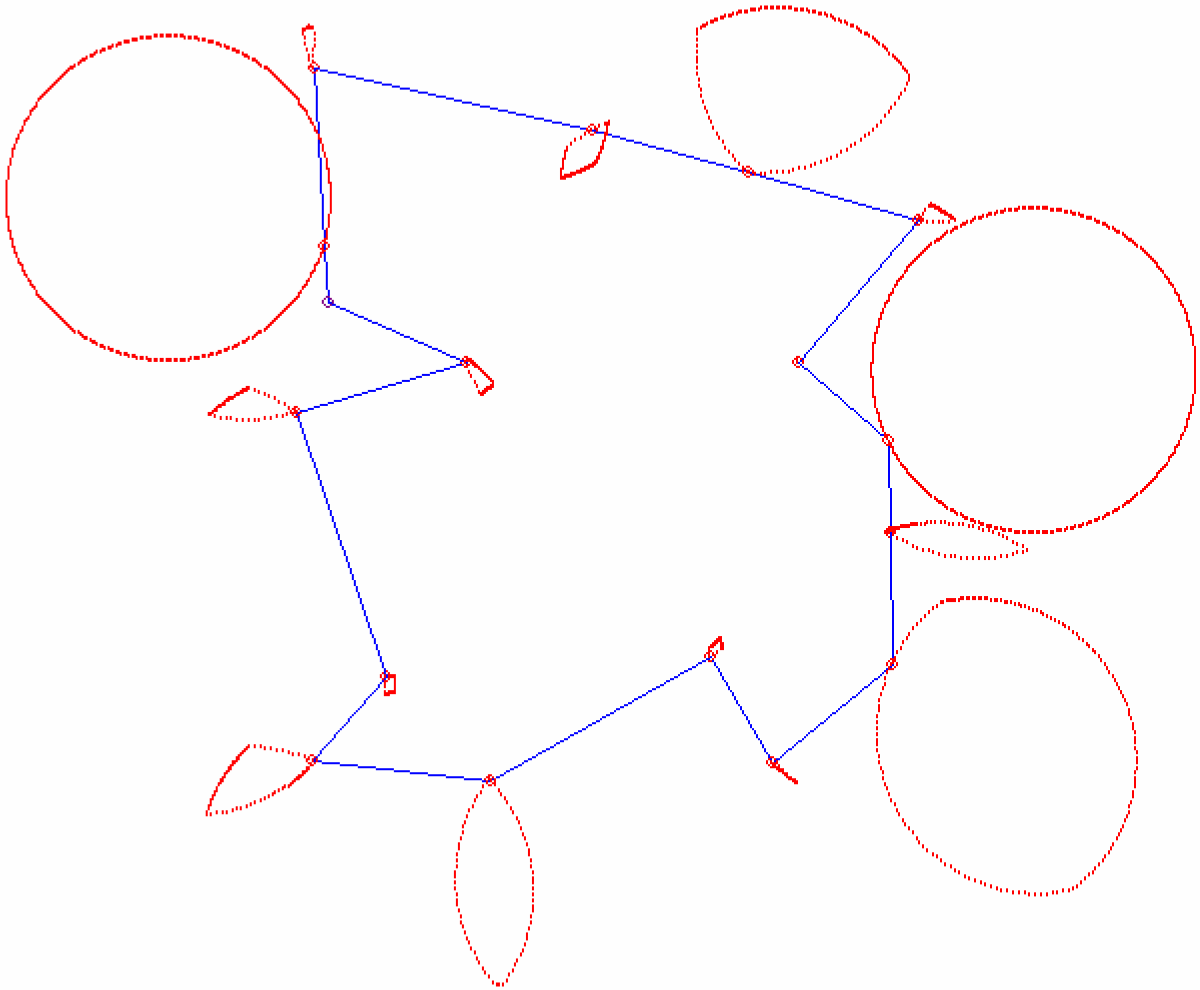
# 3. Optimize the TSP Tour

- The sequence of regions in the tour is fixed, but the location of each region's representative point is not

