The Hierarchical Traveling Salesman Problem: Some Worst-Case Results

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Introduction to the HTSP

- Consider the distribution of relief aid
  - E.g., food, bottled water, blankets, or medical packs
- The goal is to satisfy demand for relief supplies at many locations
  - Try to minimize cost
  - Take the urgency of each location into account

A Simple Model for Humanitarian Relief Routing

- Suppose we have a single vehicle which has enough capacity to satisfy the needs at all demand locations from a single depot
- Each node (location) has a known demand (for a single product called an aid package) and a known priority
  - Priority indicates urgency
  - Typically, nodes with higher priorities need to be visited before lower priority nodes

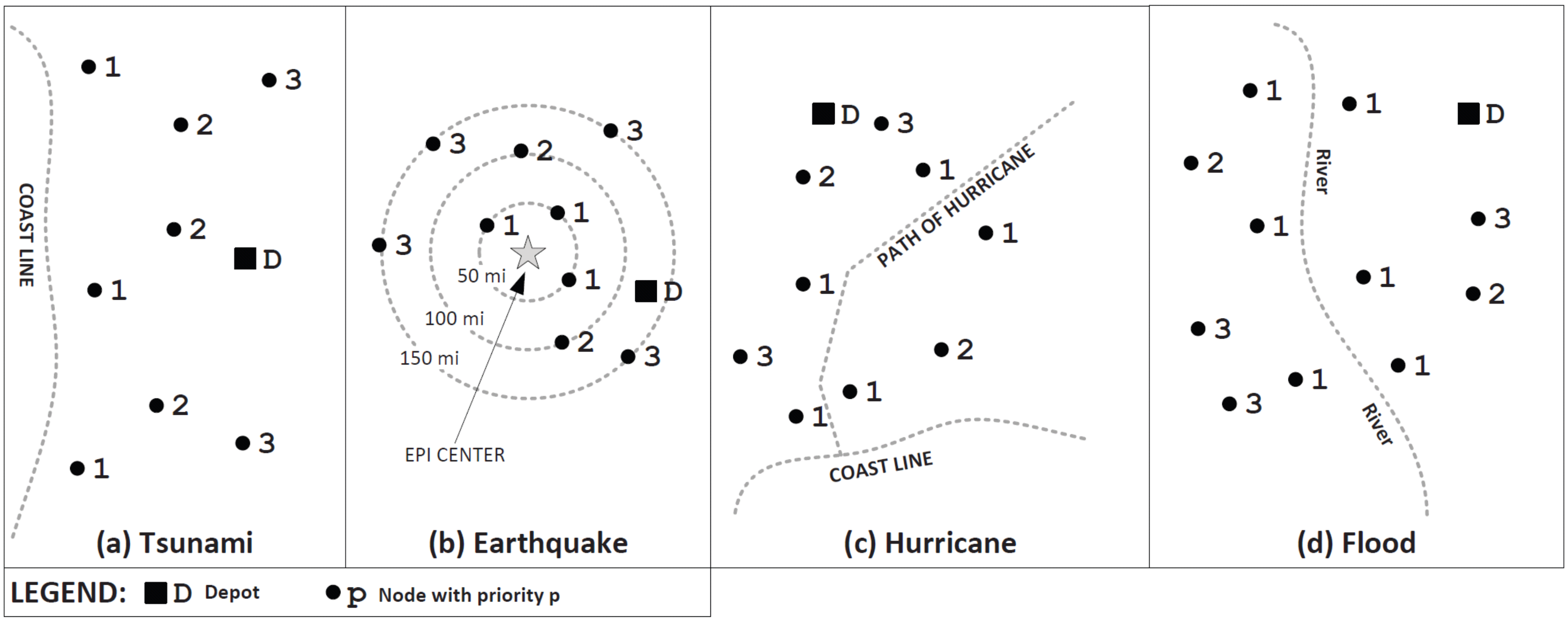
Node Priorities

- Priority 1 nodes are in most urgent need of service
- To begin, we assume
  - Priority 1 nodes must be served before priority 2 nodes
  - Priority 2 nodes must be served before priority 3 nodes, and so on
  - Visits to nodes must strictly obey the node priorities

