Multi-Period Vehicle Routing: Some New Applications

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Introduction

- In the literature, the single-period vehicle routing problem still receives most of the attention
- But, multi-period vehicle routing is also important
  - Decisions span multiple time periods
  - Decisions made for one period impact outcomes in other periods
- Our goal here is to introduce several well-known problems in multi-period vehicle routing and then focus on one of these

Multi-Period Vehicle Routing Problems

- Traditional period vehicle routing problem (PVRP)
- PVRP variants
- Inventory routing problem (IRP)
- IRP variants
- Other problems

Traditional Period Vehicle Routing Problem

- We work with a planning horizon of T days
- Each customer has frequency of visit requirements (e.g., k out of T days)
- Visits to customers must occur on allowed k-day combinations
  - If T = 5 for a 5-day week, possible 2-day combinations are M & W and T & Th
- Decision variables
  - Assign visitation schedule to each customer
  - Solve a vehicle routing problem for each day

Evolution of the PVRP as of 2007

Problem identification
Beltrami and Bodin (1974)

↓

Formal definitions / heuristic solution methods
Russell and Igo (1979)
Christofides and Beasley (1984)

↓

Classical heuristics
Tan and Beasley (1984)
Russell and Gribbin (1991)
Gaudiosio and Paletta (1992)

Metaheuristics
Chao et al. (1995)
Cordeau et al. (1997)
Drummond et al. (2001)

Mathematical programming based methods
Francis et al. (2006)
Mourgaya and Vanderbeck (2006)

Above figure borrowed from Francis, Smilowitz, and Tzur (2008)

PVRP Variants

- Multiple depots
- Intermediate facilities for capacity replenishment
- Time windows
- Service choice (i.e., service frequency becomes a variable)
- Multiple routes (i.e., a vehicle can do more than one route per day)

The Inventory Routing Problem

- The IRP involves the repeated distribution of a single product from a single facility to n customers over T days
- Customers consume the product on a daily basis and maintain a small, local inventory
- The objective is to minimize the sum of transportation and inventory-related costs (stockouts can be costly)
- Distribution is often vendor managed in the petrochemical, industrial gas, and a growing number of other industries
- This is a very rich multi-period problem

The Inventory Routing Problem

- Decisions to be made by vendor
  - When to serve a customer (proximity vs. urgency)?
  - How much to deliver?
  - What are the delivery routes?
- Usage rates can be modeled in many ways
  - Deterministic usage
  - Stochastic usage
- IRP variants involve time windows and intermediate facilities for temporary storage of product

Other Multi-Period Problems

- Master-route design for small-package delivery

  Trans. Science (2009)
- Template-route design for small-package delivery

  MSOM (2009)
- The balanced billing cycle VRP for utility companies

  Networks (2009)

VRP and PVRP

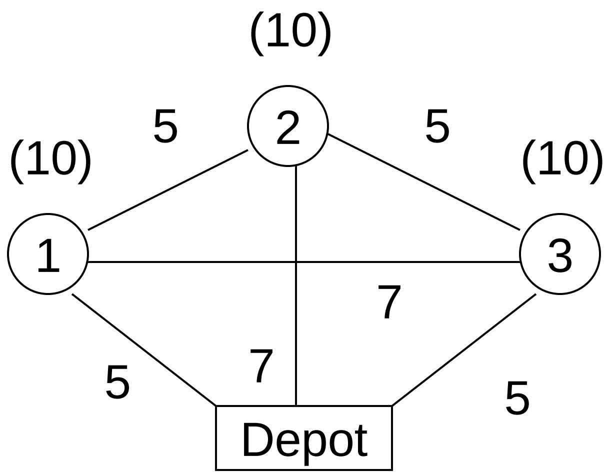
- In contrast to the classical VRP, the PVRP is a multi-period, multi-level vehicle routing problem
- In the period vehicle routing problem (PVRP), customers might require service several times during a time period