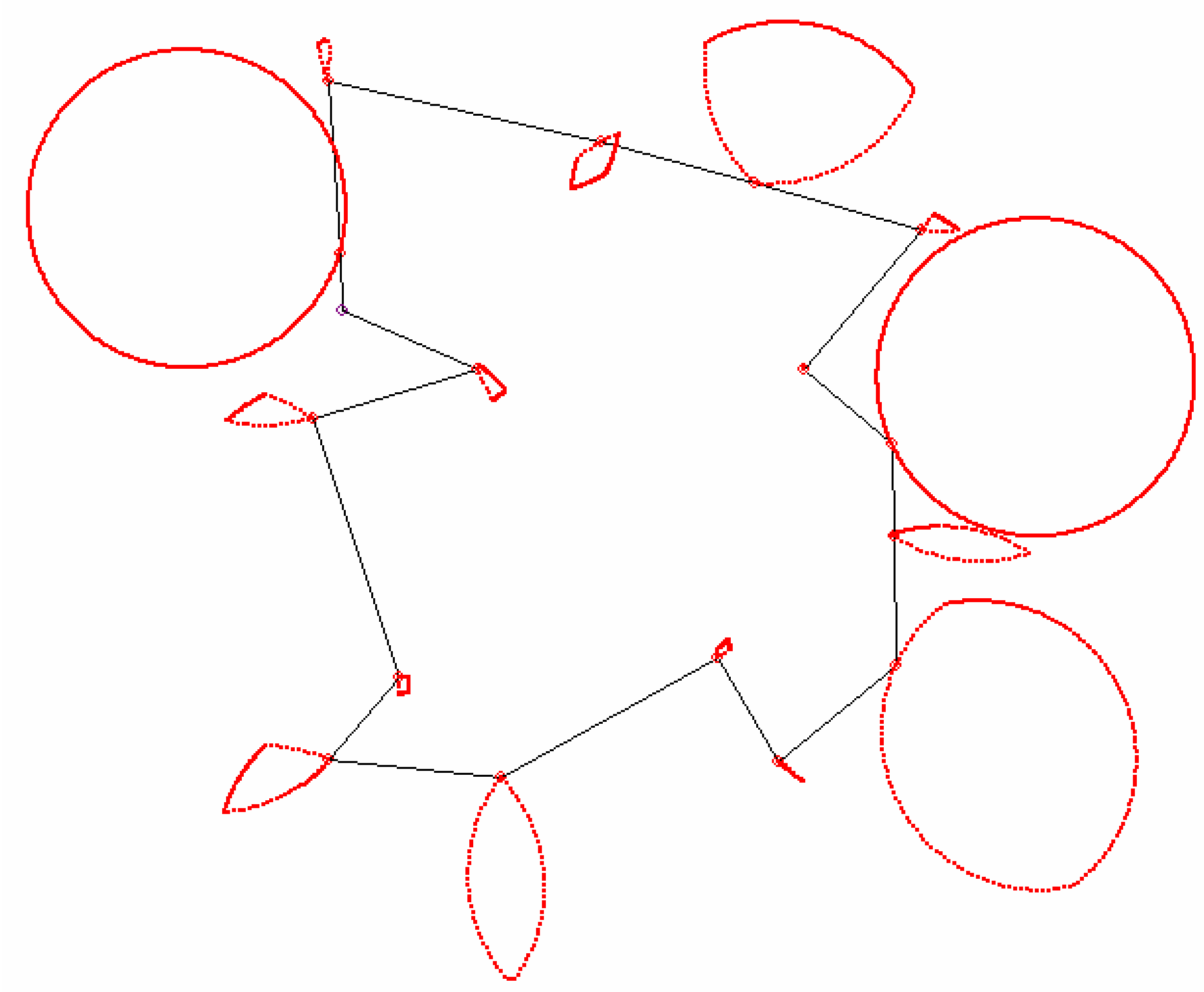
- Multiple ways to determine the locations
  1. Solve a Second Order Cone Program (SOCP)
     - Cplex can be used
  2. Approximate the SOCP – details in paper

