The Hierarchical Traveling Salesman Problem

K. Panchamgam, Y. Xiong, B. Golden*, and E. Wasil

Presented at INFORMS Annual Meeting
Charlotte, NC, November 2011

*R.H. Smith School of Business
University of Maryland

Focus of Vehicle Routing Papers

- Heuristic approach
- Exact approach
- Hybrid approach
- Case study
- Worst-case analysis

Introduction

- Consider the distribution of relief aid
  - E.g., food, bottled water, blankets, or medical packs
- The goal is to satisfy demand for relief supplies at many locations
  - Try to minimize cost
  - Take the urgency of each location into account

A Simple Model for Humanitarian Relief Routing

- Suppose we have a single vehicle which has enough capacity to satisfy the needs at all demand locations from a single depot
- Each node (location) has a known demand (for a single product called an aid package) and a known priority
  - Priority indicates urgency
  - Typically, nodes with higher priorities need to be visited before lower priority nodes

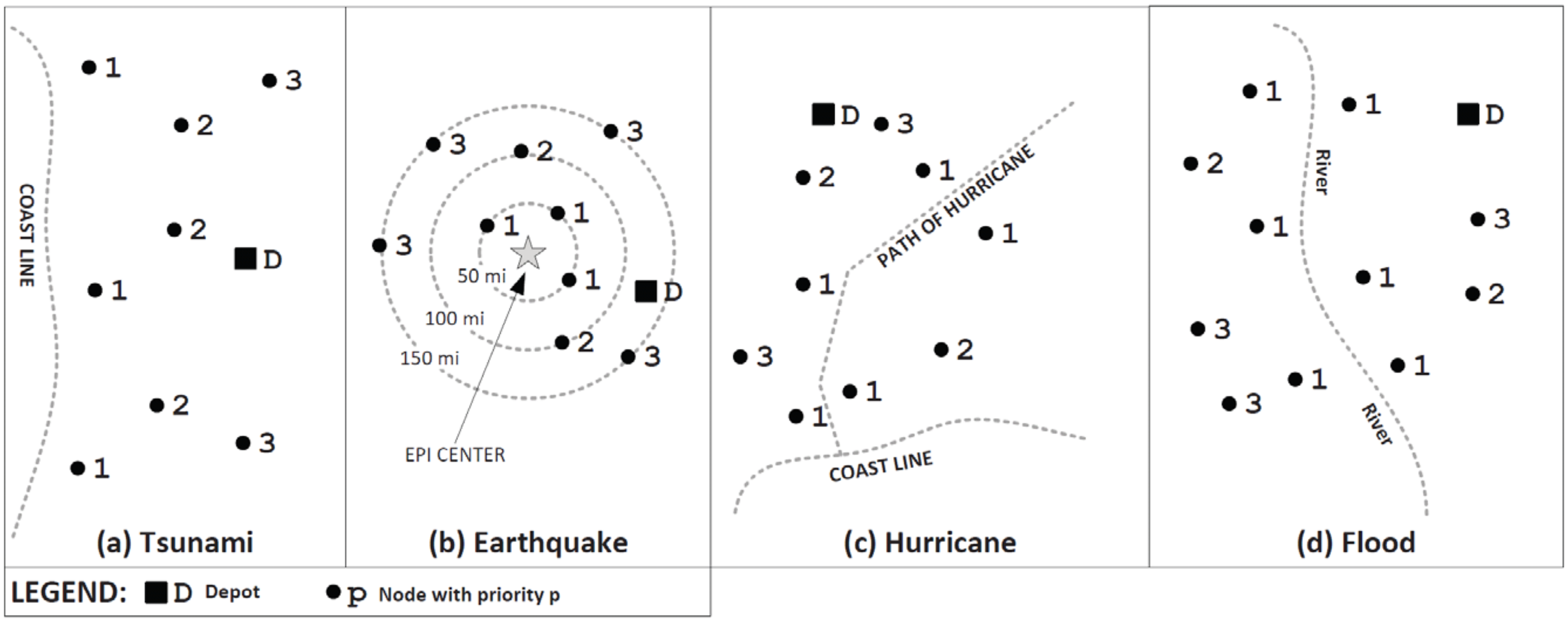
Node Priorities

- Priority 1 nodes are in most urgent need of service
- To begin, we assume
  - Priority 1 nodes must be served before priority 2 nodes
  - Priority 2 nodes must be served before priority 3 nodes, and so on
  - Visits to nodes must strictly obey the node priorities

