



An Integer Programming-Based Heuristic for the Period Vehicle Routing Problem

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VRP and PVRP

- In the vehicle routing problem (VRP) a fleet of vehicles must service the demands of customers while minimizing total distance traveled
- In the period vehicle routing problem (PVRP) customers might require service several times during a time period

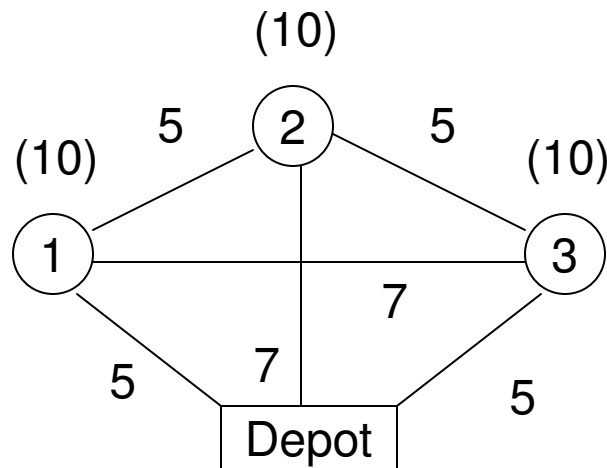


PVRP Description

- We must first assign customers to patterns (certain days of the time period) and then find routes on each day servicing the customers scheduled on that day
- We seek to minimize total distance traveled throughout the time period
- For example, a waste management company has to assign customers to certain days of the week and then create daily routes
- Some customers might only need service once a week, some might need service multiple times

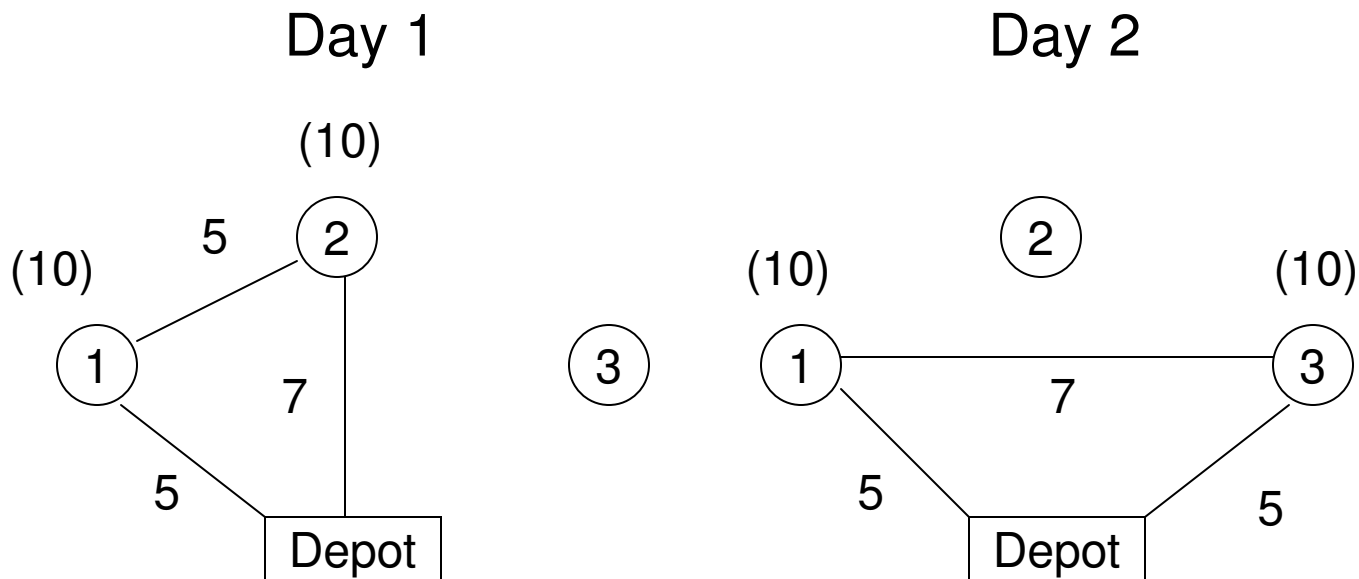
PVRP Example

- The time period is two days, $T = \{1, 2\}$
- Customer 1 must be serviced twice
- Customers 2 and 3 must be serviced once
- Node labels in parentheses are demands per delivery
- Edge labels are distances
- Vehicle capacity is 30