# Iterative Approximation

**Iterative approximation of the TSP tour**



# Final Solution



Distance:  312.3

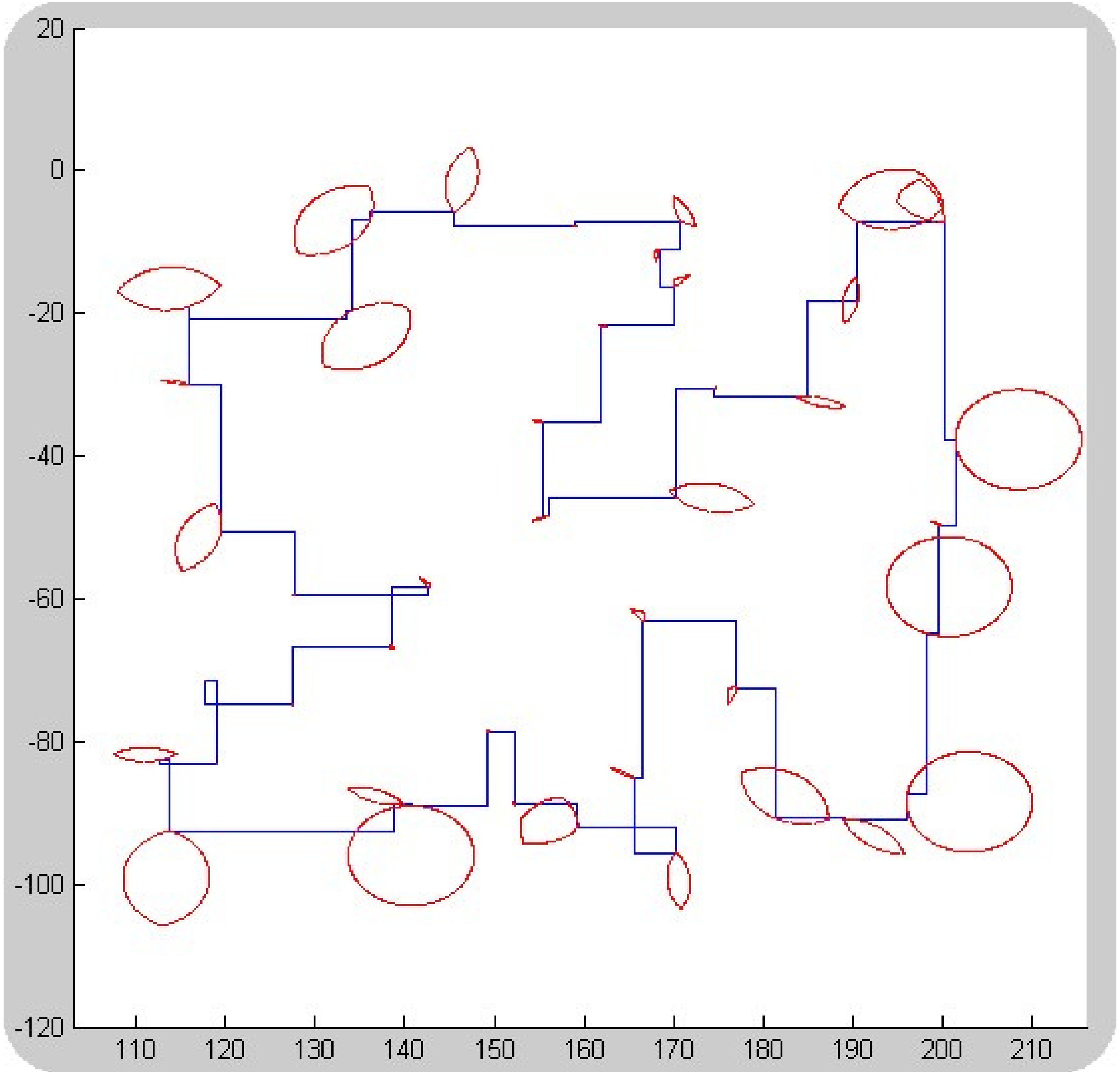
# Extensions

- Solve using Manhattan distance metric
  - Step I can use either L1 or L2 norm – results are essentially the same
  - Step II – Concorde can solve TSPs using L1 norm
  - Step III – both the heuristic and the QCP require only that edge distances are computed with L1 norm

