The Min-Max Multi-Depot Vehicle Routing Problem: Three-Stage Heuristic and Computational Results

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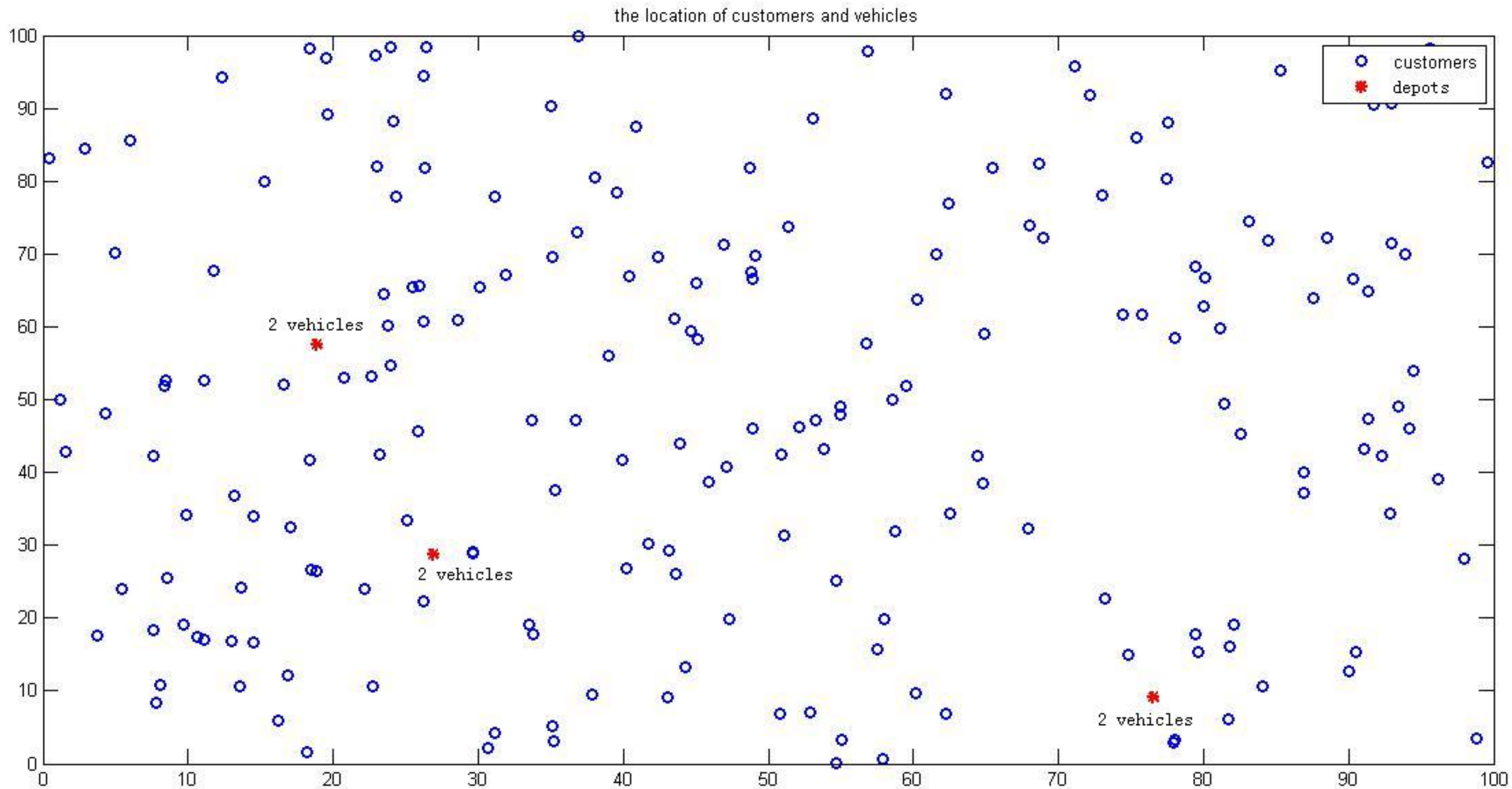
Introduction

- In the Multi-Depot VRP, the objective is to minimize the total distance traveled by all vehicles

- In the Min-Max MDVRP, the objective is to minimize the maximum distance traveled by a vehicle