The Hierarchical Traveling Salesman Problem

- We call this model the Hierarchical Traveling Salesman Problem (HTSP)
- Despite the model's simplicity, it allows us to explore the fundamental tradeoff between efficiency (distance) and priority (or urgency) in humanitarian relief and related routing problems
- A key result emerges from comparing the HTSP and TSP in terms of worst-case behavior

Four Scenarios for Node Priorities



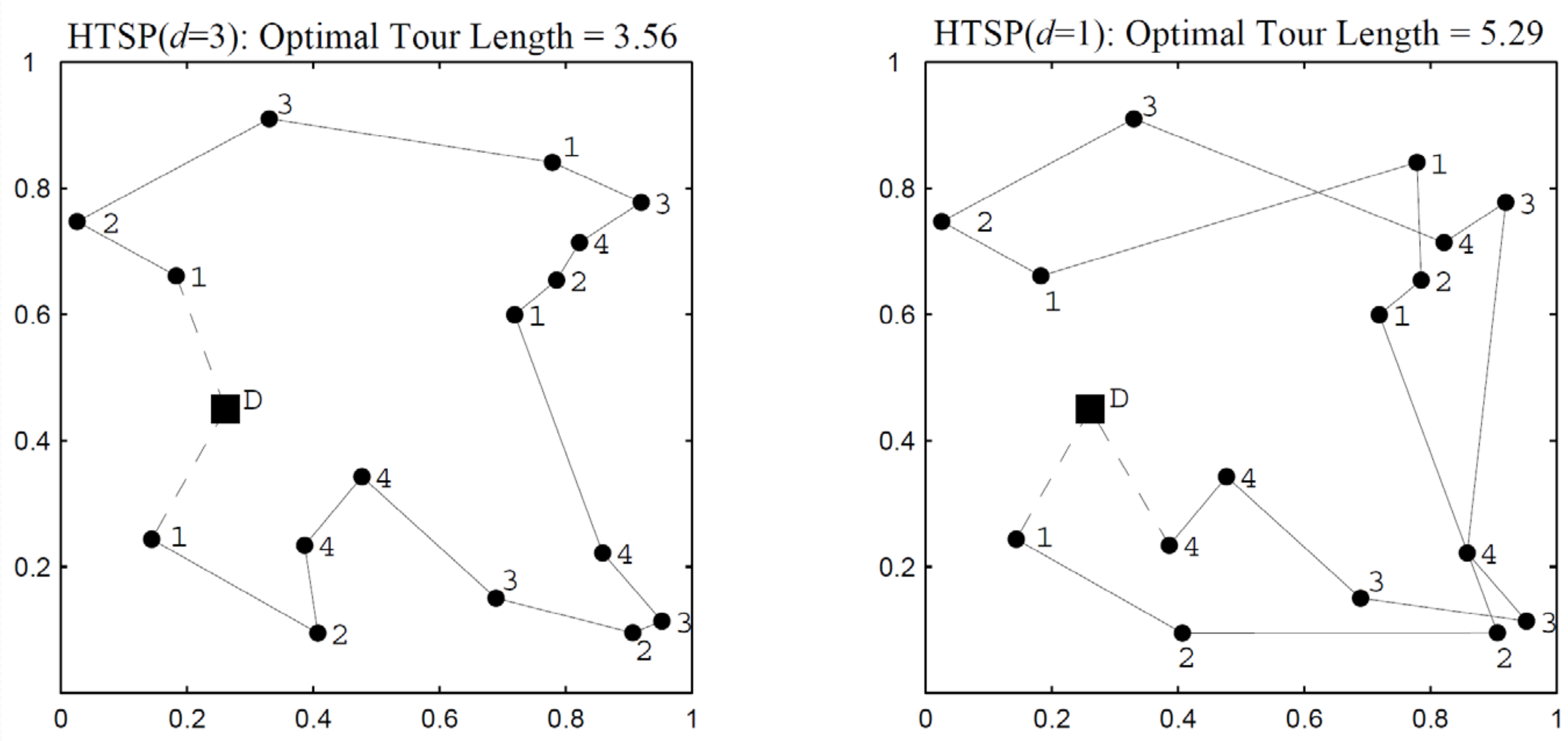
Literature Review

- Psaraftis (1980): precedence constrained TSP
- Fiala Tomlin, Pulleyblank (1992): precedence constrained helicopter routing
- Campbell et al. (2008): relief routing
- Balcik et al. (2008): last mile distribution
- Ngueveu et al. (2010): cumulative capacitated VRP

A Relaxed Version of the HTSP

- Definition: The d-relaxed priority rule adds operational flexibility by allowing the vehicle to visit nodes of priority $\pi + 1, \dots, \pi + d$ (if these priorities exist in the given instance) but not priority $\pi + d + \ell$ for $\ell \geq 1$ before visiting all nodes of priority π (for $\pi = 1, 2, \dots, P$)
- When d=0, we have the strict HTSP
- When d=P-1, we have the TSP (i.e., we can ignore node priorities)