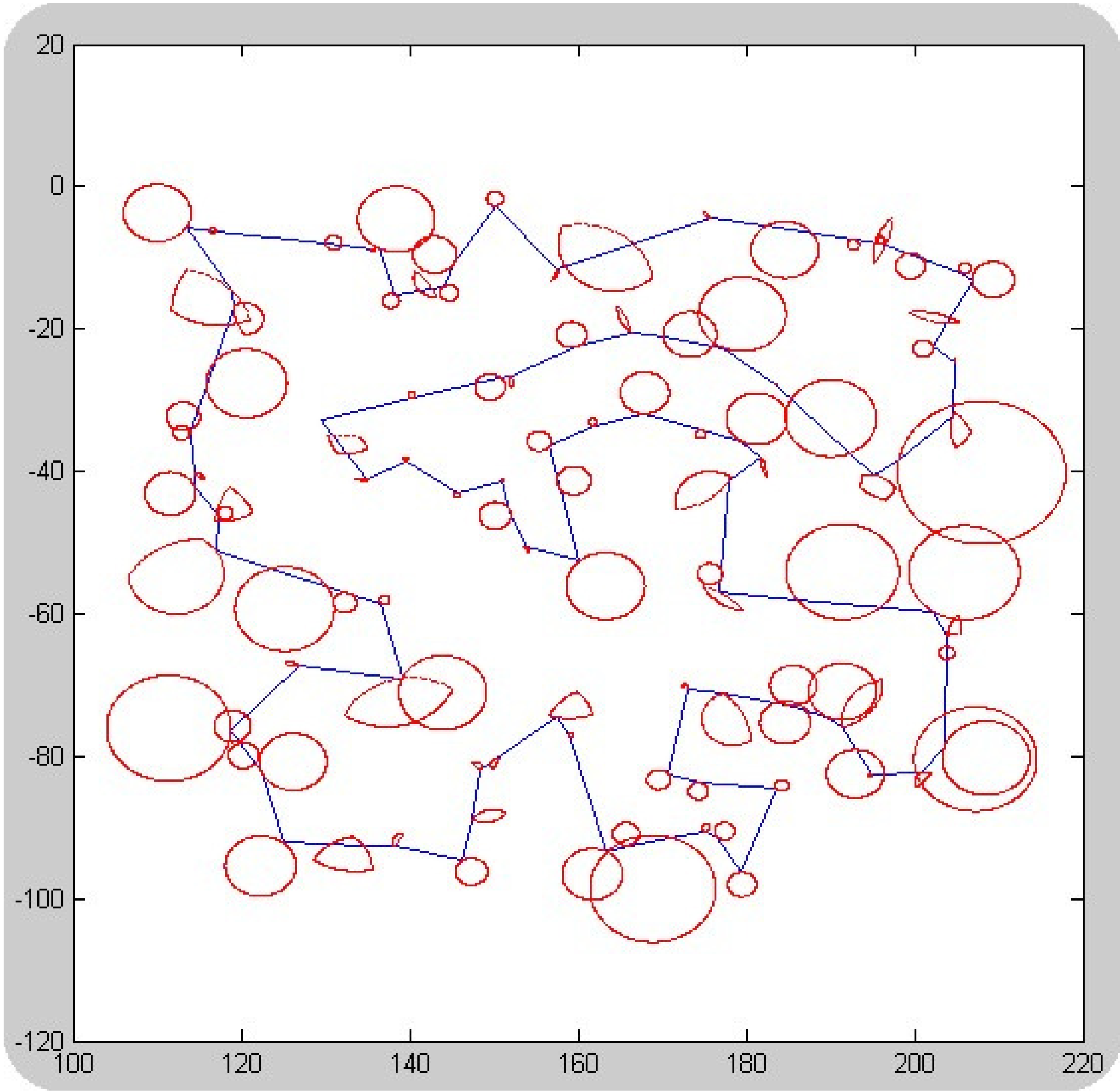
- Create test problems with an arbitrary radius for each disc
  - Radius generated uniformly from various ranges
  - Only change necessary is in Step I
    - Must handle the case when one disc lies completely inside another

- Solve in 3D
  - Two different Step I methods
  - Different ways to visualize the problem