PVRP Description

- We must first assign customers to patterns (certain days of the time period) and then find routes on each day servicing the customers scheduled on that day
- We seek to minimize total distance traveled throughout the time period
- For example, a waste management company has to assign customers to certain days of the week and then create daily routes
- Some customers might only need service once a week, some might need service multiple times

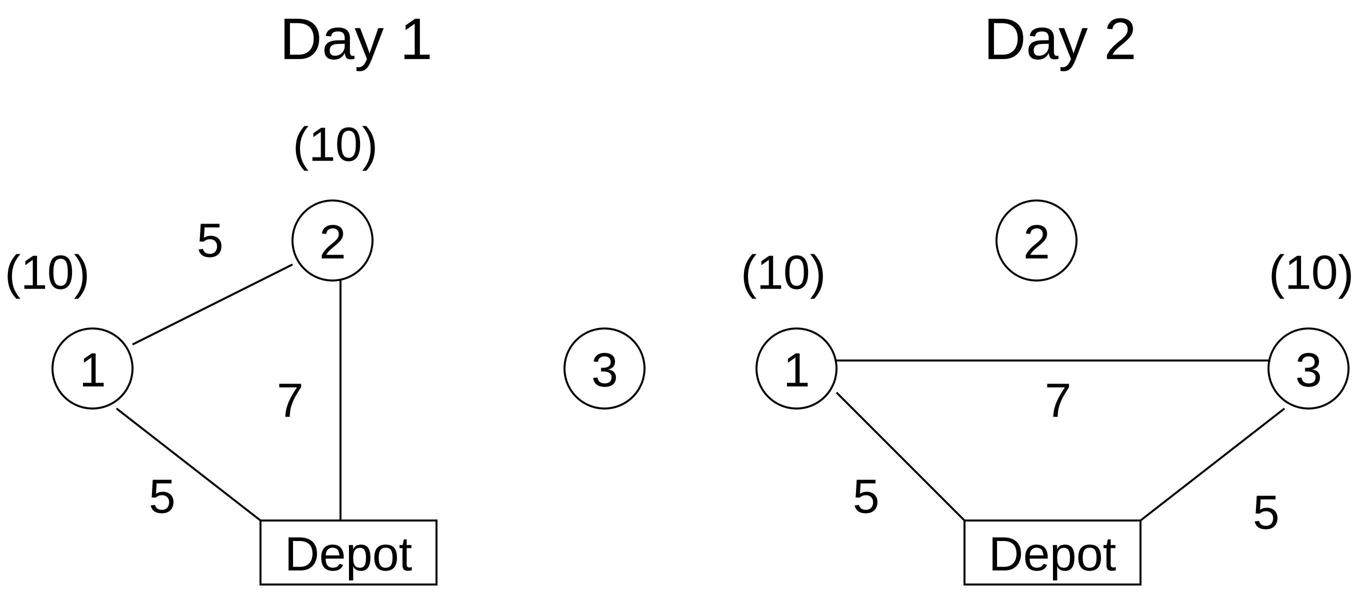
PVRP Example

- The time period is two days, $T = \{1, 2\}$
- Customer 1 must be serviced twice
- Customers 2 and 3 must be serviced once
- Node labels in parentheses are demands per delivery
- Edge labels are distances
- Vehicle capacity is 30



PVRP Example

- Customer 1 is assigned to day 1 and day 2
- Customer 2 is assigned to day 1
- Customer 3 is assigned to day 2



Total distance traveled is 34 units

PVRP Applications

- Commercial sanitation
- Grocery and soft drink distribution
- Fuel oil and industrial gas delivery
- Internal transport installation and maintenance
- Utility services
- Automobile parts distribution
- Oil collection from onshore wells

PVRP Literature

- PVRP literature dates back to the 1970s

- Recent papers on solving the PVRP
  - Cordeau, Gendreau, and Laporte (1997)
  - Alegre, Laguna, and Pacheco (2007)
  - Hemmelmayr, Doerner, and Hartl (2009)
  - Gulczynski, Golden, Wasil (2011)

IP Based PVRP Heuristic

- We develop an IP-based Heuristic algorithm for solving the PVRP, denoted IPH
- We easily adapt IPH to solve two variants later on
- We present computational results which demonstrate the effectiveness of our algorithm