The Hierarchical Traveling Salesman Problem

- We call this model the Hierarchical Traveling Salesman Problem (HTSP)
- Despite the model's simplicity, it allows us to explore the fundamental tradeoff between efficiency (distance) and priority (or urgency) in humanitarian relief and related routing problems
- A key result emerges from comparing the HTSP and TSP in terms of worst-case behavior

Four Scenarios for Node Priorities



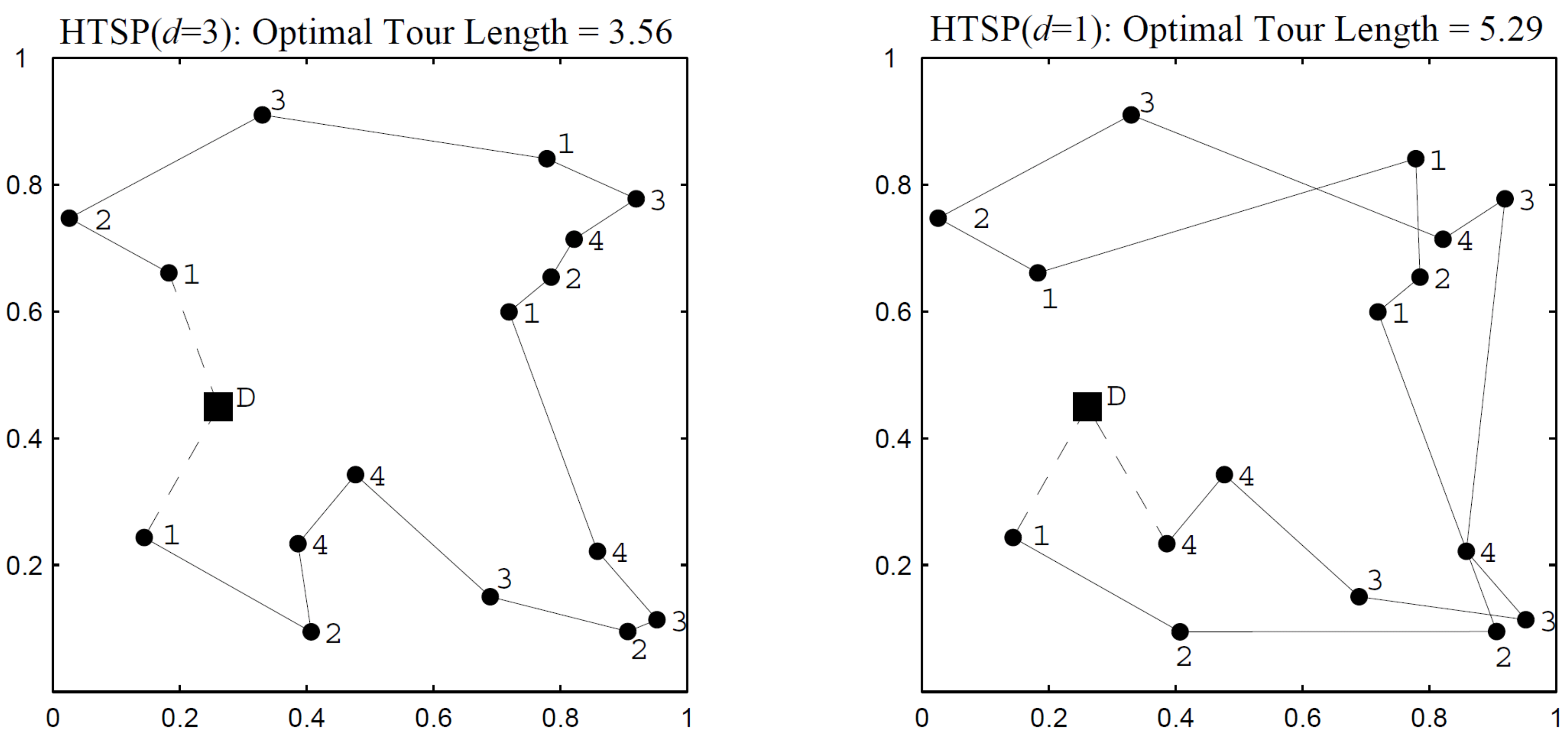
Literature Review

- Psaraftis (1980): precedence constrained TSP
- Fiala Tomlin, Pulleyblank (1992): precedence constrained helicopter routing
- Guttman-Beck et al. (2000): clustered traveling salesman problem
- Campbell et al. (2008): relief routing
- Balcik et al. (2008): last mile distribution

A Relaxed Version of the HTSP