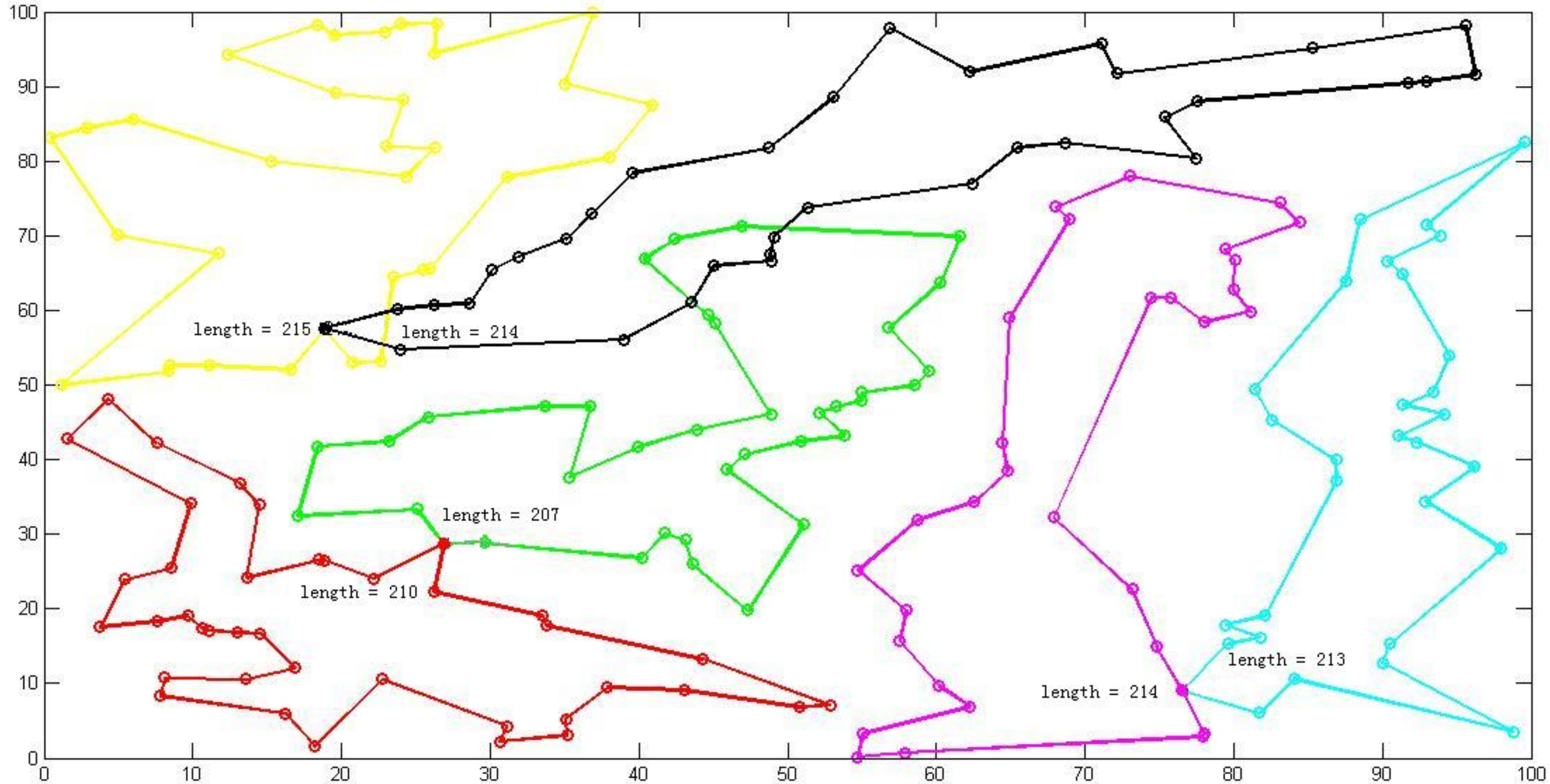
Introduction

- Min-Max Multi-Depot Vehicle Routing Problem



Introduction

- Min-max objective function



Introduction

Why is the min-max objective important?