IPH: Overview

1. Generate an initial PVRP solution *S*
2. Improve *S* by solving an IP that reassigns and reroutes customers in a way that maximizes total savings (main step)
3. Improve daily routes using a VRP heuristic
4. Remove and reinsert customers using an IP
5. Repeat steps 2-4 until a stopping condition is reached

IPH: Initial Solution

- We first assign customers to patterns in a way that balances the amount of demand serviced on each day (standard assignment IP)
- Next we find a VRP solution on each day using a quick heuristic (e.g., CW savings)
- The result is an initial PVRP solution

IPH: Improvement IP

- Given a solution *S*, we formulate an improvement IP (IMP) that maximizes the savings from reassigning customers to new service patterns, removing customers from their current routes, and inserting them into new routes
- We solve IMP repeatedly until no more improvement is achieved

IPH: IMP Formulation

Objective Function

- Maximize savings from reassignments and rerouting

Constraints

- We remove a customer from its current route if and only if we reinsert it elsewhere
- We move a customer to a new day if and only if we assign it to a pattern containing that day

IPH: IMP Formulation

- The total demand of the customers we move to a route minus the total demand of the customers we move from the route cannot exceed the residual capacity of the route
- If a customer *i* or its predecessor is removed from a route, we do not move any other customers immediately prior to *i* and we remove at most one of *i* and its predecessor (this ensures the objective function gives the savings accurately)
- We assign a customer to at most one feasible pattern