PVRP Example

- Customer 1 is assigned to day 1 and day 2
- Customer 2 is assigned to day 1
- Customer 3 is assigned to day 2



Total distance traveled is 34 units



PVRP Applications

- Waste and recycled goods collection
- Grocery and soft drink distribution
- Fuel oil and industrial gas delivery
- Internal transport installation and maintenance
- Utility services
- Automobile parts distribution
- Oil collection from onshore wells



PVRP Literature

- PVRP literature dates back to the 1970s
- Recent papers on solving the PVRP
 - Cordeau, Gendreau, and Laporte (1997)
 - Alegre, Laguna, and Pacheco (2007)
 - Hemmelmayr, Doerner, and Hartl (2009)



PVRP Variants

- In practice, companies have pre-existing solutions that over time, due to the addition and subtraction of customers and other small modifications, have become inefficient
- Some companies have fleets of thousands of vehicles that service millions of customers annually (Blakely et al., 2003)
- Not practical economically, logistically, or perhaps contractually to reroute from scratch
- Instead, the major inefficiencies in the pre-existing routes should be eliminated in a way that does not cause widespread disruption



PVRP Variants

- From a pre-existing solution S' we find an improved solution S^* while constraining the total amount of disruption
- We consider three types of constraints
 1. Hard constraint: we set a limit W and only accept solutions S^* such that the number of customers assigned to different patterns from S' to S^* is at most W
 2. Soft constraint: in the objective function we penalize S^* for each customer assigned to a different pattern than the one in S'
 3. Multi-day reassignment constraints: we fix all multi-day customers to their patterns in S' but allow one-day customers to be freely reassigned in S^*
- Collectively, we call these variants the PVRP with reassignment constraints (PVRP-RC)



PVRP-BC

- Because route balance is important in industry we also consider the period vehicle routing problem with balance constraints (PVRP-BC)
- In the PVRP-BC we start with a pre-existing solution S' that is well-balanced or has low cost routes
- In the objective function we penalize a solution for having imbalanced routes
- Starting from S' , we wish to find a solution S^* that minimizes routing distance plus an imbalance penalty



IP Based PVRP Heuristic

- We develop an IP-based Heuristic algorithm for solving the PVRP, denoted IPH
- We easily adapt IPH to solve the PVRP-RC and the PVRP-BC
- We present computational results which demonstrate the effectiveness of our algorithm



IPH: Overview

1. Generate an initial PVRP solution S
2. Improve S by solving an IP that reassigns and reroutes customers in a way that maximizes total savings (main step)
3. Improve daily routes using a VRP heuristic
4. Remove and reinsert customers using an IP
5. Repeat steps 2-4 until a stopping condition is reached



IPH: Initial Solution

- We first assign customers to patterns in a way that balances the amount of demand serviced on each day (standard assignment IP)
- Next we find a VRP solution on each day using a quick heuristic (e.g., CW savings)
- The result is an initial PVRP solution



IPH: Improvement IP

- Given a solution S we formulate an improvement IP (IMP) that maximizes the savings from reassigning customers to new service patterns, removing customers from their current routes, and inserting them into new routes
- We solve IMP repeatedly until no more improvement is achieved



IPH: IMP Formulation

Objective Function

- Maximize savings from reassignments and rerouting

Constraints

- We remove a customer from its current route if and only if we reinsert it elsewhere
- We move a customer to a new day if and only if we assign it to a pattern containing that day



IPH: IMP Formulation

- The total demand of the customers we move to a route minus the total demand of the customers we move from the route cannot exceed the residual capacity of the route
- If a customer i or its predecessor are removed from a route we do not move any other customers immediately prior to i and we remove at most one of i and its predecessor (this ensures the objective function gives the savings accurately)
- We assign a customer to at most one feasible pattern