# Manhattan Distance

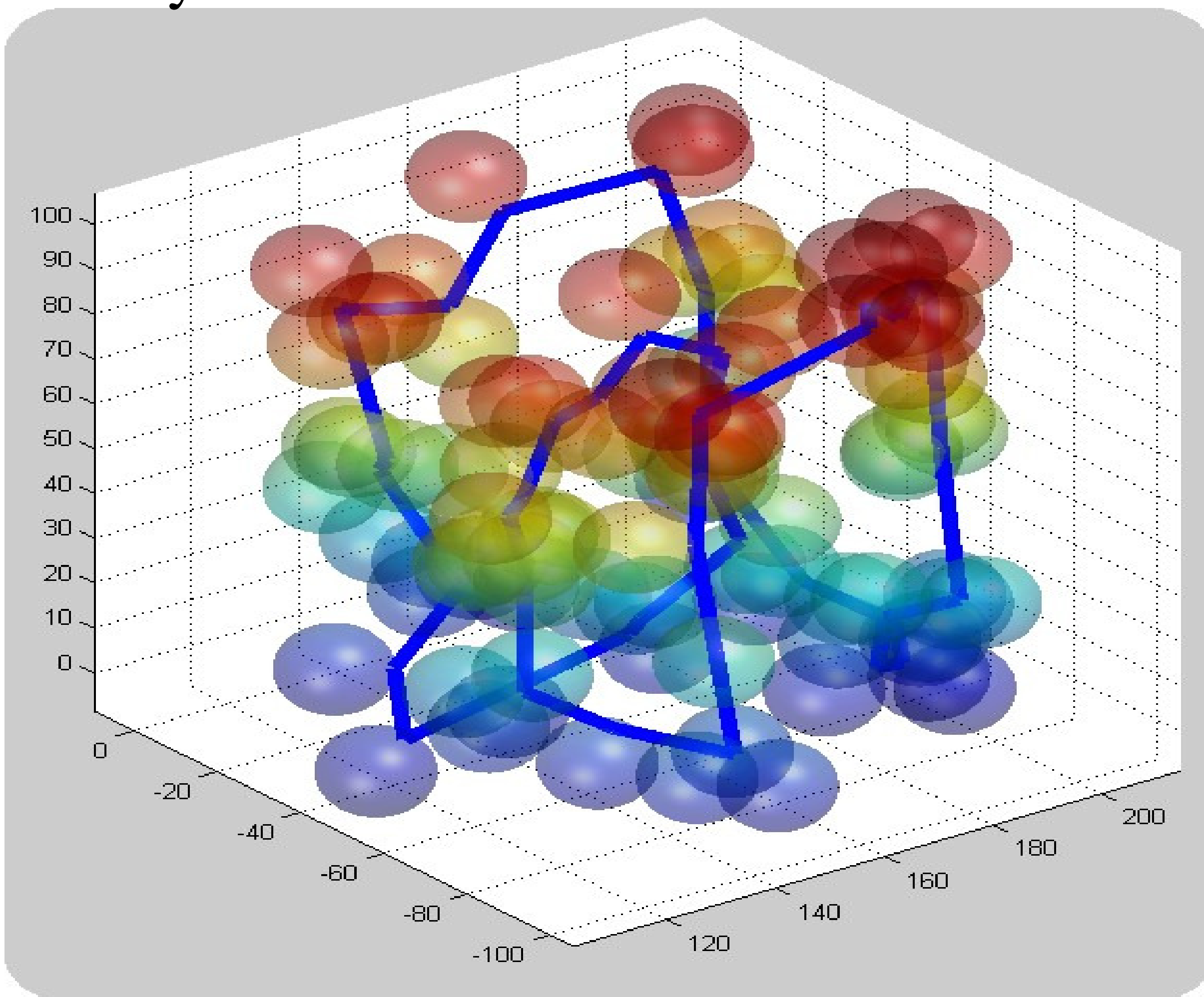


# Arbitrary Radius



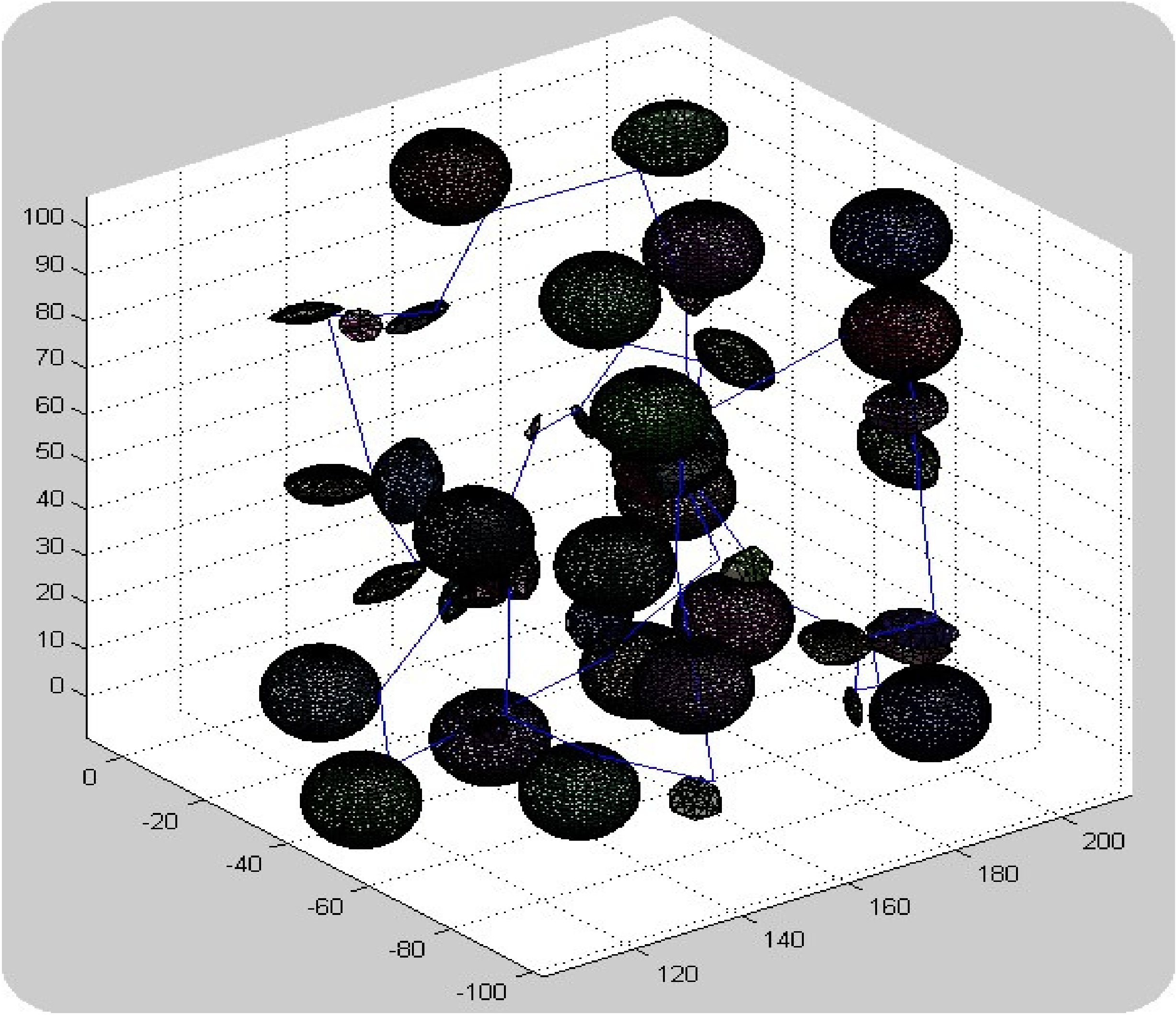
# 3D Visualizations

- Show the spheres surrounding all nodes
- Colors are only for differentiation



# 3D Visualizations

- Show polyhedral approximations of the convex hulls of  Steiner spheres – very slow



# Observations

- Greedy algorithm generates high-quality solutions

- Problems of 1000 nodes solved in less than 7 CPU seconds
  - Over half of the time spent solving the underlying TSP

- What benchmarks exist for computational comparison?
  - Genetic algorithm code of Silberholz and Golden (2007) produced excellent results on generic GTSP instances
  - No Step I → Lin-Kernighan + Step III
  - Greedy
    - Must know all Steiner Zones