- Applications
  - Disaster relief efforts
    - Serve all victims as soon as possible
  - Computer networks
    - Minimize maximum latency between a server and a client
  - Workload balance
    - Balance amount of work among drivers or across time horizon

Introduction
Other considerations

- Fixed number of vehicles available

- Basic model
  - There is no demand associated with the customers

- Capacitated model
  - Customers have demands in terms of quantity

- Service time model
  - Customers require service in terms of time

Literature Review

- Carlsson et al. (2009) proposed an LP-based balancing approach (LB) to solve the Min-Max MDVRP
  - Assignment of customers to vehicles using an LP
  - TSP solved by Concorde
  - These steps are repeated and the best feasible solution is reported

Literature Review

- LB is not expected to work very well when customers are not uniformly distributed

- It is not easy to extend LB to solve problems with customer service times

Solving the Min-Max MDVRP

- We develop a heuristic (denoted by MD)
- MD has three phases
  1. Initialization
  2. Local search
  3. Perturbation

Phase 1: Initialization

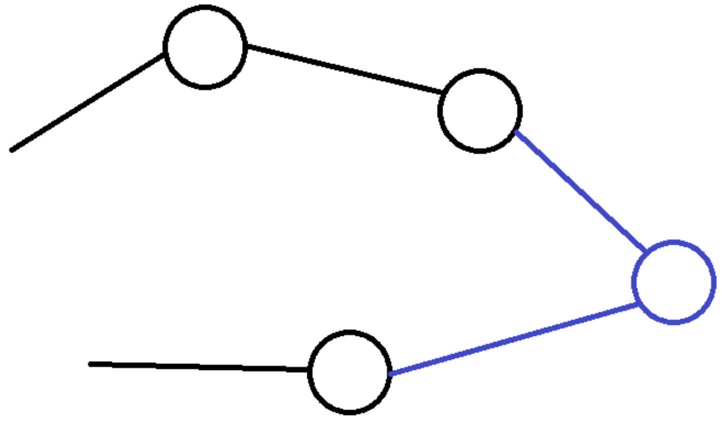
- Assign customers evenly to vehicles

$$\begin{aligned} \min \quad & \sum_{i,j} c_{ij} x_{ij} \\ s.t. \quad & \sum_{j} x_{ij} = 1 \quad \forall i \\ & \sum_{i} x_{ij} = \left\lfloor \frac{n}{m} \right\rfloor \quad or \quad \left\lfloor \frac{n}{m} \right\rfloor + 1 \quad \forall j \\ & x_{ij} \in \{0, 1\} \quad \forall i,\ j \end{aligned}$$

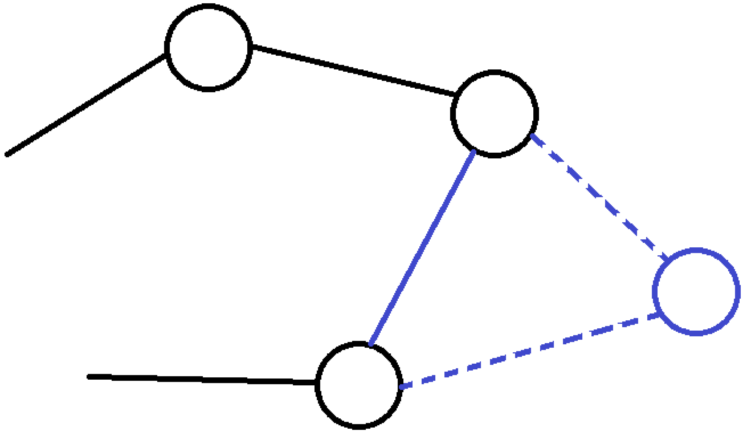
- Solve a TSP on each route using the Lin-Kernighan heuristic

Phase 2: Local Search

- Step 1. From the maximal route, identify the customer to remove (savings estimation)