IPH: IMP Example

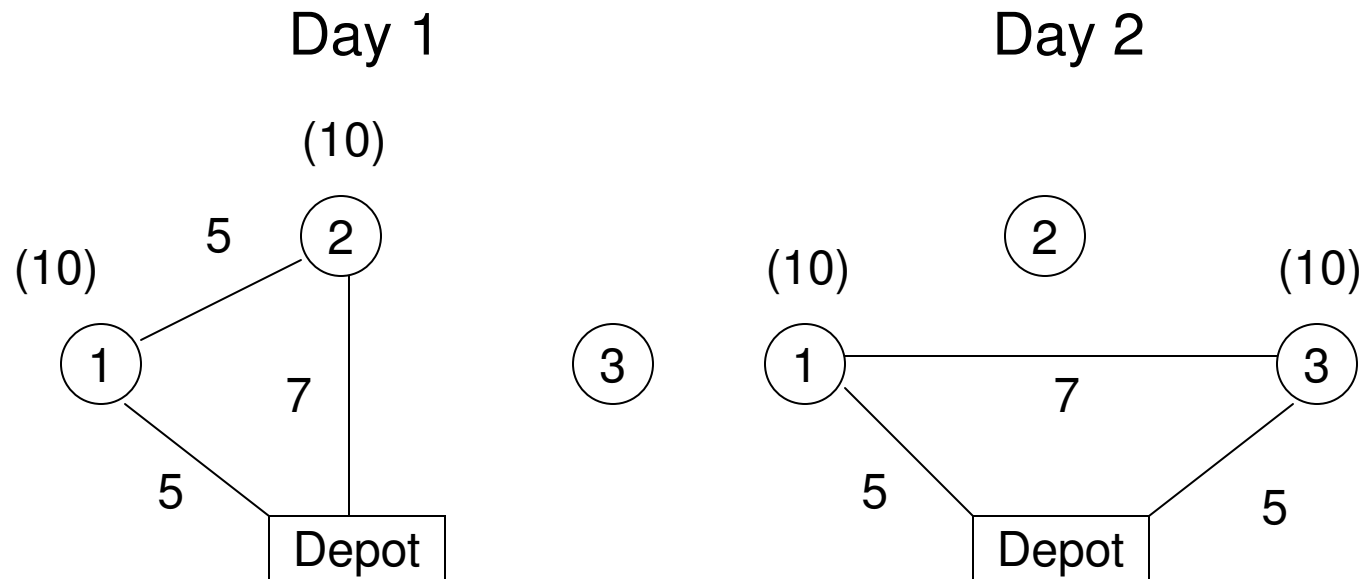
■ Initial Solution

Customer 1 is assigned to day 1 and day 2

Customer 2 is assigned to day 1

Customer 3 is assigned to day 2

Vehicle Capacity is 30 units

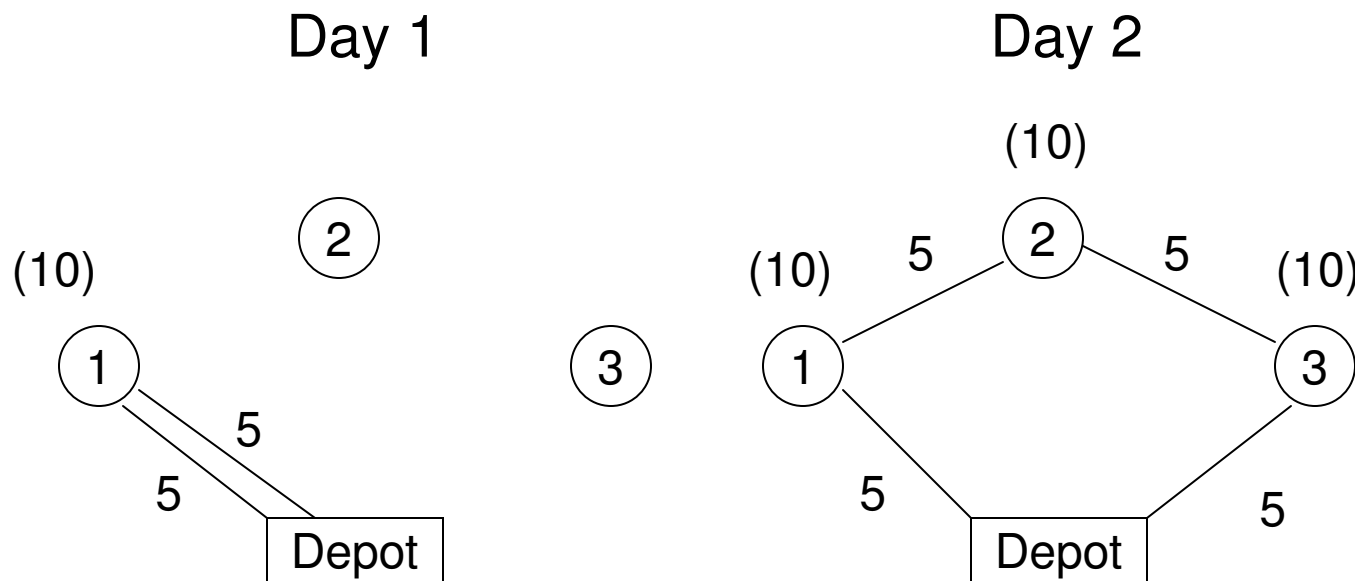


Total distance traveled is 34 units

IPH: IMP Example

- Improved Solution

Customer 2 is removed from its route on day 1, reassigned to day 2 and inserted immediately prior to customer 3, for a savings of 4 units





IPH: ERTR

- We improve daily routes by solving VRPs using the enhanced record-to-record travel algorithm (ERTR)
- ERTR was developed by Groer, Golden, and Wasil (2008)
- One-point move, two-point exchange, two-opt move
- Three-point move, OR-opt move



IPH: Re-initialization

- We remove some customers randomly from their routes
- We create a fictitious day F and assign all removed customers to routes on day F
- We solve IMP adding a constraint forcing all customers visited on F to be reassigned to a feasible pattern
- Re-initialization allows us to further explore the solution space from a new starting solution



IPH: Best Solution

- We iterate IMP, ERTR, and the re-initialization procedure until a stopping condition is reached
- The best solution found throughout this process is returned at the end



IPH Results

- On 32 problems from the PVRP literature, IPH had an average deviation from the best-known solution of 1.14%
- For Cordeau, Gendreau, and Laporte (1997) it was 1.58%
- For Alegre, Laguna, and Pacheco (2007) it was 1.36%
- For Hemmelmayr, Doerner, and Hartl (2009) it was 1.39%



IPH Run-times

- On the 32 problems IPH had an average run-time of 686 seconds
- For Cordeau, Gendreau, and Laporte it was 139 (comparable machine)
- For Alegre, Laguna, and Pacheco it was 1449 (slower machine)
- For Hemmelmayr, Doerner, and Hartl it was 149 (comparable machine)



IPH Virtues

- Comparable performance to the best algorithms on PVRPs
- Easily modified to handle real-world variants, including customer reassignment constraints and route balance constraints



PVRP-RCH and IPH-RCH

- We modify IPH to solve the PVRP-RC with a hard constraint (PVRP-RCH)
- In the PVRP-RCH, we allow no more than W customer reassignments from the pre-existing initial solution S'
- In practice, it helps to do this in stages
- We denote the modified IPH algorithm by IPH-RCH