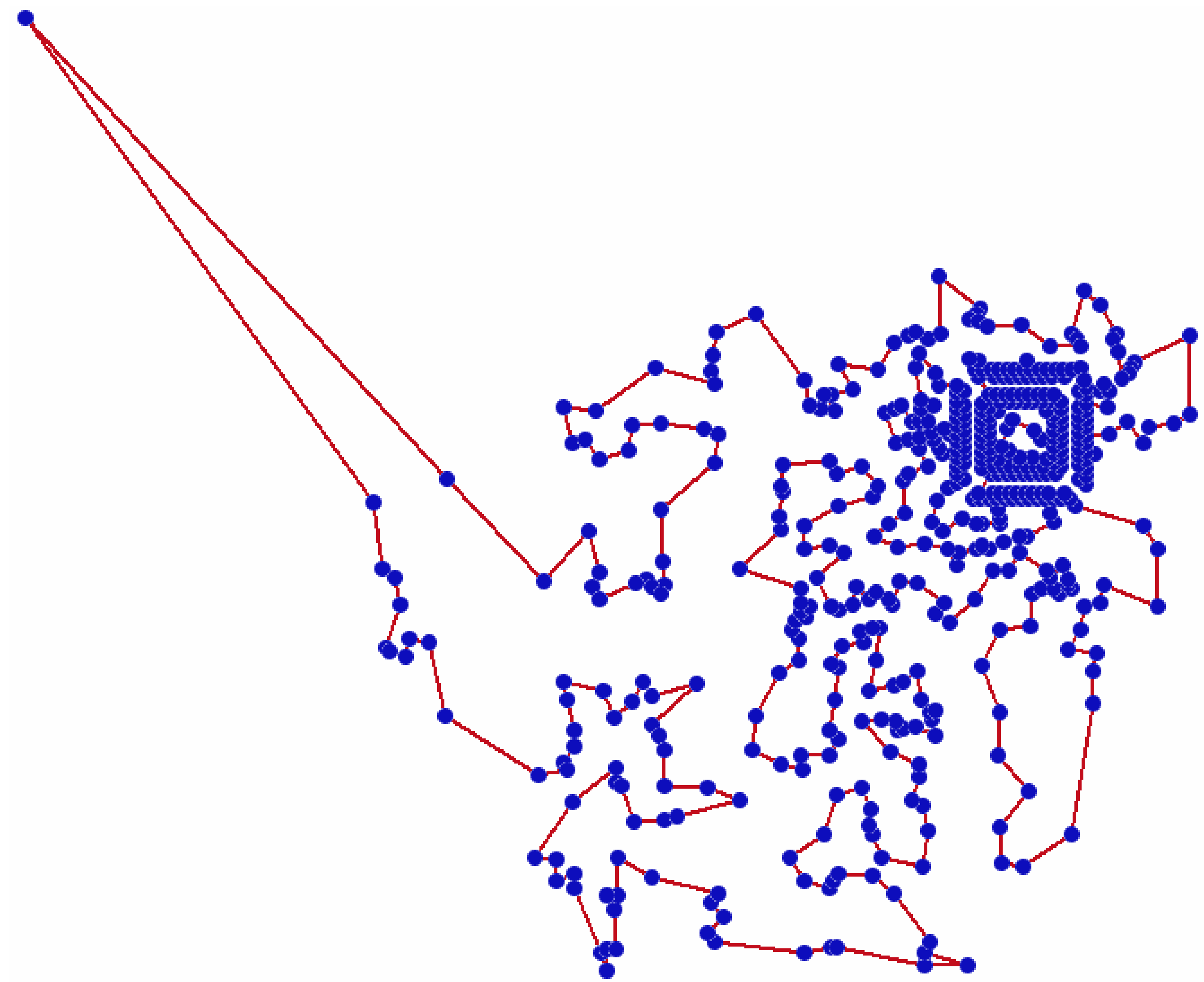
# Results

- Moderate and high-overlap problems
  - SZ heuristic outperforms GA in solution quality
    - Up to 150% better

- Low overlap problems
  - GA usually outperforms SZ by 5% - 7%
  - Steiner Zones were often half the highest degree found by our heuristic

- GA takes much more time and lots of memory
  - Must store a full or partial distance matrix
  - Few hundred nodes → days
  - 1000 nodes → weeks or months

# Results



d493.tsp from TSPLIB optimal solution: 350.19

# Results

| | | | | |
|---|---|---|---|---|
| | d493 | | | |
| | radius | 0.74904 | 3.7452 | 11.2356 |
| | overlap | 0.02 | 0.1 | 0.3 |
| GTSP-GA-24 | | 206.77 | 133.81 | 148.97 |
| continuous solution | | 204.49 | 112.55 | 82.83 |
| | CPU sec. | 309067 | 44862 | 59105 |
| SZ heuristic | | 215.46 | 106.24 | 71.16 |
| | CPU sec. | 4 | 1 | 1 |

- GTSP-GA-24 → each disc is approximated by 24 equally spaced points.
  - Solved using the genetic algorithm
  - Apply Step III to discrete GTSP solution to make it continuous

# Conclusions

- The Steiner Zone heuristic:
  - Produces consistently high-quality solutions quickly
  - Simple in concept
  - Readily adaptable to problem variants
- We have prepared the first benchmark instances for this problem
- Cannot conclusively judge any CETSP heuristic until optimal solutions are known

# Future Research Directions

- Generate SZ solutions that are better than the GA solutions for low-overlap problems without sacrificing speed
  - Local search using a pool of SZs
  - Use a GA to pick the best set of Steiner Zones
- Devise good lower bounding procedures