Customer to remove



Savings estimation

Phase 2: Local Search

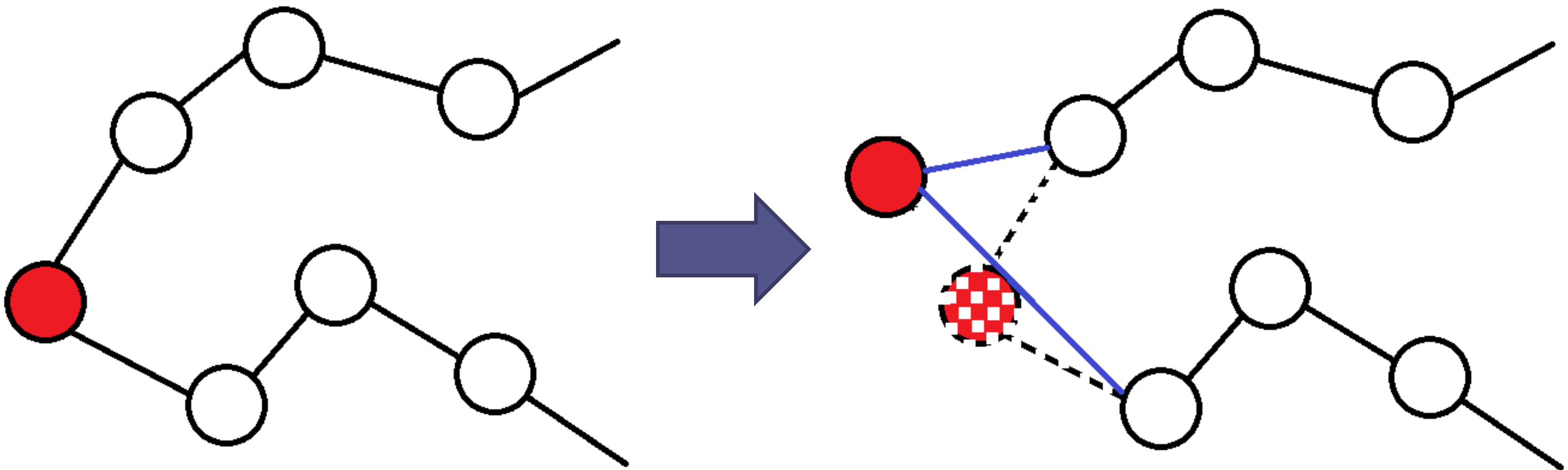
- Step 2. Identify the route on which to insert the removed customer (cost estimation)

- Step 3. Try inserting the customer in the cheapest way
  - Successful – go back to Step 1
  - Unsuccessful – try moving another customer

- Step 4. Stop if we have tried to move every customer on the maximal route

Phase 3: Perturbation

- Perturb the locations of the depots

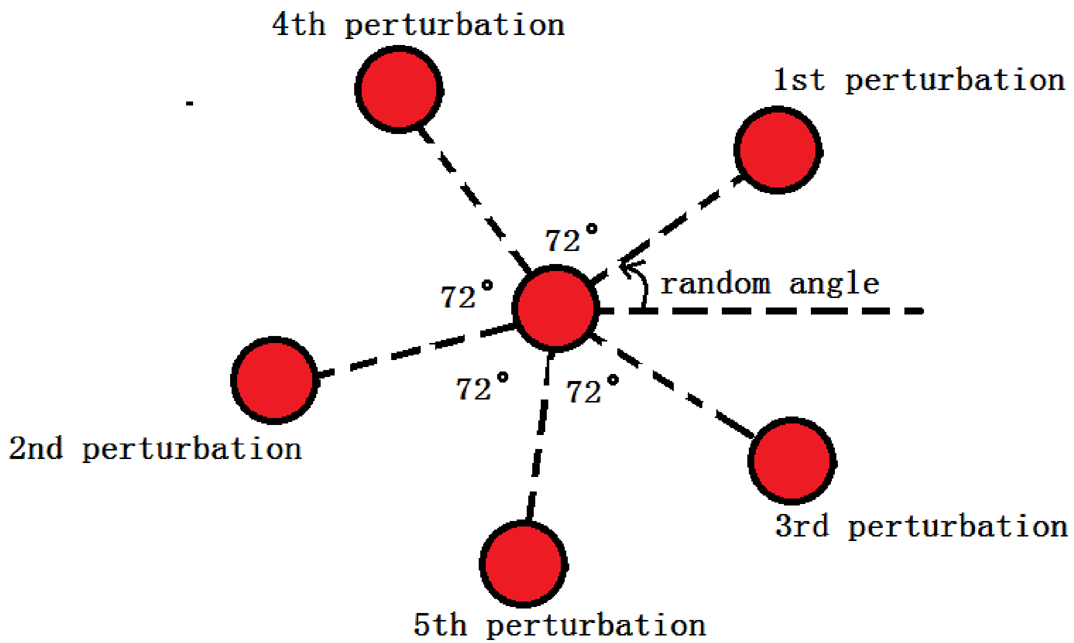


Phase 3: Perturbation

- Solve the new problem
- Set the depots back the their original positions
- Solve the problem and update the solution
- Repeat the process until there is no improvement for five consecutive perturbations

Phase 3: Perturbation

- The angle of subsequent perturbation depends on the angle of the previous perturbation.