IPH-RCH Results

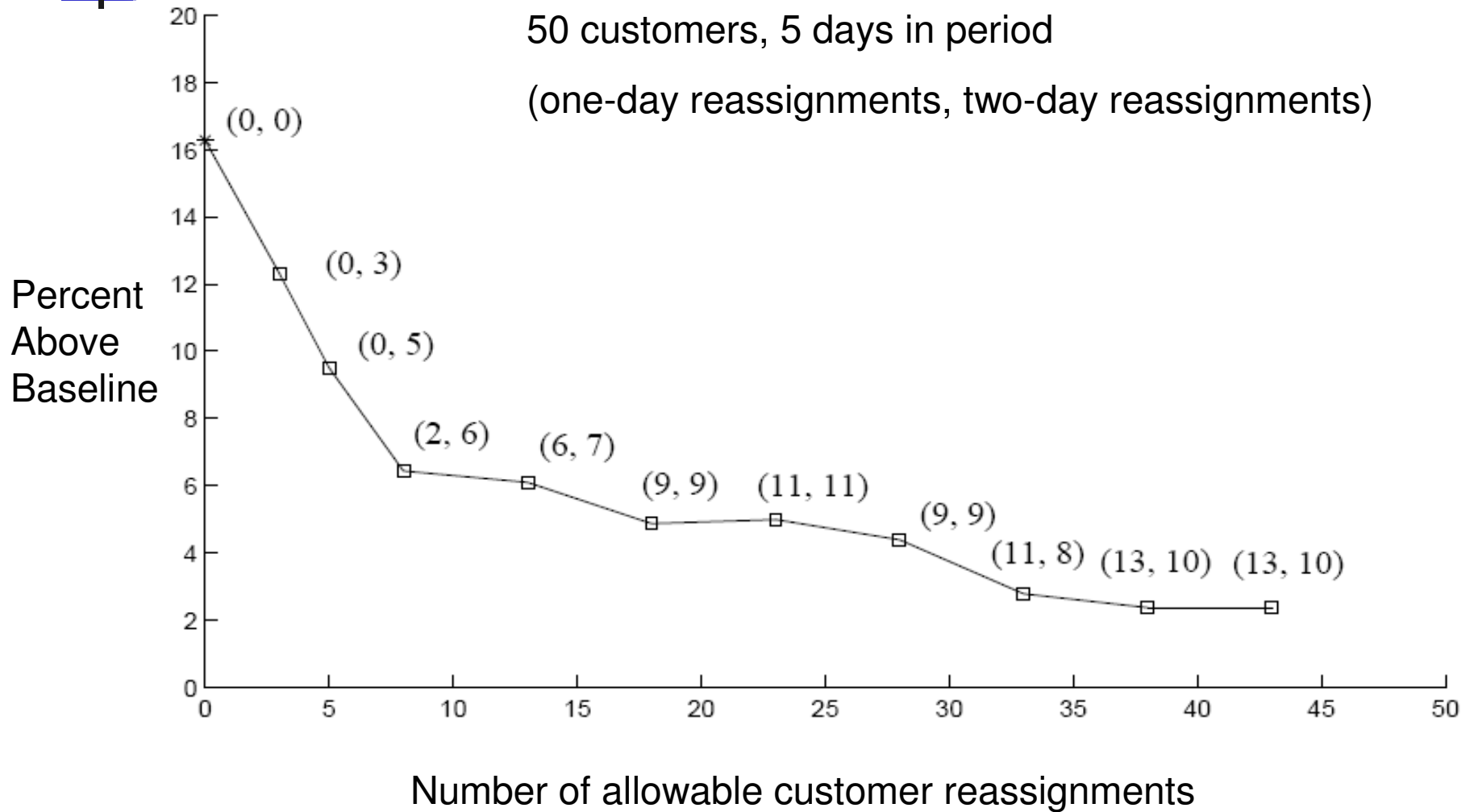
- For comparison, we implement a greedy algorithm that randomly selects a reassignment from the current three best-savings reassignments and repeats until W reassignments are made or there is no improvement
- On 26 problems, with $W = 10\%$ of the number of customers, we run IPH-RCH once and the greedy algorithm 151 times, recording the best solution and the median solution, respectively
- We started with an initial solution S' that was on average 16.64% above the baseline (solution in which no restriction was put on the number of reassignments)
- For IPH-RCH the solution was 9.91% above, for Greedy Best it was 11.51% above, and for Greedy Median it was 12.62% above