- Definition: The d-relaxed priority rule adds operational flexibility by allowing the vehicle to visit nodes of priority $\pi + 1, \ldots, \pi + d$ (if these priorities exist in the given instance) but not priority $\pi + d + \ell$ for $\ell \geq 1$ before visiting all nodes of priority π (for $\pi = 1, 2, \ldots, P$)
- When d=0, we have the strict HTSP
- When d=P-1, we have the TSP (i.e., we can ignore node priorities)

Efficiency vs. Priority



Main Results (*Optimization Letters*, forth.)

- Let P be the number of priority classes
- Assume the triangle inequality holds
- Let $Z^*_{d,P}$ and Z^*_{TSP} be the optimal tour length (distance) for the HTSP with the d-relaxed priority rule and for the TSP (without priorities), respectively
- We obtain the following results below (and the bounds are tight)

$$(a) \quad Z^*_{0,P} \leq P Z^*_{TSP}$$

$$(b) \quad Z^*_{d,P} \leq \left\lceil \frac{P}{d+1} \right\rceil Z^*_{TSP}$$

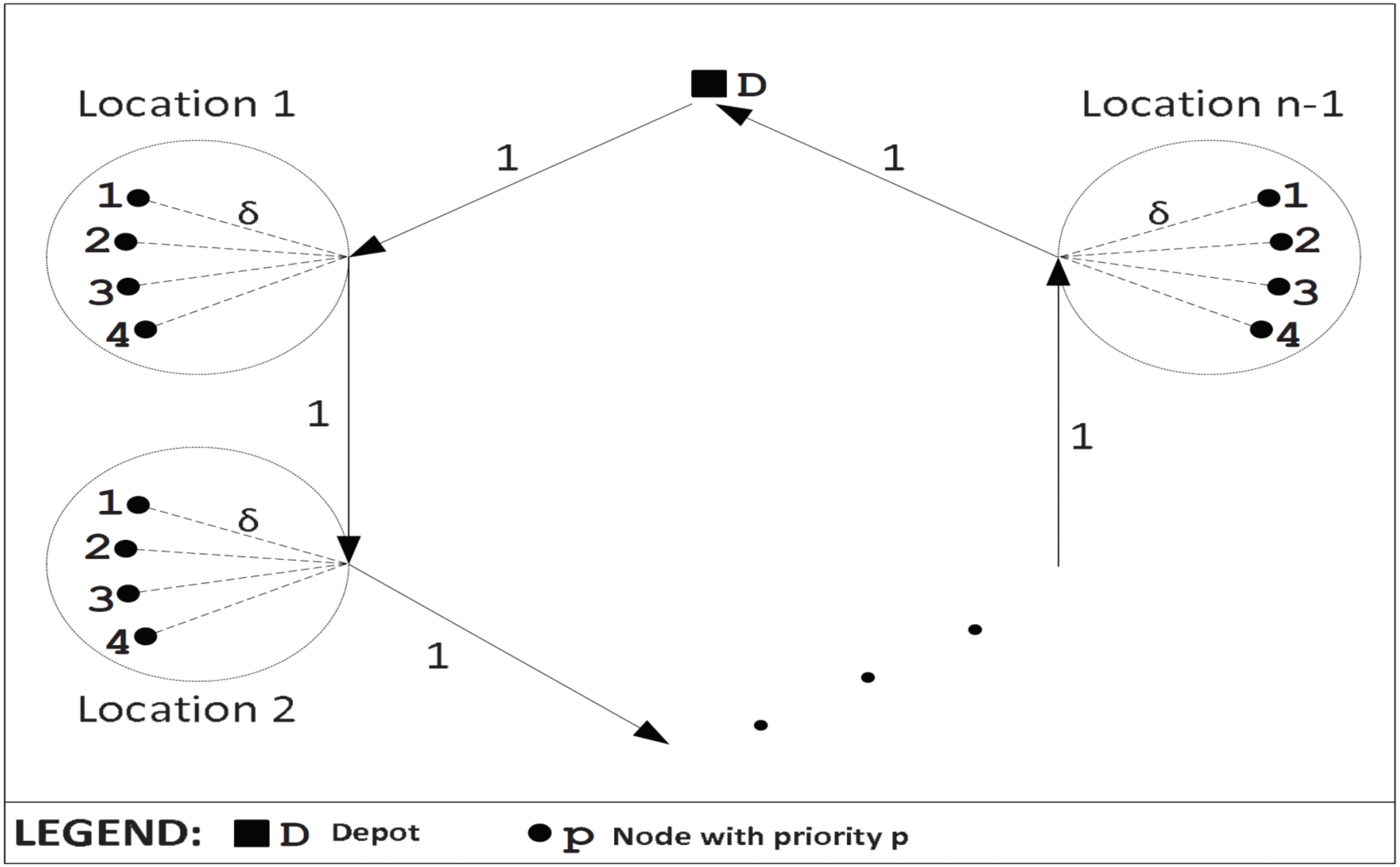
The General Result and Two Special Cases