Computational Results

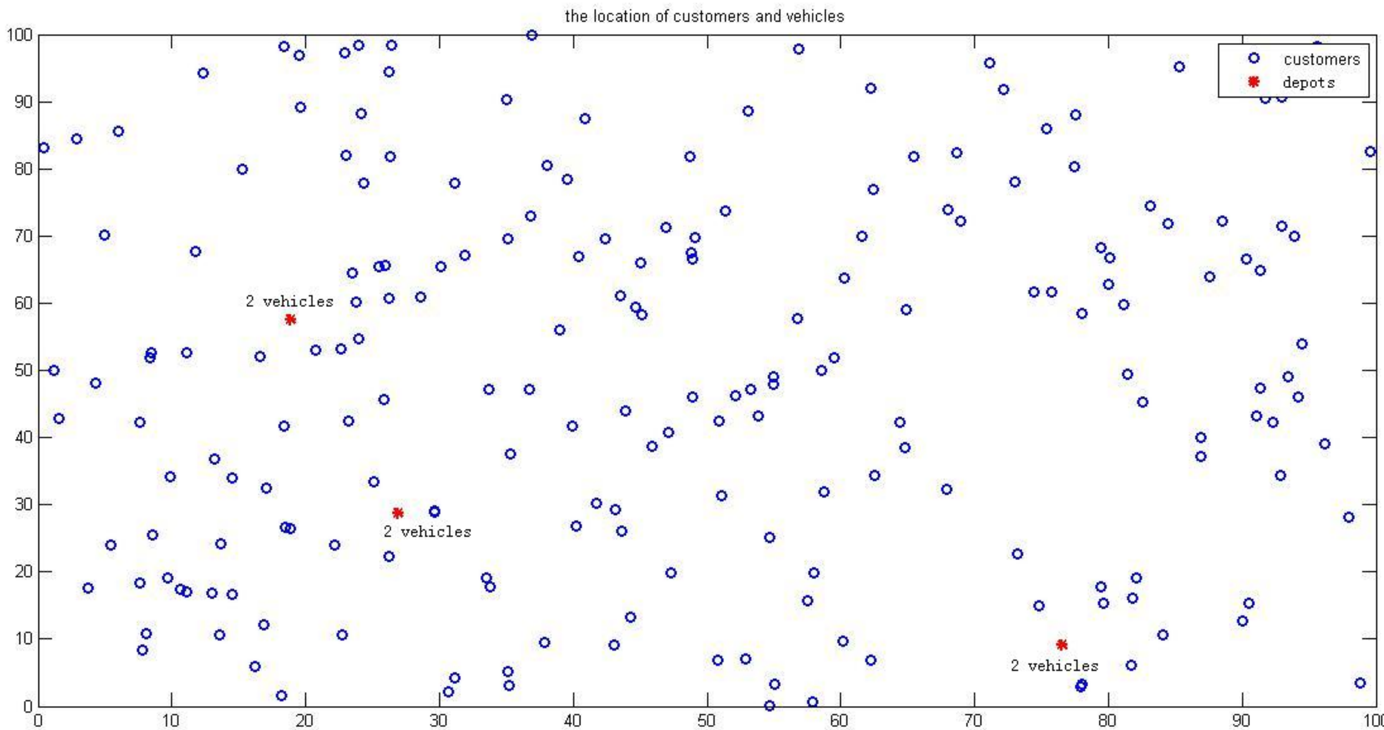
- 20 test problems
  - 10 to 500 customers
  - 3 to 20 depots
  - Problems have uniform and non-uniform customer locations

- MD used an Intel Pentium CPU with 2.20 GHz processor

- Code for LB required a 32-bit machine (Intel Core i5 with 2.40 GHz processor)