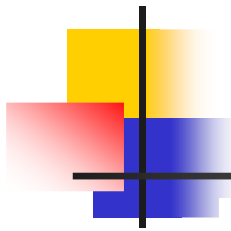


IPH-RCH and IPH-RCS

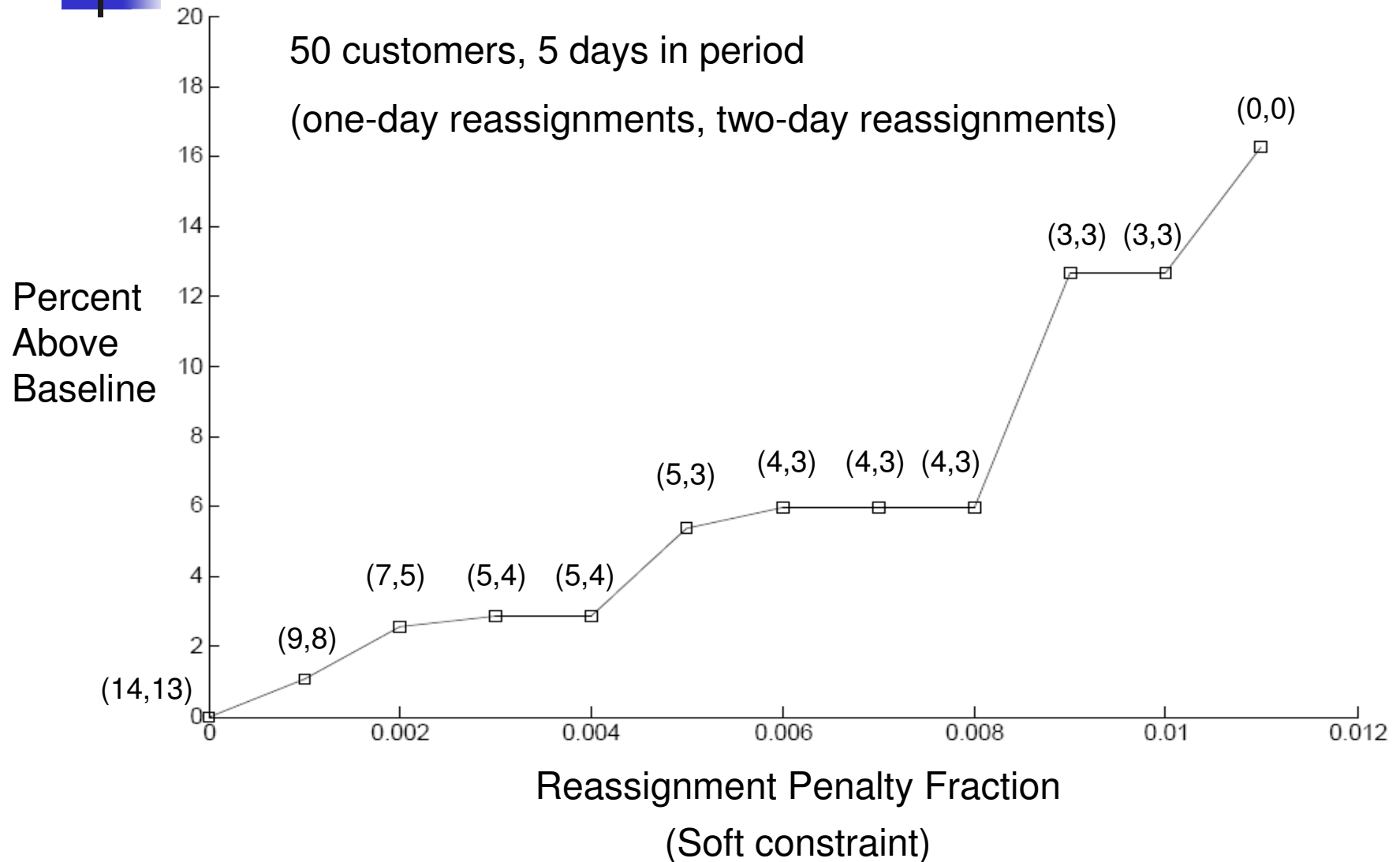
- For an example problem, we run IPH-RCH for different W values and see how the solution is affected
- We also modify IPH for the PVRP-RCS (IPH-RCS)
- For an example problem, we run IPH-RCS for different reassignment penalties and see how the solution is impacted