- $Z^*_{d,P} \leq \left\lceil \frac{P}{d+1} \right\rceil Z^*_{TSP}$

- If d=0, we have part (a)

- If d=P-1, then $Z^*_{d,P} = Z^*_{TSP}$

Worst-Case Example



Several Observations

- Observation 1. The worst-case example shows that the bounds in (a) and (b) are tight and cannot be improved

- Observation 2. We can "solve" a TSP over the entire set of nodes using our favorite TSP heuristic and obtain a feasible tour for the HTSP by traversing the TSP tour back and forth

- Observation 3. Suppose we select Christofides' heuristic and let $Z^h_{d,P}$ be the length of the resulting feasible solution to the HTSP, then we have $Z^h_{d,P} \le \frac{3}{2} \cdot \left\lceil \frac{P}{d+1} \right\rceil Z^*_{TSP}$

Observations and Extensions

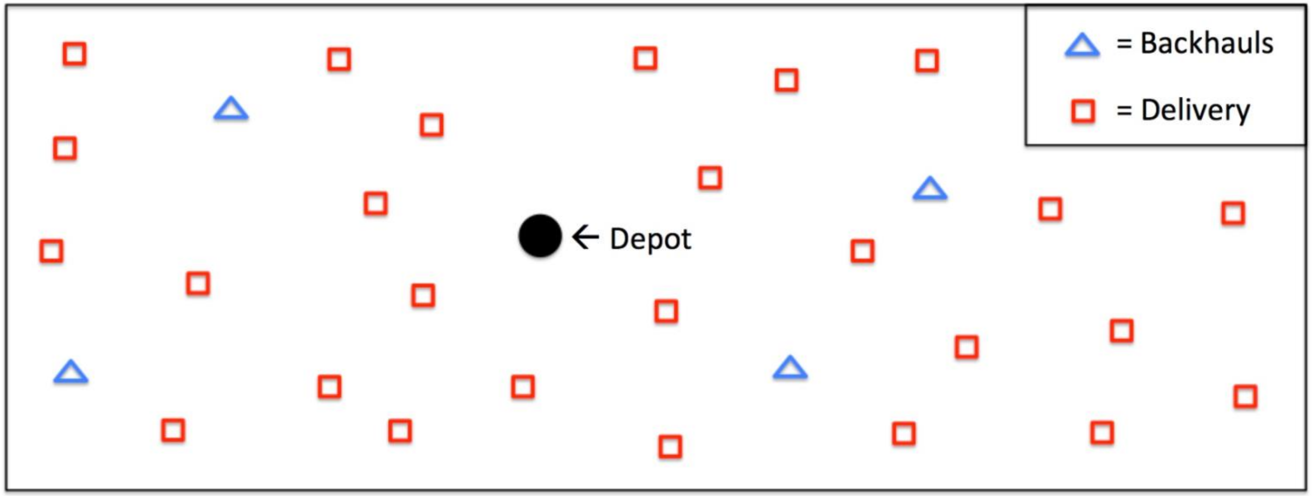
- Observation 4. The HTSP (with d=0) can be modeled and solved as an ATSP
- Observation 5. Other applications of the HTSP include routing of service technicians, routing of unmanned aerial vehicles, and vehicle routing with backhauls
- We can obtain similar worst-case results (with tight bounds) for the HTSP on the line and the Hierarchical Chinese Postman Problem (HCPP)

Vehicle Routing with Backhauls

- Problem statement
  - Find a set of vehicle routes that services the delivery and backhaul (pickup) customers such that vehicle capacity is not violated and total distance traveled is minimized
  - Such a customer mix occurs in many industries (e.g., the grocery industry)
    - supermarkets – delivery points