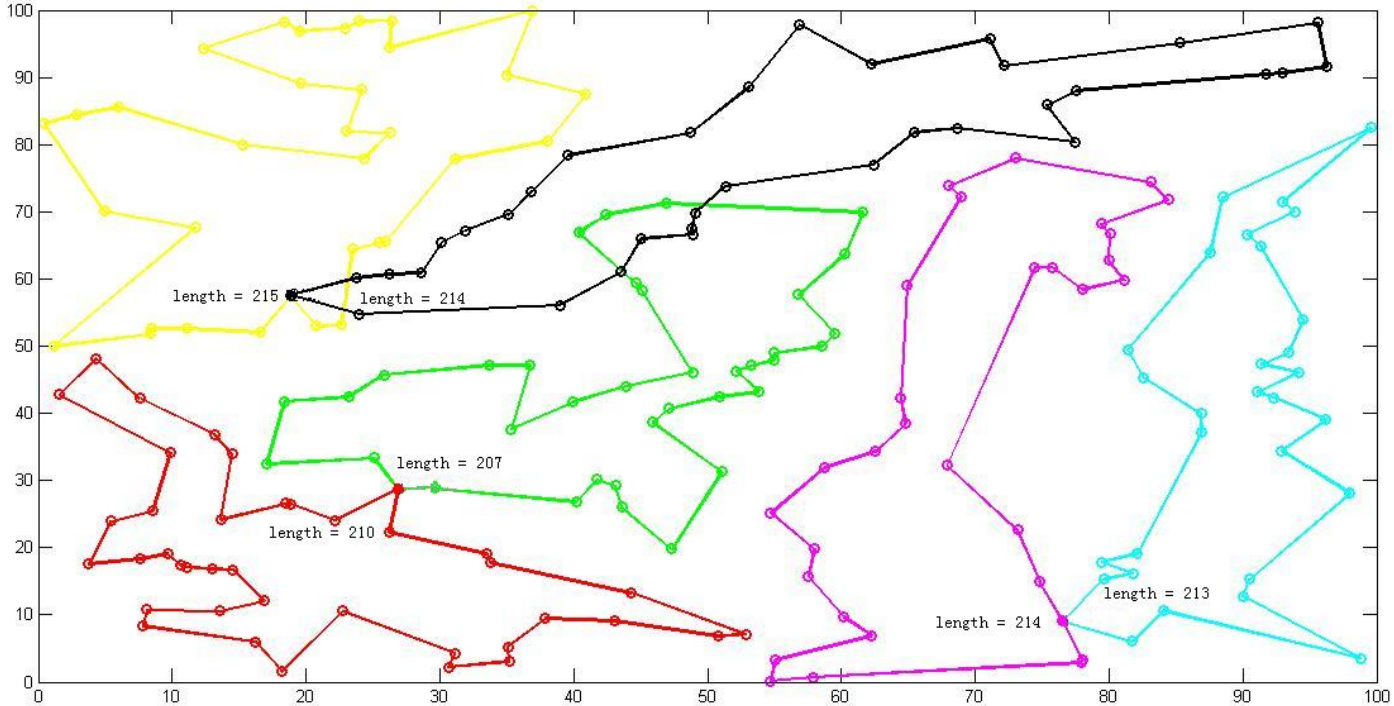
Computational Results

Problem MM8 (3 depots, 200 customers, 2 vehicles)



Computational Results

Problem MM8 (3 depots, 200 customers, 2 vehicles)



Computational Results

Uniform Customer Locations

- MD outperforms the LB-based heuristic by 12.5% on average

Identifier	LB Objective	LB Time (s)	MD Objective	MD Time (s)	Improvement (%)	VRPH*
MM2	149.225	38.2	129.863	11.1	12.98	136.105
MM3	265.349	61.4	238.973	18.1	9.94	243.511
MM7	222.071	14.5	189.015	1.5	14.89	189.633
MM8	242.730	73.2	213.501	29.8	12.04	226.135
MM10	197.594	32.9	197.39	3.5	0.10	194.172
MM11	119.658	78.5	102.391	2.6	14.43	103.954
MM12	114.826	37.9	78.307	2.5	31.80	79.747
MM13	138.823	35.8	121.535	4.9	12.45	125.760
MM14	146.492	35.5	134.612	8.2	8.11	140.841
MM15	110.963	41.0	99.805	5.2	10.06	111.571
MM16	115.744	60.2	101.328	22.8	12.46	113.925
MM18	439.606	68.4	393.428	619.1	10.50	444.173

Computational Results

Non-uniform Customer Locations

- MD outperforms the LB-based heuristic by 19.0% on average

	LB		MD			
Identifier	Objective	Time (s)	Objective	Time (s)	**Improvement (%)**	**VRPH**
MM4	569.453	43.9	481.889	18.0	15.38	485.551
MM5	398.970	40.2	319.941	32.8	19.81	402.885
MM9	183.157	36.8	151.789	111.6	17.13	158.417
MM17	325.708	56.8	243.895	235.1	25.12	263.195
MM19	474.935	68.4	365.939	616.0	22.95	388.128
MM20	385.297	92.1	333.050	359.6	13.56	340.817

*VRPH: Groer, C., Golden, B., and Wasil, E. (2010). A library of local search heuristics for the vehicle routing problem. *Mathematical Programming Computation*, 2, 79-101.

Conclusions

- On the 20 test problems, MD outperforms the LB-based heuristic by 13.2% on average
- In future work, we want to investigate the quality of the MD solution when applied to the service time model
- We also hope to enhance MD in order to produce even better solutions