IPH-RCH Results





IPH-RCS Results





PVRP-RCMD

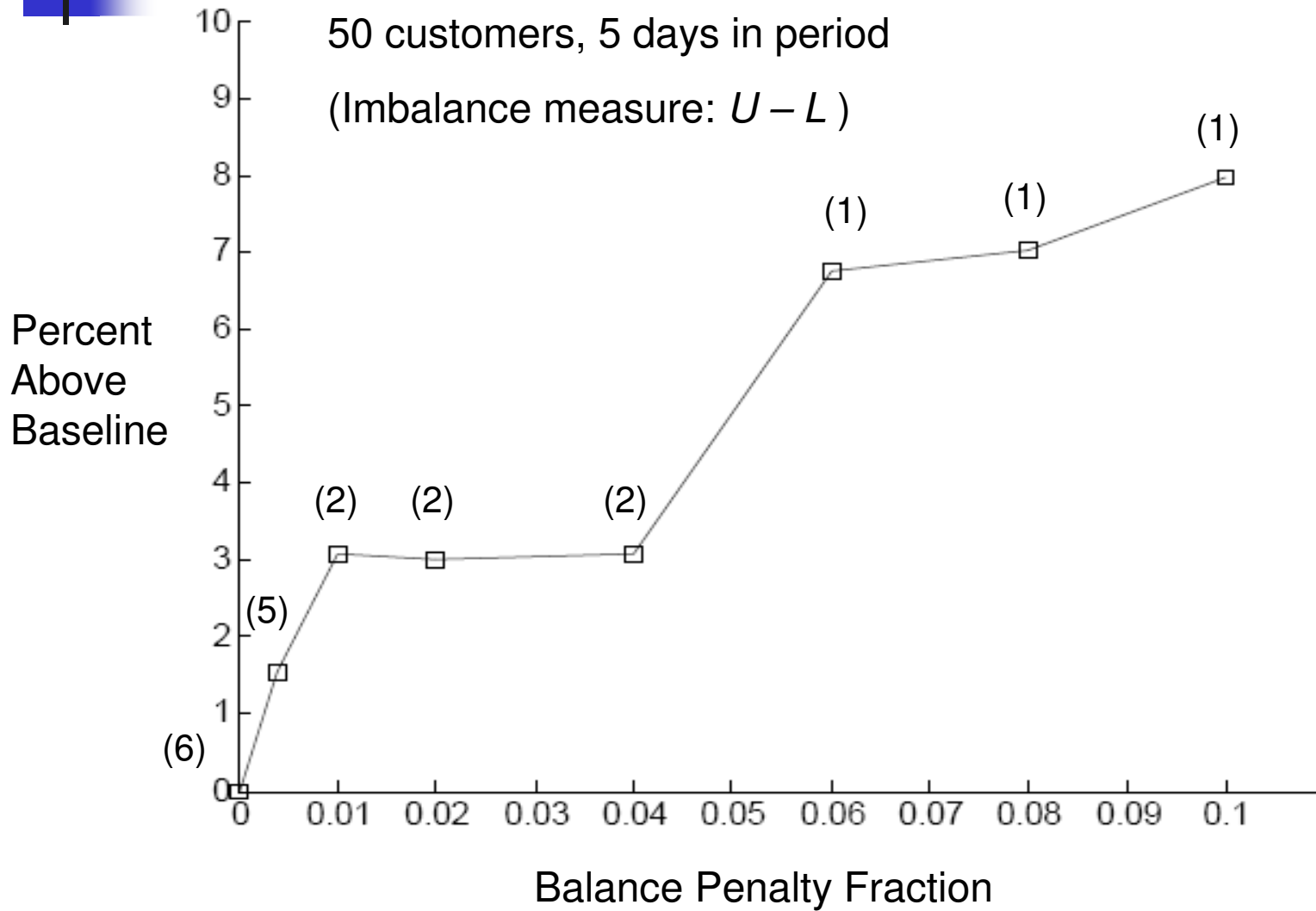
- One-day customers are the easiest and most convenient to reassign
- We consider the PVRP-RC in which we fix all multi-day customers, but allow one-day customers to be reassigned freely
- In our computational experiments, the initial solution can be substantially improved using this practical compromise



PVRP-BC

- Route balance is of key importance in industry
- We consider the problem of improving a maximally balanced initial solution or a low cost initial solution without inducing too much imbalance
- Minimize: routing distance + $\rho C(U - L)$
 - ρ = imbalance penalty fraction
 - C = distance of the initial solution
 - U = the most customers on a route in a solution
 - L = the fewest customers on a route in a solution
- We denote this problem PVRP-BC
- We modify IPH for the PVRP-BC (IPH-BC)

IPH-BC Results





Conclusion

- We develop a new IP-based algorithm for solving the PVRP
- We adapt our algorithm to solve several variants
- We consider the PVRP in which we start with an initial solution S' and then improve this solution with minimal disruption



Conclusion

- We also considered a variant that takes into account route balance
- We believe the PVRP variants are important in accurately modeling industrial problems