Efficiency vs. Priority



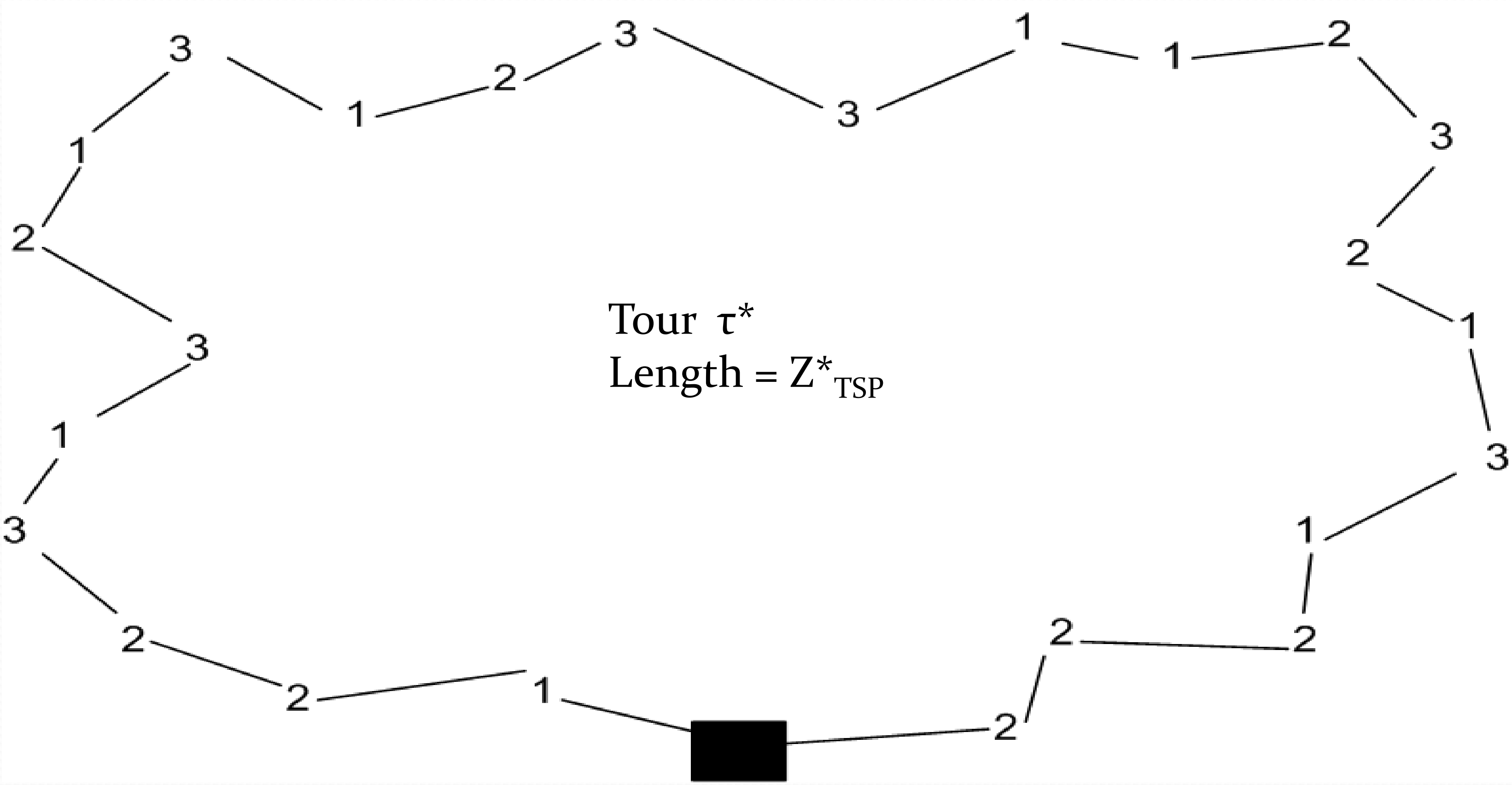
Main Results

- Let P be the number of priority classes
- Assume the triangle inequality holds
- Let $Z^*_{d,P}$ and Z^*_{TSP} be the optimal tour length (distance) for the HTSP with the d-relaxed priority rule and for the TSP (without priorities), respectively
- We obtain the following results