IPH: IMP Example

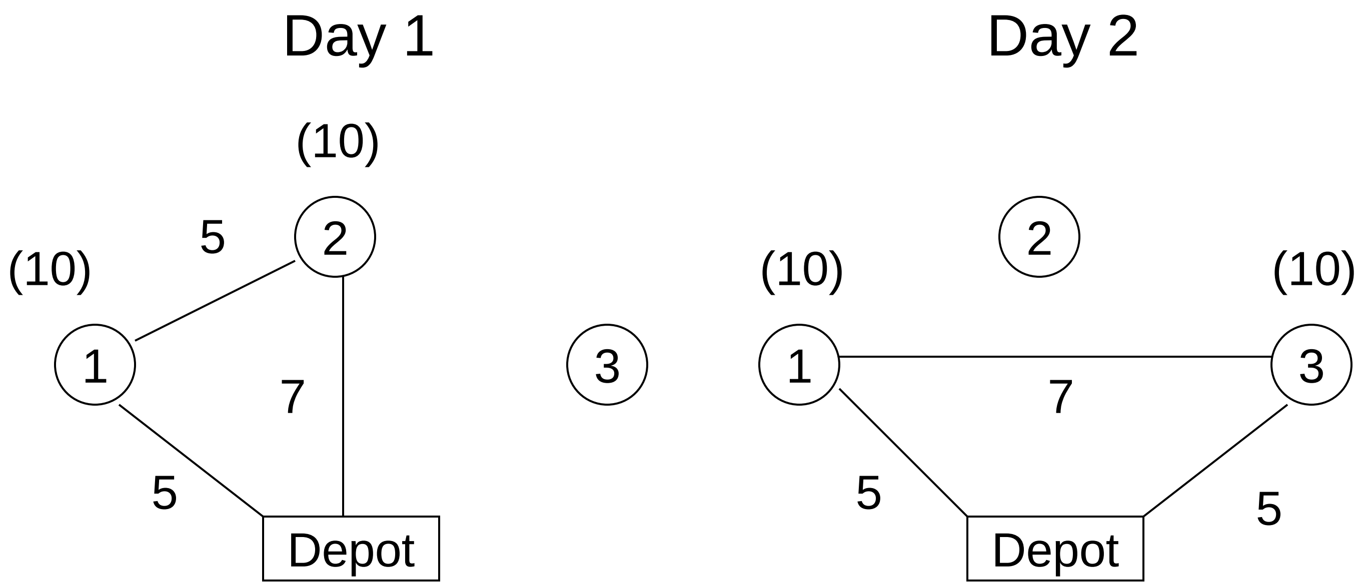
- Initial Solution

Customer 1 is assigned to day 1 and day 2

Customer 2 is assigned to day 1

Customer 3 is assigned to day 2

Vehicle Capacity is 30 units

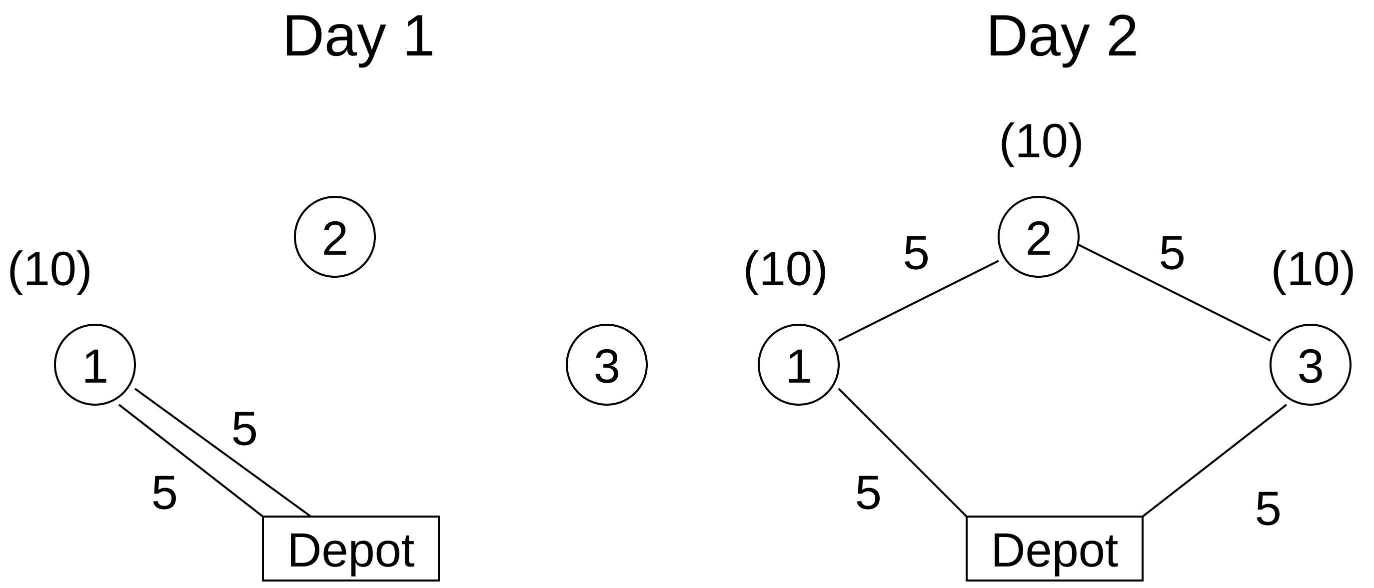


Total distance traveled is 34 units

IPH: IMP Example

- Improved Solution

  Customer 2 is removed from its route on day 1, reassigned to day 2 and inserted immediately prior to customer 3, for a savings of 4 units



Total distance traveled is 30 units

IPH: ERTR

- We improve daily routes by solving VRPs using the enhanced record-to-record travel algorithm (ERTR)
- ERTR was developed by Groer, Golden, and Wasil (2008)
- One-point move, two-point exchange, two-opt move
- Three-point move, OR-opt move

IPH: Re-initialization

- We remove some customers randomly from their routes
- We create a fictitious day *F* and assign all removed customers to routes on day *F*
- We solve IMP adding a constraint forcing all customers visited on *F* to be reassigned to a feasible pattern
- Re-initialization allows us to further explore the solution space from a new starting solution

IPH: Best Solution

- We iterate IMP, ERTR, and the re-initialization procedure until a stopping condition is reached