    - poultry processors, fruit vendors – backhauls

Vehicle Routing with Backhauls

- Backhauls are serviced at the end of a route
  - Deliveries are high-priority stops
  - Small number of backhauls
  - Difficult to rearrange on-board load in rear-loaded vehicles



What Next?

- We tried to generalize the previous worst-case bound
- What is the worst-case bound for

$$\frac{Z^*_{d_1,P}}{Z^*_{d_2,P}} \quad \text{where } d_1 < d_2 < P-1?$$

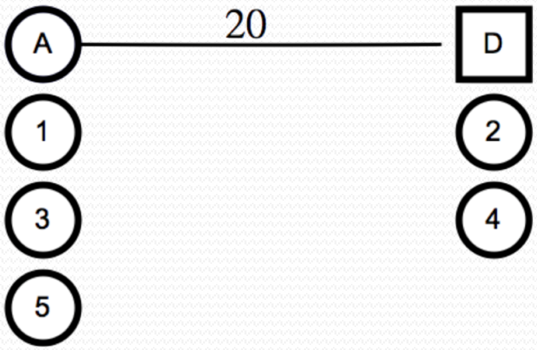
- We have only been able to derive a partial result
- New result (Xiong & Golden, 2013): For any HTSP problem where the triangle inequality holds,

$$\boxed{Z^*_{0,P} \leq 3Z^*_{1,P}}$$

and the bound is tight

New HTSP Result

- We developed a worst-case example, but a simpler one was found by two of my students (Kim & Park, 2013)
- Consider the example below with P=5



$$Z^*_{0,5} = 120$$

$$Z^*_{1,5} = 40$$

$$Ratio = 3$$

- I would be happy to see someone at this conference extend our work

Further Remarks

- The HTSP and several generalizations have been formulated as mixed integer programs
- HTSP instances with 30 or so nodes were solved to optimality using CPLEX
- Future work
  - New worst-case results
  - The Hierarchical Vehicle Routing Problem (HVRP)
  - A multi-day planning horizon
  - Uncertainty with respect to node priorities

Reference

- "The Hierarchical Traveling Salesman Problem" (by K. Panchamgam, Y. Xiong, B. Golden, B. Dussault, and E. Wasil), forthcoming in *Optimization Letters*, 2013.