$$\text{(a) } Z^*_{0,P} \leq P\, Z^*_{TSP}$$

$$\text{(b) } Z^*_{d,P} \leq \left\lceil \frac{P}{d+1} \right\rceil Z^*_{TSP}$$

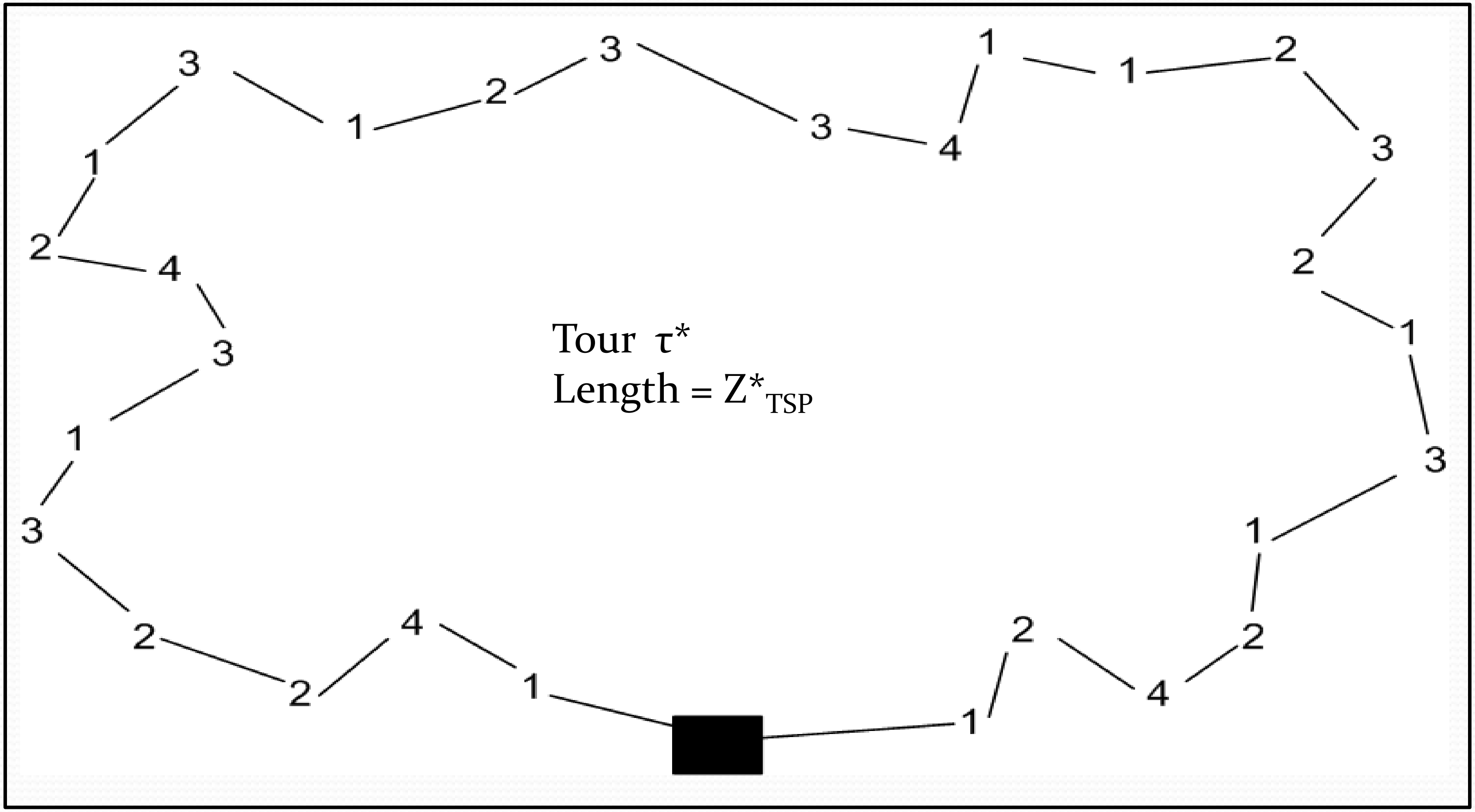
Sketch of Proof (a)



Sketch of Proof (a)

- Construct tours τ(1), τ(2), and τ(3)
- Visit nodes in the same order as they appear in τ*
- From the triangle inequality, the lengths of τ(1), τ(2), and τ(3) are each ≤ Z^*_{TSP}
- It is easy to construct a feasible solution τ to the HTSP from τ(1), τ(2), and τ(3)
- The length of τ ≤ $\sum_{i=1}^{3}$ {length of τ (i)} ≤ $3Z^*_{TSP}$

Sketch of Proof of (b)



Sketch of Proof of (b)

- In our example, P=4 and d=1
- In the worst case, we can't visit a priority 3 node until we have visited all priority 1 and 2 nodes
- Visit nodes in the same order as they appear in τ^*
- $\tau(1)$ includes priority 1 and 2 nodes
- $\tau(2)$ includes priority 3 and 4 nodes
- As before, we can construct τ from $\tau(1)$ and $\tau(2)$
- The length of $\tau \leq \sum_{i=1}^{2}$ {length of τ (i)} $\leq 2Z*_{TSP}$