- The best solution found throughout this process is returned at the end

IPH Results

- On 32 problems from the PVRP literature, IPH had an average deviation from the best-known solution of 0.90%
- For Cordeau, Gendreau, and Laporte (1997) it was 1.58%
- For Alegre, Laguna, and Pacheco (2007) it was 1.37%
- For Hemmelmayr, Doerner, and Hartl (2009) it was 1.39%

IPH Run-times

- On the 32 problems IPH had an average run-time of 623 seconds
- For Cordeau, Gendreau, and Laporte it was 139 (comparable machine)
- For Alegre, Laguna, and Pacheco it was 1449 (slower machine)
- For Hemmelmayr, Doerner, and Hartl it was 149 (comparable machine)
- Problem size ranges from 50 to 417 customers

IPH Virtues

- Comparable performance to the best algorithms on PVRPs
- Easily modified to handle real-world variants
- In the real world, we rarely start from scratch

PVRP Variants

- In practice, companies have pre-existing solutions that over time, due to the addition and deletion of customers and other small modifications, become inefficient
- Some companies have fleets of thousands of vehicles that service millions of customers annually
- Not practical economically, logistically, or perhaps contractually to reroute from scratch
- Instead, the major inefficiencies in the pre-existing routes should be eliminated in a way that does not cause widespread disruption

PVRP Variants

- From a pre-existing solution *S'* we find an improved solution *S** while constraining the total amount of disruption

- We consider three types of constraints

  1. Hard constraint: we set a limit *W* and only accept solutions *S** such that the number of customers assigned to different patterns from *S'* to *S** is at most *W*
  2. Soft constraint: in the objective function we penalize *S** for each customer assigned to a different pattern than the one in *S'*
  3. Restricted reassignment constraints: we fix all multi-day customers to their patterns in *S'* but allow one-day customers to be freely reassigned in *S**

- Collectively, we call these variants the PVRP with reassignment constraints (PVRP-RC)

PVRP-BC

- Because route balance is important in industry we also consider the period vehicle routing problem with balance constraints (PVRP-BC)
- In the PVRP-BC we start with a pre-existing solution *S'* that is well-balanced or has low cost routes (but not both)
- In the objective function, we penalize a solution for having imbalanced routes
- Starting from *S',* we wish to find a solution *S** that minimizes routing distance plus an imbalance penalty

PVRP-RCH and IPH-RCH

- We modify IPH to solve the PVRP-RC with a hard constraint (PVRP-RCH)
- In the PVRP-RCH, we allow no more than *W* customer reassignments from the pre-existing initial solution *S'*
- In practice, it helps to do this in stages
- We denote the modified IPH algorithm by IPH-RCH

IPH-RCH Results

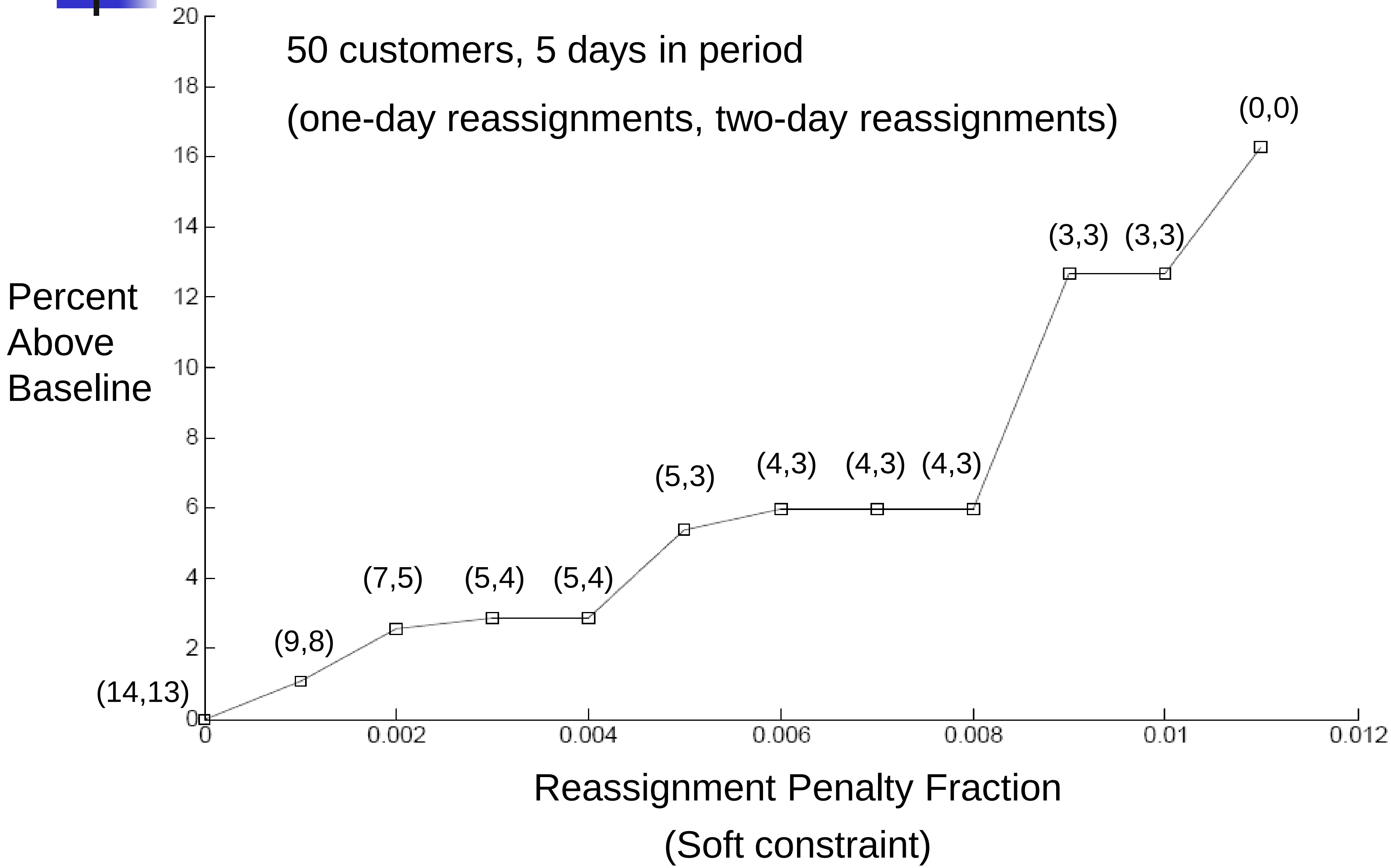
- For comparison, we implement a greedy algorithm that randomly selects a reassignment from the current three best-savings reassignments and repeats until *W* reassignments are made or there is no improvement
- On 26 problems, with *W* = 10% of the number of customers, we run IPH-RCH once and the greedy algorithm 151 times, recording the best solution and the median solution, respectively
- We started with an initial solution *S'* that was on average 16.64% above the baseline (solution in which no restriction was put on the number of reassignments)
- For IPH-RCH the solution was 9.91% above, for Greedy Best it was 11.51% above, and for Greedy Median it was 12.62% above

Tradeoff Between Distance and Disruption

- We modify IPH for the PVRP with soft reassignment constraints (IPH-RCS)

- For an example problem, we run IPH-RCS for different reassignment penalties and see how the solution is impacted

IPH-RCS Results



Reassigning one-day customers

- One-day customers are the easiest and most convenient to reassign
- We consider the PVRP in which we fix all multi-day customers to their initial patterns, but allow one-day customers to be reassigned freely
- In our computational experiments, the initial solution can be substantially improved using this practical compromise

PVRP-BC

- Route balance is of key importance in industry
- We consider the problem of improving a maximally balanced initial solution or a low cost initial solution without inducing too much imbalance
- Minimize: routing distance + $\rho C(U - L)$