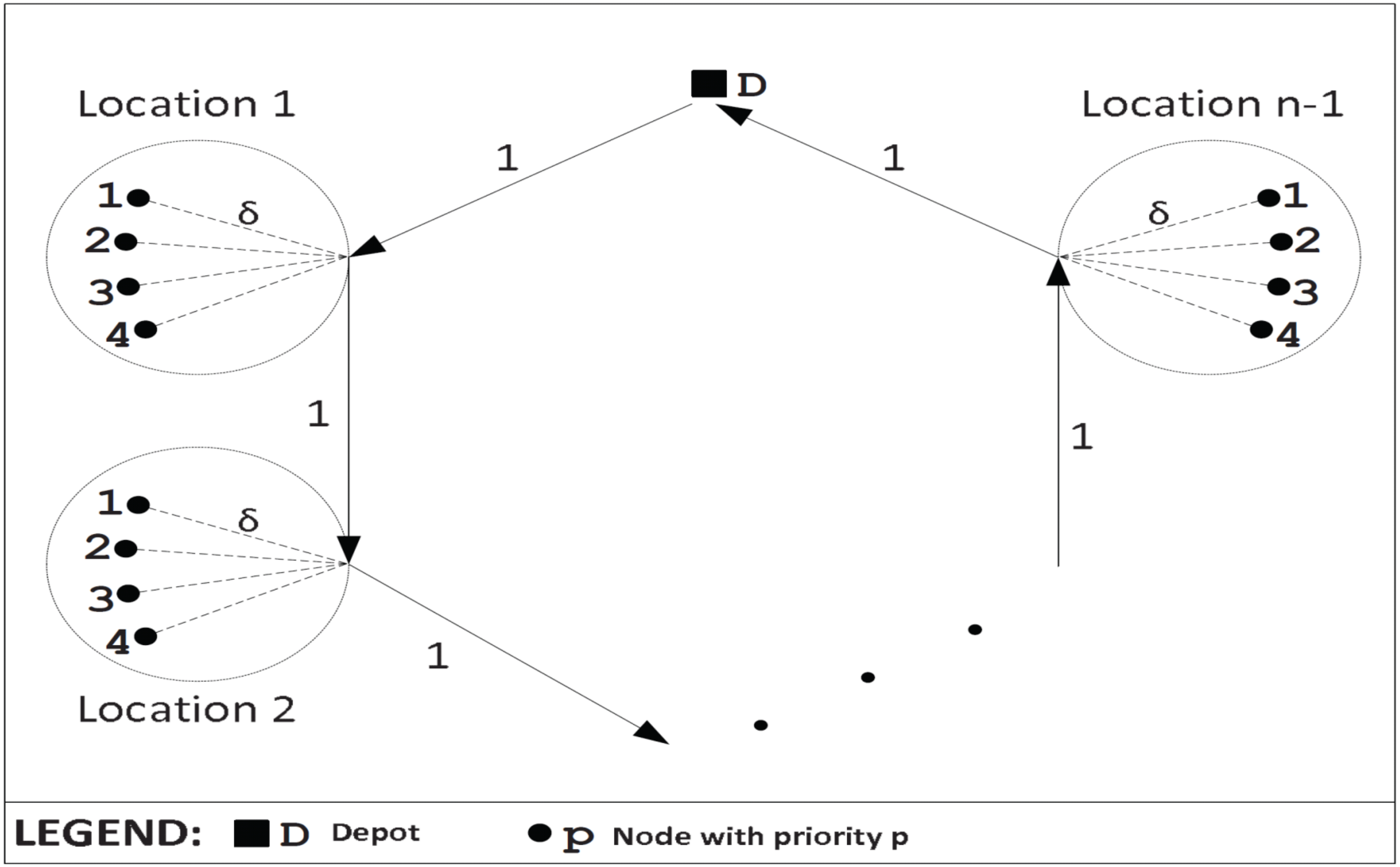
The General Result and Two Special Cases

- $Z^*_{d,P} \leq \left\lceil \frac{P}{d+1} \right\rceil Z^*_{TSP}$

- If d=0, we have part (a)

- If d=P-1, then $Z^*_{d,P} = Z^*_{TSP}$

Worst-case Example



Conclusions and Extensions

- The worst-case example shows that the bounds in (a) and (b) are tight and cannot be improved
- The HTSP and several generalizations have been formulated as mixed integer programs
- HTSP instances with 30 or so nodes were solved to optimality using CPLEX
- Future work: capacitated vehicles, multiple products, a multi-day planning horizon, uncertainty with respect to node priorities