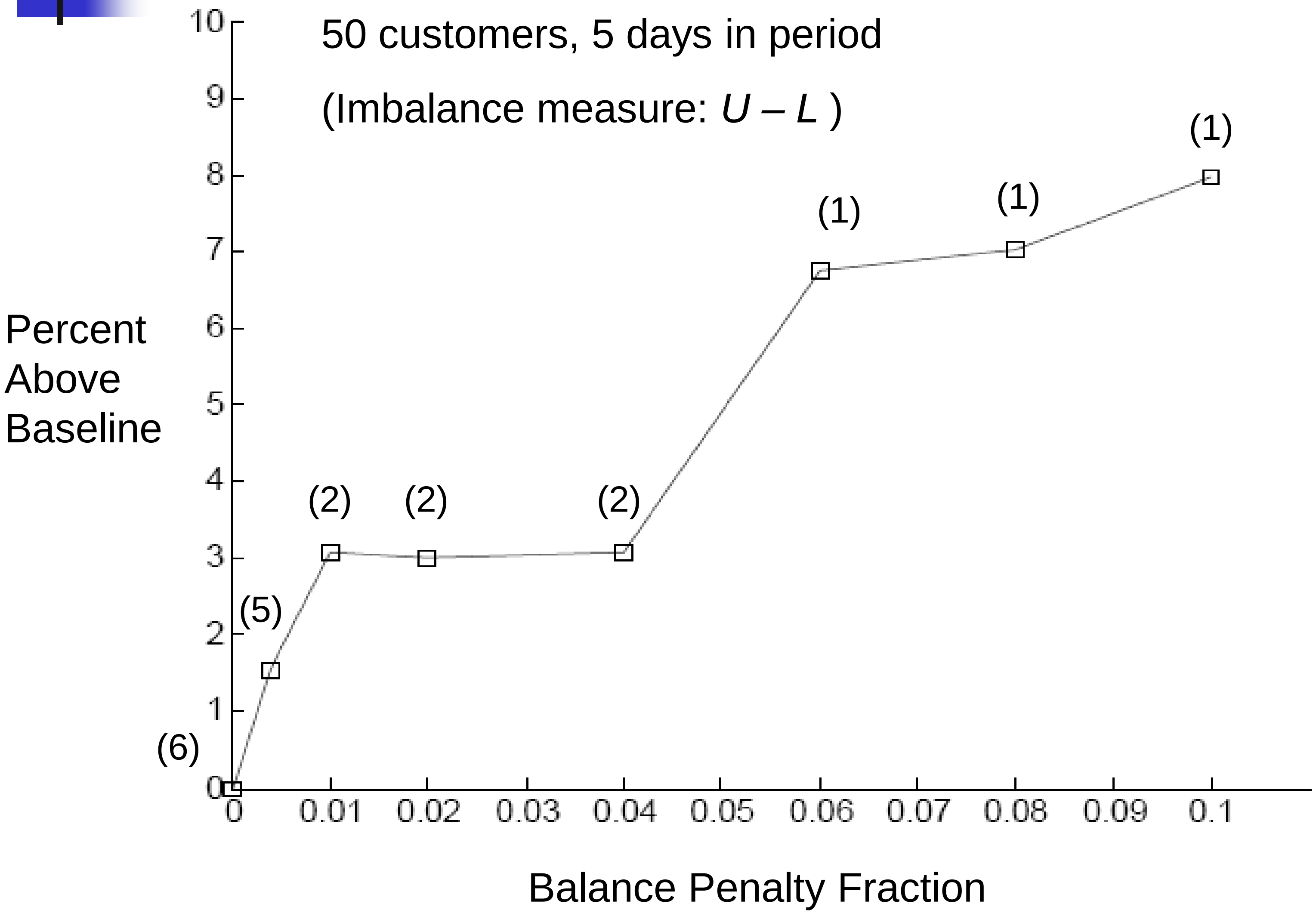
  ρ = imbalance penalty fraction

  C = distance of the initial solution

  U = the most customers on a route in a solution

  L = the fewest customers on a route in a solution
- We denote this problem PVRP-BC
- We modify IPH for the PVRP-BC (IPH-BC)

IPH-BC Results



Conclusion

- We develop a new IP-based algorithm for solving the PVRP
- We adapt our algorithm to solve several variants – the PVRP-RC, the PVRP-BC, and both
- These PVRP variants are important in modeling real-world problems